# Policy Evaluation Summary: Policy Evaluation

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# Bias/Variance of Model-free Policy Evaluation Algorithms

- Return  $G_t$  is an unbiased estimate of  $V^{\pi}(s_t)$
- TD target  $[r_t + \gamma V^{\pi}(s_{t+1})]$  is biased estimate of  $V^{\pi}(s)$
- ullet But often TD much lower variance than a single return  $G_t$ 
  - ► MC: Return function of multi-step sequence of random actions, states & rewards
  - ▶ TD target only has one random action, reward and next state



# Bias/Variance of Model-free Policy Evaluation Algorithms

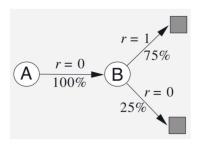
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  - ► High variance
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- MC:
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  - Consistent (converges to true) even with function approximation
- TD
  - Some bias
  - Lower variance
  - ightharpoonup TD(0) converges to true value with tabular representation
  - ightharpoonup TD(0) does not always converge with function approximation



#### AB Example [Sutton & Barto, 2018]



- Two states A, B with  $\gamma = 1$
- Given 8 episodes of experience:
  - ► A, 0, B, 0
  - ightharpoonup B, 1 (observed 6 times)
  - ▶ B, 0
- Under batch (offline) solution for this finite set of observations, what do MC and TD(0) converge to?
- Imagine run TD updates over data infinite number of times



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  - $\triangleright$  B, 1 (observed 6 times)
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- For *B*:
  - ► MC:  $V(B) = \frac{6}{8} = 0.75$ ► TD:  $V(B) = \frac{6}{8} = 0.75$

### AB Example [Sutton & Barto, 2018]

- Given 8 episodes of experience:
  - ► A, 0, B, 0
  - ▶ B, 1 (observed 6 times)
  - ▶ B, 0
- For B:
  - ► MC:  $V(B) = \frac{6}{8} = 0.75$ ► TD:  $V(B) = \frac{6}{8} = 0.75$
- For A:
  - ▶ MC: only one episode with  $A \rightsquigarrow V(A) = 0$
  - ▶ TD: bootstraps from  $V(B) \rightsquigarrow V(A) = 0.75$

$$V^{\pi}(s) = V^{\pi}(s) + \alpha(\underbrace{[r_t + \gamma V^{\pi}(s_{t+1})]}_{\text{TD target}} - V^{\pi}(s))$$

- → Monte Carlo in batch setting converges to minimal MSE (mean squared error)
- $\rightarrow$  TD(0) converges to DP policy  $V^{\pi}$  for the MDP with the maximum likelihood model estimates



# Efficiency

- Data efficiency & Computational efficiency
- ullet In simplest TD, use  $(s,a,r,s^\prime)$  once to update V(s)
  - ightharpoonup O(1) operation per update
  - ▶ In an episode of length L, O(L)
- In MC have to wait till episode finishes, then also O(L)
- MC can be more data efficient than simple TD in non-Markov domains
- TD can exploit Markov structure → leveraging this is helpful



# Summary: Policy Evaluation

Estimating the expected return of a particular policy if don't have access to true MDP models. Example: evaluating average purchases per session of new product recommendation system

- Dynamic Programming
- Monte Carlo policy evaluation
  - Policy evaluation when we don't have a model of how the world works
- Temporal Difference (TD)
- Metrics to evaluate and compare algorithms
  - Robustness to Markov assumption
  - Bias/variance characteristics
  - Data efficiency
  - Computational efficiency

