Computational Finance and Risk Management

Assignment 2
Risk based and Robust Portfolio Selection Strategies

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By

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Objective:

This assignment is to compare the computational investment strategies based on minimizing portfolio variance and maximizing the Sharpe ratio considering the effect of transaction costs. 20 stocks were traded over a period of two years withholding period lasting for two months and having 12 holding periods. Thus, the portfolio was rebalanced at most 12 times on the first trading day of each 2-month holding period. This is designed to verify if the strategy is feasible
i.e. the user has enough budget to re-balance the portfolio. The strategies implemented are as follows:

1. Portfolio Optimization Strategies

1.1 *Buy and Hold Strategy:*

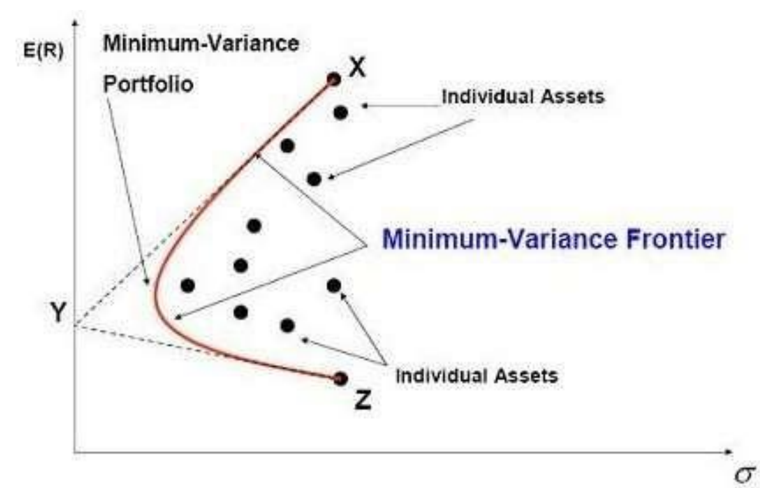
Buy and hold is a passive investment strategy in which an investor buys stocks (or other types of securities such as ETFs) and holds them for a long period regardless of fluctuations in the market. An investor who uses a buy-and-hold strategy actively selects investments but has no concern for short-term price movements and technical indicators. Thus, in this case, the stocks bought at the beginning of the period one is not traded until the last period i.e. period 12.

1.2 *Equally Weighted Portfolio Strategy:*

This is strategy is based on the paradigm that investing equally in stocks with smaller names as well as the higher names can increase the return potential of the portfolio as compared to the weighting the stocks based on market capitalization. Thus, in this case, based on the initial portfolio value, each stock in the portfolio receives an equal amount of investment and traded later after the holding period of 2 months for two years.

1.3 *Minimum Variance Portfolio Strategy:*

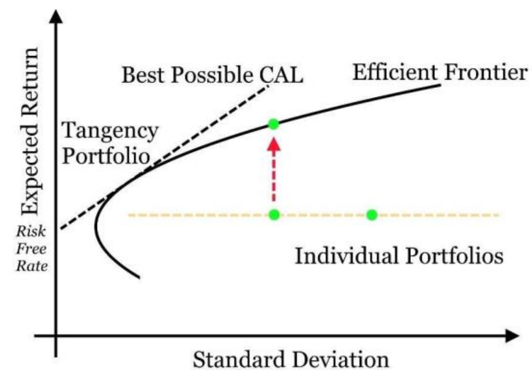
A portfolio of individually risky assets that, when taken together, result in the lowest possible risk level for the rate of expected return. Such a portfolio hedges each investment with an offsetting investment; the individual investor's choice on how much to offset investments depends on the level of risk and expected return he/she is willing to accept. The investments in a minimum variance portfolio are individually riskier than the portfolio.



1.4 *Maximum Sharpe ratio portfolio Strategy:*

Sharpe ratio is a risk-adjusted measure of return that measures the excess return (risk premium) per unit of risk (deviation) in an investment asset or a portfolio. It signifies the additional return an investor receives for the additional volatility of holding the risky asset over risk-free assets.

Thus, a portfolio with a higher Sharpe ratio is preferred. Mathematically, it is defined as follows:



$$\text{maximize} \quad \frac{\mu^T x - r_f}{\sqrt{x^T Q x}}$$

$$\text{s.t.} \quad \sum_j x_j = 1, \qquad Ax \geq b. \qquad 0 \leq x.$$

1.5 *Equal Risk Contribution Portfolio Strategy:*

Equal risk contribution indices weight stocks in such a way that they contribute equally to the risk of a portfolio. This relatively new smart beta investment strategy aims to control volatility risk, and in doing so, improve returns on a risk-adjusted basis. A greater risk is associated with a higher risk premium.

The equal risk model is a type of risk-parity strategy, but with a slightly different slant. Risk parity infers that individual investments within a fund have the same level of volatility, while an equal risk approach engineers' constituents' weighting so that they contribute equal levels of volatility.

