Policy Evaluation

TD(λ)

Marius Lindauer

Leibniz Universität Hannover

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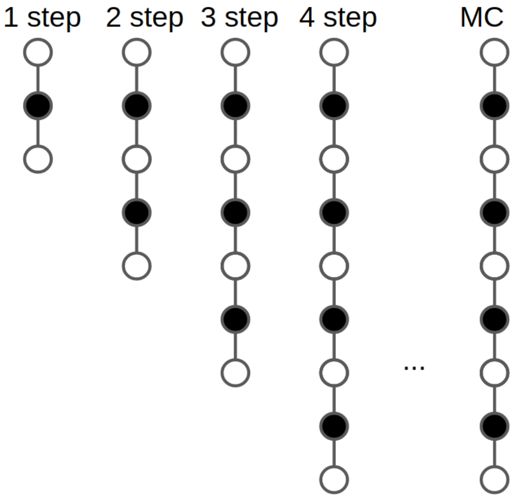
TD vs. MC



n-Step Return

- Defining n-step returns for different n

$$\begin{array}{rcl} n = 1 & (TD) & G_t^{(1)} = R_{t+1} + \gamma V(s_{t+1}) \\ n = 2 & & G_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 V(s_{t+2}) \\ \vdots & & \vdots \\ n = \infty & (MC) & G_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T \end{array}$$

- General n-step return

$$G_t^{(n)} = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{n-1} R_{t+n} + \gamma^n V(s_{t+n})$$

- n-step temporal-difference learning

$$V(s_t) \leftarrow V(s_t) + \alpha \left(G_t^{(n)} - V(s_t) \right)$$

Averaging n-Step Return

- Hard to say what best n is
- The agent plays the episode anyway and therefore, all updates are possible in principle
- One solution could be to average different n-step updates, e.g.,

$$\frac{1}{2}G^{(2)} + \frac{1}{2}G^{(4)}$$

- Combines information from two different time steps
- Could we combine information from all time steps?

λ-Return

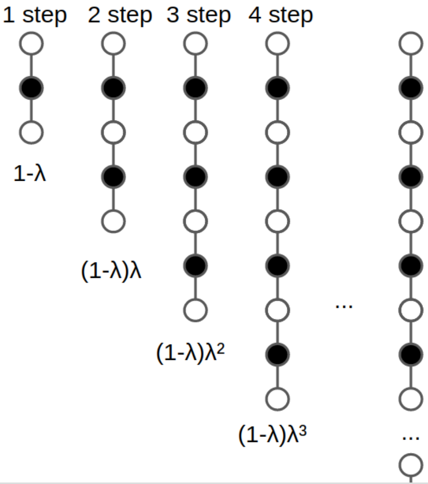
- The λ-return G_t^λ combines all n-steps returns $G_t^{(n)}$
- Using weight $(1-\lambda)\lambda^{n-1}$

$$G_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty}\lambda^{n-1}G_t^{(n)}$$

$$\sum_{n=1}^{T-t-2}(1-\lambda)\lambda^{n-1} + \lambda^{T-t-1} = 1$$

- Forward-view TD(λ)

$$V(s_t) \leftarrow V(s_t) + \alpha\left(G_t^\lambda - V(s_t)\right)$$



Backward View TD(λ)

- Forward view provides theory
- Backward view provides mechanism
- Update online, every step, from incomplete sequences

Eligibility Traces

- Episode: Bell, Bell, Bell, Light, Shock
- Credit assignment problem: Was the bell or the light responsible for the shock at the end?

Eligibility Traces

- Episode: Bell, Bell, Bell, Light, Shock
- Credit assignment problem: Was the bell or the light responsible for the shock at the end?
- Frequency heuristic: assign credit to most frequent states
- Recency heuristic: assign credit to most recent states
- Eligibility traces combine both heuristics:

$$
\begin{aligned}
E_0(s) &= 0 \\
E_t(s) &= \gamma \lambda E_{t-1}(s) + \mathbf{1}(S_t = s)
\end{aligned}
$$

$\rightsquigarrow$ decrease of importance exponentially proportional to time in the past

$\rightsquigarrow$ boost of importance for each time the state was visited

Backward View TD(λ)

- Keep an eligibility trace for every state s
- Update value $V(s)$ for every state s
- In proportion to TD-error δ_t and eligibility trace $E_t(s)$

$$
\begin{aligned}
\delta_t &= R_{t+1} + \gamma V(s_{t+1}) - V(s_t) \\
V(s) &\leftarrow V(s) + \alpha \delta_t E_t(s)
\end{aligned}
$$

MC, TD(0) and TD(λ)

- When $\lambda = 0$, only the current state is updated
- When $\lambda = 1$, the same as the total update of MC