Policy Evaluation

Monte Carlo Evaluation: Temporal Difference Learning

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Temporal Difference Learning

- "If one had to identify one idea as central and novel to reinforcement learning, it would undoubtedly be temporal-difference (TD) learning." – Sutton and Barto 2017
- Combination of Monte Carlo & dynamic programming methods
- ► Model-free
- bootstraps and samples
- ► Can be used in episodic or infinite-horizon non-episodic settings
- \blacktriangleright Immediately updates estimate of V after each (s,a,r,s') tuple

Temporal Difference Learning for Estimating V

- lacktriangle Aim: estimate $V^\pi(s)$ given episodes generated under policy π
- $\blacktriangleright \ G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots$ in MDP M under policy π
- $\blacktriangleright \ V^\pi(s) = \mathbb{E}[G_t \mid s_t = s]$

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- ► Recall Bellman operator (if known MDP models)

$$B^{\pi}V(s) = r(s,\pi(s)) + \gamma \sum_{s' \in S} p(s' \mid s,\pi(s))V(s')$$

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▶ Insight: have an estimate of V^{π} , used to estimate expected return

$$V^{\pi}(s) = V^{\pi}(s) + \alpha([r_t + \gamma V^{\pi}(s_{t+1})] - V^{\pi}(s))$$

Temporal Difference [TD(0)] Learning

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- ▶ Simplest TD learning: update value towards estimated value

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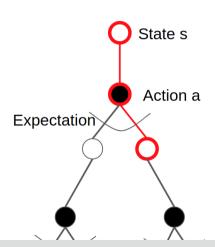
- ightharpoonup Can immediately update value estimate after (s,a,r,s') tuple
- → Don't need episodic setting

Temporal Difference [TD(0)] Learning Algorithm

Input:
$$\alpha$$
 Initialize $V^\pi(s) = 0. \forall s \in S$ Loop

- ightharpoonup Sample tuple (s_t, a_t, r_t, s_{t+1})
- $\blacktriangleright \ V^{\pi}(s) = V^{\pi}(s) + \alpha(\underbrace{[r_t + \gamma V^{\pi}(s_{t+1})]}_{\text{TD target}} V^{\pi}(s))$

Temporal Difference [TD(0)] Learning Algorithm



- \blacktriangleright TD updates the value estimate using a sample of s_{t+1} to approximate an expectation
- \blacktriangleright TD updates the value estimate by bootstrapping, uses estimates of $V(s_{t+1})$