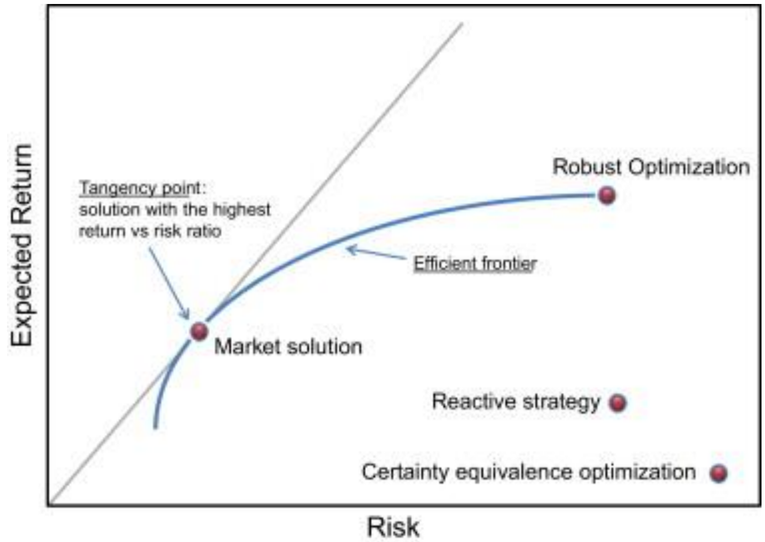
The equal risk contribution strategy is in some ways like the minimum variance approach, which gives greater weighting to low volatility, stable stocks in order to control volatility and risk.

1.6 *Leveraged Equal Risk Contribution Portfolio Strategy:*

In the Leveraged ERC strategy, our objective is to go long by 200% using the amount obtained by shorting the risk-free assets. This increases the funds available for investing and stocks. The paradigm of leveraging is that the investor expects a higher return in stocks as compared to that of the coupon payments owed after a given period. Here the shorting is implemented after every period thus after each period the coupon payment, as well as the face value, will be deducted from the portfolio value.

1.7 *Robust Mean-Variance Portfolio Optimization Strategy:*

Robust mean-variance optimization strategy fixes the error in return estimates which were overestimated in the Markowitz mean-variance models. This strategy considers the uncertainty in the estimation of the returns with a true expected return in an ellipsoidal or box uncertainty set. Thus, apart from minimizing the variance of portfolio and maximizing the returns, this strategy also minimizes the return estimation error.



2. Constraints in developing Strategies

i. Holding period lasts for two months over a period of two years, this portfolio can be rebalanced at most 12 times

ii. Transaction cost is 0.5% of the traded volume (the difference between the selling and buying price of a stock)

iii. Transaction fees are not accounted for in the optimization problem but are accounted for in computing the portfolio value.

iv. The initial cash account is zero and any cash in cash account doesn't pay interest

v. Initial portfolio value is one million dollars

vi. Trading of an only integer number of stocks is allowed

3. Implementation of Rounding Process

The optimal number of stocks obtained from the implementation of the strategies above results in a non-integer number of stocks and trading of such stocks isn't practical. Thus, the objective of the rounding procedure is to ensure that only the integer number of stocks are traded. The approach used behind the rounding procedure is to maximize the amount that is invested in the stocks thereby minimizing the amount in cash account. To ensure that the maximum amount is invested, the ceil function is used to round the non-integer to an integer. For instance, ceil (3.4) will be rounded to 3. This will lead to the maximum utilization of the amount in the cash account by investing it in the stocks.
However, the use of ceil function increases the chances of falling short of cash to trade the stock as desired i.e. ensuring that the funds in the cash account are minimum but non-negative. The strategy used for the same is elucidated below.

```
x_optimal= np.ceil(w1)            # optimal number of shares for initial period
```

4. Implementation of the Validation Process

The approach used to ensure that there is enough budget to re-balance the portfolio after considering the transaction cost is as follows:

1. Finding the optimal value of the number of stocks based on the strategy
2. Implementing the rounding procedure
```
x_optimal= np.ceil(w1)
```
3. Computing the transaction cost as follows:
```
tc=sum(cur_prices*abs(x_init- x_optimal))*0.005
Transaction cost= sum(current_prices*abs(initial quantity of stocks-
                                     Optimal quantity of stocks))*0.005
```
4. Computing the cash in the account after deducting the above transaction cost
```
cash_optimal= PV-sum(cur_prices*x_optimal)-tc
```
5. Checking if the cash in account is negative. If it is negative, then the budget is not enough to rebalance the portfolio, thus decrease the number of stocks purchased. If it is positive, then the optimal number of stocks as obtained by the rounding procedure in step 2 is used.
6. Approach to increase the cash in account from negative to non-negative is to decrease the stock with the minimum current price to ensure the cash in the account is minimal and most of the cash in the account is invested:
    a. Searching for the stock with the minimum current price
    b. Checking if the number of this stock in the portfolio after implementing the rounding procedure is non-zero. If its zero to step 6g
    c. Decreasing the number of this stock obtained in step 6b by 1
    d. Finding the new transaction cost as per the formula in step 3 adding the old transaction cost in the cash account since it will be replaced by the new transaction cost in step 6e
    e. Computing the cash in accounting by adding the current price of the share, which was decreased by 1 in step 6c and subtracting the new transaction cost obtained in step 6d
    f. Repeating step 5 to step 6e until the cash in the account is positive
    g. If the number of stocks in the portfolio in step 6b is already zero it cannot be reduced further to increase the budget in the cash account. Thus, search for the maximum number of stocks in the portfolio and implement step 6c to step 6e and keeping on repeating until the cash in the account is positive.

