RL: Policy Search

Analytic Gradient

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Computing the gradient analytically

- We now compute the policy gradient analytically
- Assume policy π_θ is differentiable whenever it is non-zero and we know the gradient $\nabla_\theta \pi_\theta(s, a)$
- Denote a state-action trajectory as

$$\tau = (s_0, a_0, r_0, \ldots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$$

- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for a trajectory τ

$\rightsquigarrow$ Focusing for now on $V(s_0, \theta) = \sum_\tau P(\tau; \theta) R(\tau)$

Likelihood Ratio Policy Gradient I

- Denote a state-action trajectory as

$$\tau = (s_0, a_0, r_0, \dots, s_{T-1}, a_{T-1}, r_{T-1}, s_T)$$

- Use $R(\tau) = \sum_{t=0}^{T} R(s_t, a_t)$ to be the sum of rewards for τ
- Policy value is

$$V(\theta) = \mathbb{E}\left[\sum_{t=0}^{T} R(s_t, a_t); \pi_\theta\right] = \sum_{\tau} P(\tau; \theta) R(\tau)$$

  where $P(\tau;\theta)$ is used to denote the probability over trajectories when executing policy π_θ
- In this new notation, our goal is to find the policy parameters θ^*

$$\theta^* \in \arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau)$$

Likelihood Ratio Policy Gradient II

- Our goal is to find the policy parameters θ^*

$$\theta^* \in \arg\max_\theta V(\theta) = \arg\max_\theta \sum_\tau P(\tau;\theta)R(\tau)$$

- Take the gradient with respect to θ:

$$\begin{aligned}
\nabla_\theta V(\theta) &= \nabla_\theta \sum_\tau P(\tau;\theta)R(\tau) \\
&= \sum_\tau \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau \frac{P(\tau;\theta)}{P(\tau;\theta)} \nabla_\theta P(\tau;\theta)R(\tau) \\
&= \sum_\tau P(\tau;\theta)R(\tau) \underbrace{\frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)}}_{\text{likelihood ratio}}
\end{aligned}$$

Likelihood Ratio Policy Gradient III

- Our goal is to find the policy parameters θ^*

$$\theta^* \in \arg\max_{\theta} V(\theta) = \arg\max_{\theta} \sum_{\tau} P(\tau;\theta)R(\tau)$$

- Take the gradient with respect to θ:

$$\nabla_\theta V(\theta) = \sum_{\tau} P(\tau;\theta)R(\tau)\nabla_\theta \log P(\tau;\theta)$$

- Approximate with empirical estimate for m sample trajectories under policy π_θ:

$$\nabla_\theta V(\theta) \approx \frac{1}{m}\sum_{i=1}^{m} R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$$

Decomposing the Trajectories Into States and Actions

- Approximate with empirical estimate for m sample trajectories under policy π_θ:

$$\nabla_\theta V(\theta) \approx \frac{1}{m}\sum_{i=1}^{m} R(\tau^{(i)})\nabla_\theta \log P(\tau^{(i)};\theta)$$

$$\begin{aligned}
\nabla_\theta \log P(\tau^{(i)};\theta) &= \nabla_\theta \log \left[\mu(s_0)\prod_{t=0}^{T-1}\pi_\theta(a_t \mid s_t)P(s_{t+1} \mid s_t, a_t)\right] \\
&= \nabla_\theta \left[\log \mu(s_0) + \sum_{t=0}^{T-1} \log \pi_\theta(a_t \mid s_t) + \log P(s_{t+1} \mid s_t, a_t)\right] \\
&= \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t \mid s_t)
\end{aligned}$$

$\rightsquigarrow$ No dynamics model required!