

RL: Basics

Value Iteration

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MDP: Computing Optimal Policy and Optimal Value

- Policy iteration computes optimal value and policy
- Value iteration is another technique
 - ▶ Idea: Maintain optimal value of starting in a state s if we have a finite number of steps k left in the episode
 - ▶ Iterate to consider longer and longer episode

Bellman Equation and Bellman Backup Operators

- Value function of a policy must satisfy the Bellman equation

$$V^\pi(s) = R^\pi(s) + \gamma \sum_{s' \in S} P^\pi(s' | s) V^\pi(s')$$

- Bellman backup operator B
 - ▶ Applied to a value function
 - ▶ Returns a new value function
 - ▶ Improves the value if possible

$$BV(s) = \max_a [R(s, a) + \gamma \sum_{s' \in S} p(s' | s, a) V(s')]$$

- ▶ BV yields a value function over all states s
- ▶ Note: Read B as an operator applied to V

Value Iteration (VI)

- Set $k = 1$
- Initialize $V_0(s) = 0$ for all states s
- Loop until convergence
 - ▶ For each state s

$$V_{k+1}(s) = \max_{a \in A} R(s, a) + \gamma \sum_{s' \in S} P(s' \mid s, a) V_k(s')$$

- ▶ View as Bellmann backup on value function

$$V_{k+1} = BV_k$$

$$\pi_{k+1}(s) \in \arg \max_{a \in A} R(s, a) + \gamma \sum_{s' \in S} P(s' \mid s, a) V_k(s')$$

Policy Iteration as Bellman Operations

- Bellman backup operator B^π for a particular policy is defined as

$$B^\pi V(s) = R^\pi(s) + \gamma \sum_{s' \in S} P^\pi(s' | s) V(s')$$

- Policy evaluation amounts to computing the fixed point of B^π
- To do policy evaluation, repeatedly apply operator until V stops changing

$$V^\pi = B^\pi B^\pi B^\pi B^\pi \dots B^\pi V$$

Policy Iteration as Bellman Operations

- Bellman backup operator B^π for a particular policy is defined as

$$B^\pi V(s) = R^\pi(s) + \gamma \sum_{s' \in S} P^\pi(s' | s) V(s')$$

- To do policy improvement

$$\pi_{k+1}(s) \in \arg \max_{a \in A} R(s, a) + \gamma \sum_{s' \in S} P(s' | s, a) V^{\pi_k}(s')$$

Going back to Value Iteration (VI)

- Set $k = 1$
- Initialize $V_0(s) = 0$ for all states s
- Loop until convergence
 - ▶ For each state s

$$V_{k+1}(s) = \max_{a \in A} R(s, a) + \gamma \sum_{s' \in S} P(s' \mid s, a) V_k(s')$$

- ▶ Equivalent in Bellman backup notation

$$V_{k+1} = BV_k$$

- ▶ To extract optimal policy if we can act for $k + 1$ more steps

$$\pi_{k+1}(s) \in \arg \max_{a \in A} R(s, a) + \gamma \sum_{s' \in S} P(s' \mid s, a) V_{k+1}(s')$$

What you should know

- Define MP, MRP, MDP, Bellman operator, Q-value, policy
- Be able to implement
 - ▶ Value Iteration
 - ▶ Policy Iteration
- Which policy evaluation methods require the Markov assumption?