Below mentioned is the snippet of a code mentioning the validation process for period 1. This is implemented for every strategy in the function and for any period >=2, the function is validated considering all strategies.

```python
x_optimal= np.ceil(w1)                          # optimal number of shares for initial period
tc=sum(cur_prices*abs(x_init-x_optimal))*0.005  # Calculating transaction cost
cash_optimal= PV-sum(cur_prices*x_optimal)-tc   # Computing optimal cash
if cash_optimal<0:                              # If the ceiling of no. of shares makes optimal cash negative
  while cash_optimal < 0 :
    index_min = np.argsort(cur_prices)          # Sorting current prices based on the increasing order into an index
    for mi in index_min:                        # Now this loop takes every stock into account by checking if they > 0
      if (x_optimal[mi])>0:
        x_optimal[mi]= x_optimal[mi] - 1        # Here the optimal shares are reduced by '1' and the loop continues till optimal
        cash_optimal+=tc
        tc=sum(cur_prices*abs(x_init-x_optimal))*0.005
        cash_optimal+=(cur_prices[mi]-tc)
        break
      else:
        continue
```

For period >=2, the code for the validation process is mentioned below:

```python
if period>=2:
    PV=sum(cur_prices*x[strategy, period-1])+ cash[strategy, period-1]
    tc=np.sum(cur_prices*abs(x[strategy, period-1]-x[strategy, period-1]))*0.005
    cash[strategy, period-1]= PV-sum(cur_prices*x[strategy, period-1])-tc
    if cash[strategy, period-1]<0:
      while cash[strategy, period-1] < 0:
        index_min = np.argsort(cur_prices)
        for mi in index_min:
          if (x[strategy, period-1][mi])>0:
            while (x[strategy, period-1][mi])>0:
              x[strategy, period-1][mi]= x[strategy, period-1][mi] - 1
              cash[strategy, period-1]-=tc
              tc=sum(cur_prices*abs(x[strategy, period-1]-x[strategy, period-2]))*0.005
              cash[strategy, period-1]= cash[strategy, period-1]+cur_prices[mi]-tc
              break
            else:
              continue
```

Cash optimal for each strategy corresponding to all periods in 2015-2016 is depicted below and furthermore investing would lead to an imbalance in optimization strategy.

	period 1	period 2	period 3	period 4	period 5	period 6	period 7	period 8	period 9	period 10	period 11	period 12
Buy and Hold	0	0	0	0	0	0	0	0	0	0	0	0
Equally Weighted Portfolio	1.89373	2.72024	1.98465	0.87597	0.347389	1.09121	0.530864	1.48785	0.414314	2.89412	1.10186	4.23712
Minimum Variance Portfolio	2.58157	0.32238	3.8167	6.42919	0.70084	0.811049	0.688342	6.5211	3.3227	26.3436	2.17452	0.467083
Maximum Sharpe Ratio Portfolio	3.7603	18.0214	39.4964	1.7092	1.17486	0.60499	1.93657	30.1773	2.98423	4.6129	4.29328	21.4731
Equal Risk Contribution	0.485554	1.79645	0.0864018	1.0677	0.503007	0.926621	2.53668	1.14058	1.47711	3.77684	6.74052	5.73343
Leveraged Equal Risk	1.97773	0.495376	1.24838	0.916716	0.64189	1.2553	0.519117	1.62892	1.64871	2.23249	6.67367	6.40719
Robust Mean Variance	2.33048	1.48853	11.3288	9.17917	1.53925	1.3481	7.41432	4.4639	11.1973	17.4229	3.19855	7.45271

Cash optimal for each strategy corresponding to all periods in 2008-2009 is depicted below and furthermore investing would lead to an imbalance in optimization strategy.

	period 1	period 2	period 3	period 4	period 5	period 6	period 7	period 8	period 9	period 10	period 11	period 12
Buy and Hold	0	0	0	0	0	0	0	0	0	0	0	0
Equally Weighted Portfolio	0.096812	4.56319	5.35442	0.539678	2.47249	1.16501	1.37285	0.271438	1.68941	2.26946	3.69321	3.17021
Minimum Variance Portfolio	3.85617	1.53059	20.7985	2.37936	10.7038	1.98944	14.4453	1.19473	2.4389	14.4308	5.75524	7.43743
Maximum Sharpe Ratio Portfolio	15.3065	18.9253	6.44854	0.429153	1.24304	8.91976	1.00366	1.16533	0.679209	0.807347	3.23726	23.2152
Equal Risk Contribution	0.91063	4.07222	2.86259	4.42723	0.409176	1.71846	0.401665	1.85925	2.05031	1.69212	0.841526	1.75201
Leveraged Equal Risk	4.32274	3.26362	0.10858	0.0631944	2.67723	0.262564	1.52037	0.783398	1.63155	3.82551	0.581065	1.02052
Robust Mean Variance	5.61286	0.370053	2.56851	3.79866	9.85674	1.42492	6.11702	0.180375	1.75689	3.18151	11.278	4.27291

It is observed that cash remaining in the account is minimal and had been found to be invested effectively.

5. <u>Gradient Computation for Equal Risk Contribution</u>

The IPOPT solver requires the gradient to be solved to compute ERC strategy. The gradient for the objective function is mentioned below.

