

# Policy Evaluation

## Summary: Policy Evaluation

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Automated  
Machine Learning  
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# Bias/Variance of Model-free Policy Evaluation Algorithms

- Return  $G_t$  is an unbiased estimate of  $V^\pi(s_t)$
- TD target  $[r_t + \gamma V^\pi(s_{t+1})]$  is biased estimate of  $V^\pi(s)$
- But often TD much lower variance than a single return  $G_t$ 
  - ▶ MC: Return function of multi-step sequence of random actions, states & rewards
  - ▶ TD target only has one random action, reward and next state

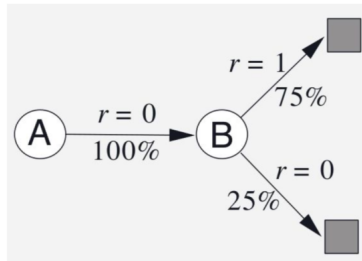
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  - ▶ Unbiased (for first visit MC)
  - ▶ High variance
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- TD
  - ▶ Some bias
  - ▶ Lower variance
  - ▶  $TD(0)$  converges to true value with tabular representation
  - ▶  $TD(0)$  does not always converge with function approximation

# AB Example [Sutton & Barto, 2018]



- Two states  $A$ ,  $B$  with  $\gamma = 1$
- Given 8 episodes of experience:
  - ▶  $A, 0, B, 0$
  - ▶  $B, 1$  (observed 6 times)
  - ▶  $B, 0$
- Under batch (offline) solution for this finite set of observations, what do MC and TD(0) converge to?
- Imagine run TD updates over data infinite number of times?

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- Given 8 episodes of experience:

- ▶  $A, 0, B, 0$
- ▶  $B, 1$  (observed 6 times)
- ▶  $B, 0$

- For  $B$ :

- ▶ MC:  $V(B) = \frac{6}{8} = 0.75$
- ▶ TD:  $V(B) = \frac{6}{8} = 0.75$

# AB Example [Sutton & Barto, 2018]

- Given 8 episodes of experience:
  - $A, 0, B, 0$
  - $B, 1$  (observed 6 times)
  - $B, 0$
- For  $B$ :
  - MC:  $V(B) = \frac{6}{8} = 0.75$
  - TD:  $V(B) = \frac{6}{8} = 0.75$
- For  $A$ :
  - MC: only one episode with  $A \rightsquigarrow V(A) = 0$
  - TD: bootstraps from  $V(B) \rightsquigarrow V(A) = 0.75$

$$V^\pi(s) = V^\pi(s) + \underbrace{\alpha([r_t + \gamma V^\pi(s_{t+1})] - V^\pi(s))}_{\text{TD target}}$$

- $\rightsquigarrow$  Monte Carlo in batch setting converges to minimal **MSE** (mean squared error)
- $\rightsquigarrow$  TD(0) converges to DP policy  $V^\pi$  for the MDP with the **maximum likelihood model estimates**

- Data efficiency & Computational efficiency
- In simplest TD, use  $(s, a, r, s_0)$  once to update  $V(s)$ 
  - ▶  $O(1)$  operation per update
  - ▶ In an episode of length  $L$ ,  $O(L)$
- In MC have to wait till episode finishes, then also  $O(L)$
- MC can be more data efficient than simple TD in non-Markov domains
- TD can exploit Markov structure  $\rightsquigarrow$  leveraging this is helpful



# Summary: Policy Evaluation

Estimating the expected return of a particular policy if don't have access to true MDP models. Example: evaluating average purchases per session of new product recommendation system

- Dynamic Programming
- Monte Carlo policy evaluation
  - ▶ Policy evaluation when we don't have a model of how the world works
- Temporal Difference (TD)
- Metrics to evaluate and compare algorithms
  - ▶ Robustness to Markov assumption
  - ▶ Bias/variance characteristics
  - ▶ Data efficiency
  - ▶ Computational efficiency