Objective Function:

$$\begin{array}{ll} \min\limits_{w} & \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(w_i(Qw)_i - w_j(Qw)_j\right)^2 \\ \text{s.t.} & \sum_{i=1}^{n} w_i = 1 \\ & w \geq 0 \end{array}$$

Gradient Computation:

```python
def gradient(self, x):
    # The callback for calculating the gradient
    grad = np.zeros(n)
    y = x * np.dot(Q, x)
    RC= np.dot(Q,x)
    # Insert your gradient computations here
    for i in range(n):
      for j in range(n):
        for k in range(j+1,n):
          if i==j:
            grad[i]=grad[i]+ 4*(((Q[i][j]*x[i])+RC[i]-(Q[k][j]*x[k]))*(y[j]-y[k]))
          elif i==k:
            grad[i]=grad[i]+ 4*(((Q[i][j]*x[i])-RC[j]-(Q[j][j]*x[j]))*(y[j]-y[k]))
          else:
            grad[i]=grad[i]+ 4*(((Q[i][k]*x[i])-(Q[j][k]*x[j]))*(y[j]-y[k]))
    # You can use finite differences to check the gradient
    return grad
```

6. Implementation of Leveraged Equal Risk Contribution

In the Leveraged ERC strategy, we go long by 200% using the amount obtained by shorting the risk-free assets. This increases the funds available for investing and stocks. However, the paradigm of leveraging is that the investor expects a higher return in stocks as compared to that of the coupon payments owed after a given period. Here the shorting is implemented after every period thus after each period the coupon payment as well as the face value is deducted from the portfolio value.

Coupon Payment= Rate of Interest per period * price of the bond

Coupon Payment for period 2015-16= (0.025/6) *1000002.12

Coupon Payment for period 2008-09= (0.045/6) * 548247.97

```
# Get current portfolio positions
if period == 1:
    curr_positions = init_positions
    curr_cash = 0
    portf_value[strategy] = np.zeros((N_days, 1))
    if strategy==5:
      curr_positions=2*init_positions       # 200% long
else:
    curr_positions = x[strategy, period-2]
    curr_cash = cash[strategy, period-2]
```

```
# Annual risk-free rate for years 2015-2016 is 2.5%
r_rf = 0.025
# Annual risk-free rate for years 2008-2009 is 4.5%
r_rf2008_2009 = 0.045
```

```
cash_optimal= PV-sum(cur_prices*x_optimal)-tc-(r_rf*init_value)         # Computing optimal cash
```

Computing Portfolio Value for all periods:

```
# Compute portfolio value
if strategy==5:
  p_values = np.dot(data_prices[day_ind_start:day_ind_end+1,:], x[strategy, period-1]) + cash[strategy, period-1]-init_value
  portf_value[strategy][day_ind_start:day_ind_end+1] = np.reshape(p_values, (p_values.size,1))
else:
  p_values = np.dot(data_prices[day_ind_start:day_ind_end+1,:], x[strategy, period-1]) + cash[strategy, period-1]
  portf_value[strategy][day_ind_start:day_ind_end+1] = np.reshape(p_values, (p_values.size,1))
```

7. Implementing Robust Mean Variance Optimization

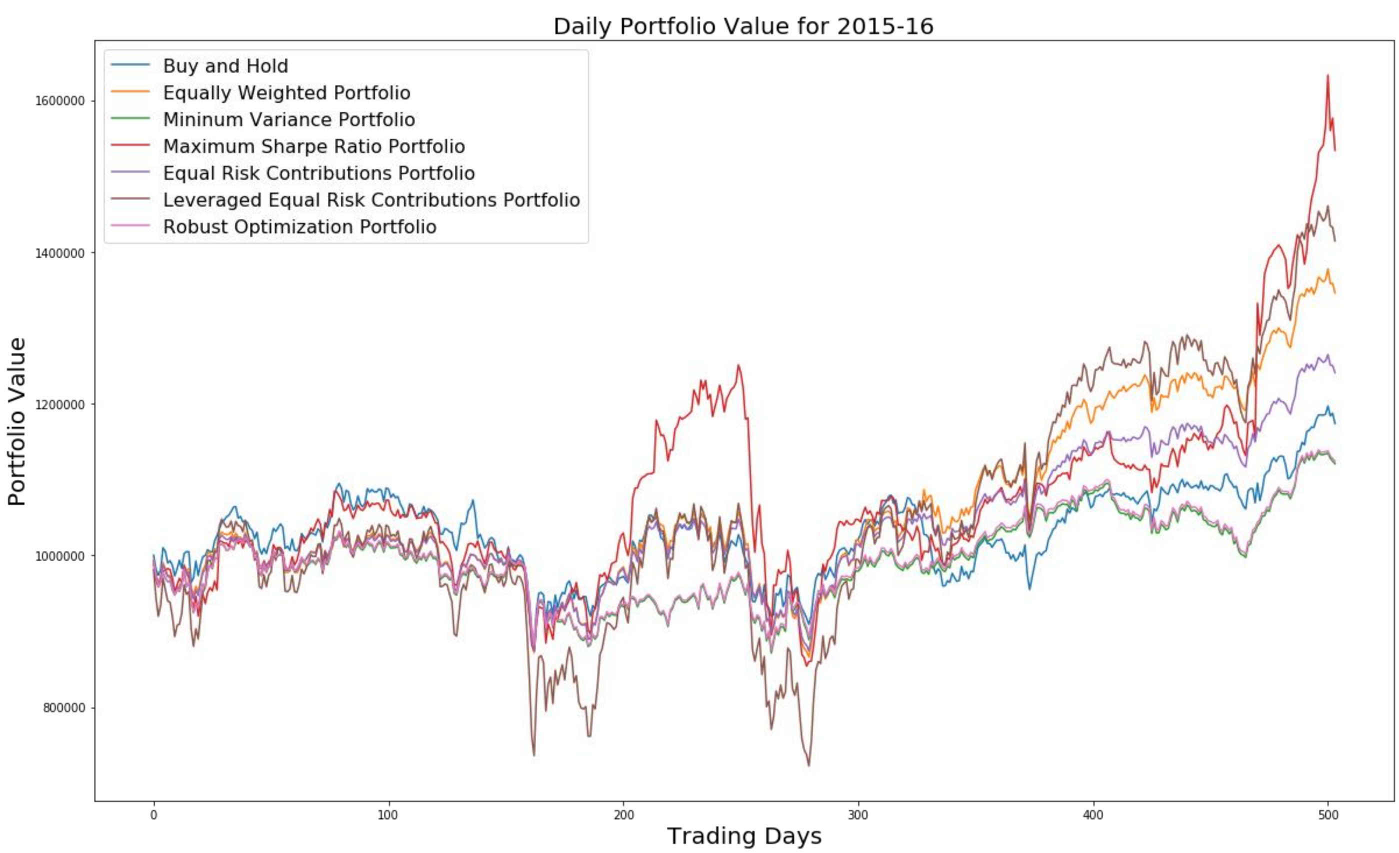
Robust mean-variance optimization strategy fixes the error in return estimates which was overestimated in the Markowitz mean-variance model. In other words, this strategy reduces the gap between the Markowitz's efficient frontier and the actual frontier as shown below. The target returns and the estimation error (ϵ) in the case is derived from the minimum variance portfolio. This is because the target vector must be chosen to consider the worst possibility, which is case of the minimum variance model as it gives the lowest returns as also observed from the portfolio values of the first four strategies. It considers the uncertainty in estimation of the returns with a true expected return in an ellipsoidal or box uncertainty set. Thus, apart from minimizing the variance of portfolio and maximizing the returns, this strategy also minimizes the return estimation error.

This solves the below mentioned optimization problem with the constraints (ϵ) for both returns and variance which is computed from the minimum variance strategy.

$$
\begin{array}{lrcr}
\min & w^T Q w & & \\
\text{s.t.} & \sum_{i=1}^{n} w_i & = & 1 \\
 & \mu^T w & \geq & \varepsilon_{\text{ret}} \\
 & w^T \Theta w & \leq & \tilde{\varepsilon}_{\text{rob}} \\
 & w & \geq & 0
\end{array}
$$

8. Analysis and Plots

8.1 Daily Portfolio Values 2015-2016 - All Strategies



```
Period 12: start date 11/1/2016, end date 12/30/2016
  Strategy "Buy and Hold", value begin = $ 1077523.53, value end = $ 1173675.24
  Strategy "Equally Weighted Portfolio", value begin = $ 1209299.04, value end = $ 1345698.12
  Strategy "Mininum Variance Portfolio", value begin = $ 1005921.56, value end = $ 1120591.99
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1155798.45, value end = $ 1533592.38
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1131912.57, value end = $ 1240531.12
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1204359.22, value end = $ 1414171.60
  Strategy "Robust Optimization Portfolio", value begin = $ 1010600.71, value end = $ 1123753.44
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```
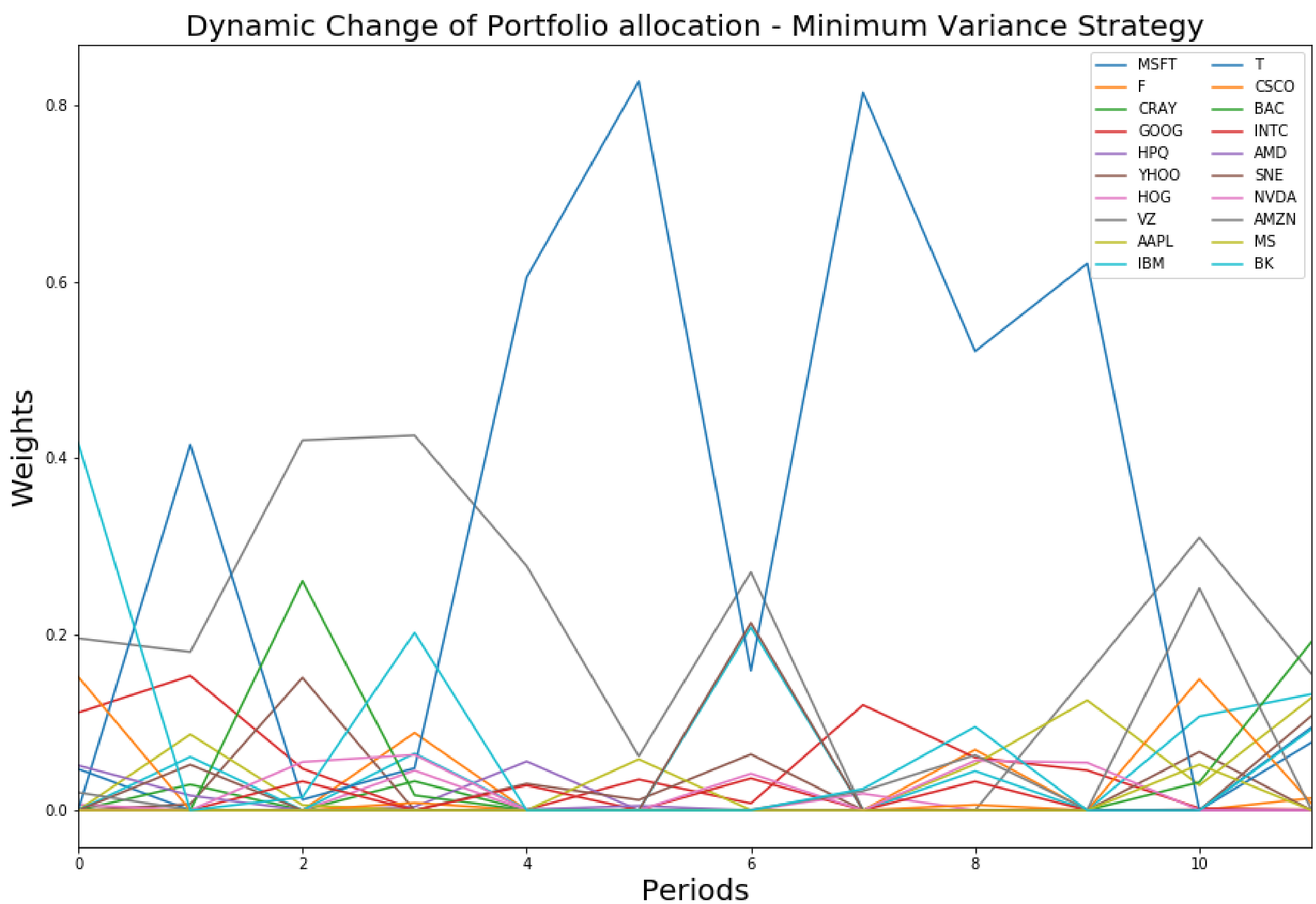

From the above plot and portfolio values mentioned, it can be easily concluded that the Sharpe ratio has the maximum portfolio value or the highest returns as compared to the other strategies. On the other hand, the minimum variance portfolio has the least portfolio value or the least returns. This is since in the minimum variance portfolio selection strategy, in minimizing the risk, the returns are minimized. This strategy selects the portfolio on the efficient frontier obtained by the Markowitz principle with the minimum variance or minimizes risk. In the Sharpe ratio portfolio selection strategy, the portfolio which has the largest return per unit of variance/ risk is preferred. The buy and hold strategy didn't perform better as compared to the others as it is a passive investment strategy and yields only on a term basis. We can also observe the Robust optimization strategy being the second last above minimum variance as it uses the error values based on the minimum variance strategy.

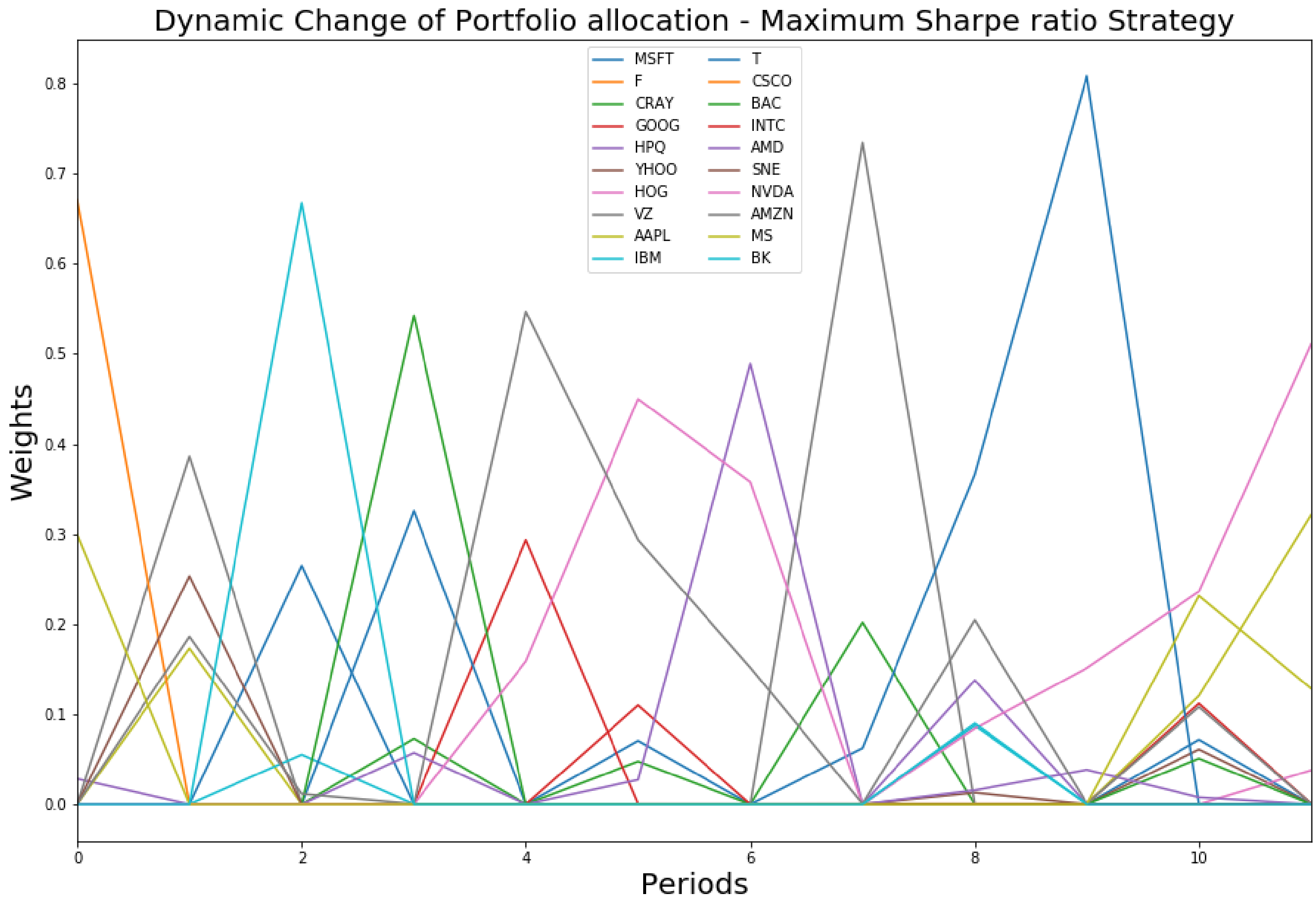
8.2 Dynamic Change of Portfolio allocation – (2015-2016)

8.2.1 "Minimum Variance Strategy"



The above plot signifies the weight allocation to each stock over 12 holding period using the minimum variance portfolio selection strategy. Thus, the strategy gives higher weights to the stocks with minimum variance.

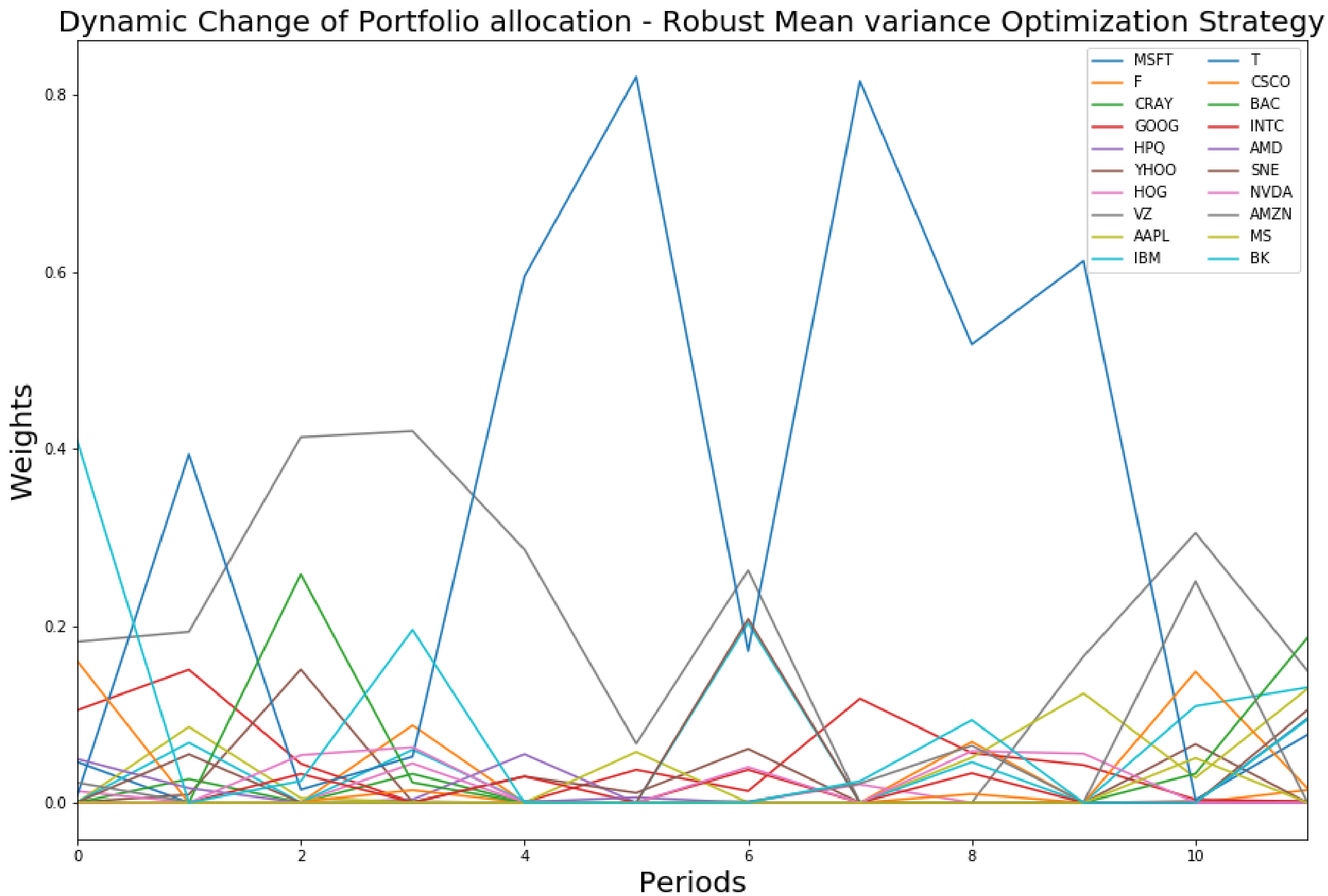
8.2.2 "Maximum Sharpe Ratio Strategy"



The above plot signifies the weight allocation to each stock over 12 holding period using the maximum Sharpe ratio portfolio selection strategy. In this strategy, the portfolio which has the largest return per unit of variance/ risk is preferred. Thus, it considers risk as well as the return. Unlike the minimum variance portfolio selection strategy where the weight of Google's stock in the portfolio was predominantly higher, this strategy has almost distributed the weights diversely based on the ratio for each stock obtained. This led to its highest return as seen from plot 1 as compared to the other strategies.

8.2.3 "Robust Mean Variance Optimization Strategy"

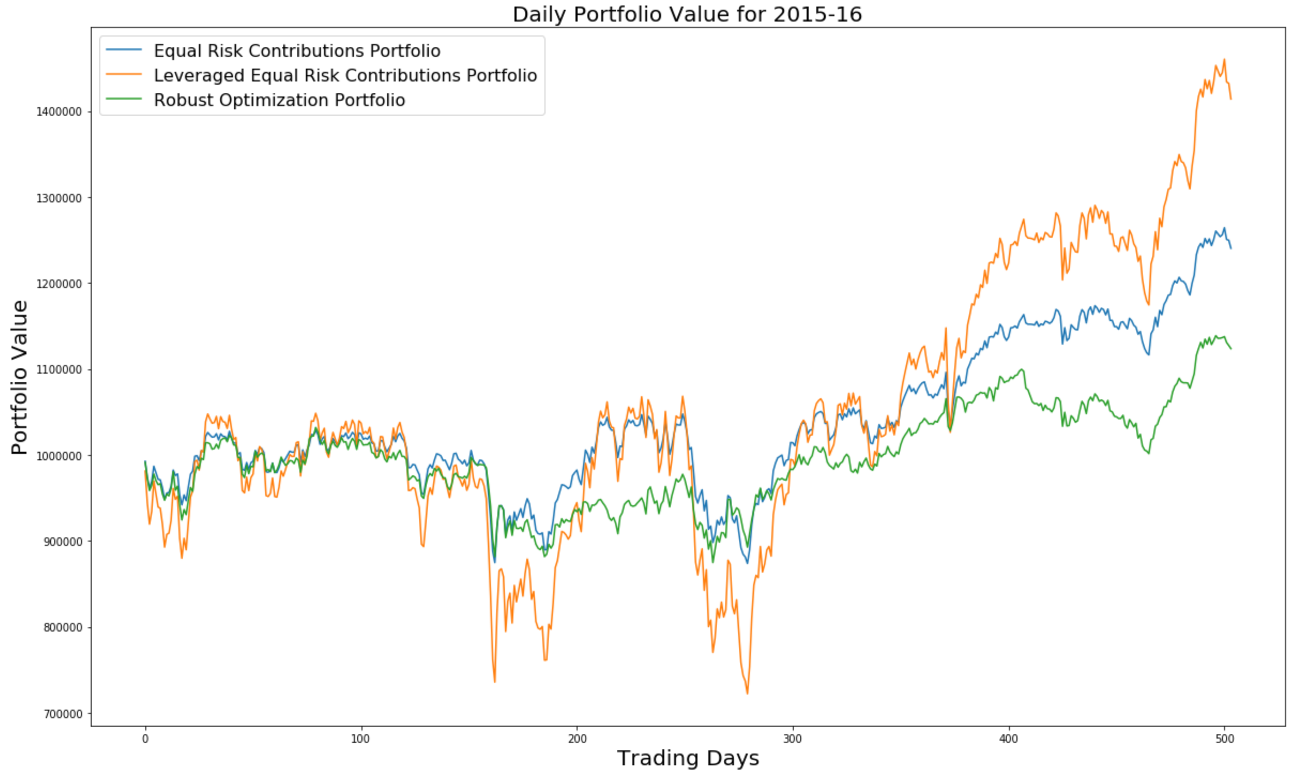
The plot below signifies the weight allocation to each stock over 12 holding period using the Robust mean variance portfolio selection strategy. In this strategy, the portfolio which has the return >=ϵ and risk <=ϵ is preferred. Thus, it considers risk as well as the return. Like the minimum variance portfolio selection strategy where the weight of Google's stock in the portfolio was predominantly higher, this strategy also has almost distributed the weights based on the minimum variance for each stock obtained. This led to its lowest return as seen from plot 1 as compared to the other strategies.



The changes in the weight allocation between the consecutive periods signifies the amount of trading involved in the strategy. Thus, in can be concluded that the robust mean variance optimization strategy reduces the trading drastically as compared to the maximum Sharpe ratio strategy and the minimum variance strategy. Moreover, maximum Sharpe ratio strategy involves very high level of trading as the dynamic changes in the weight allocation between the consecutive periods it too high.

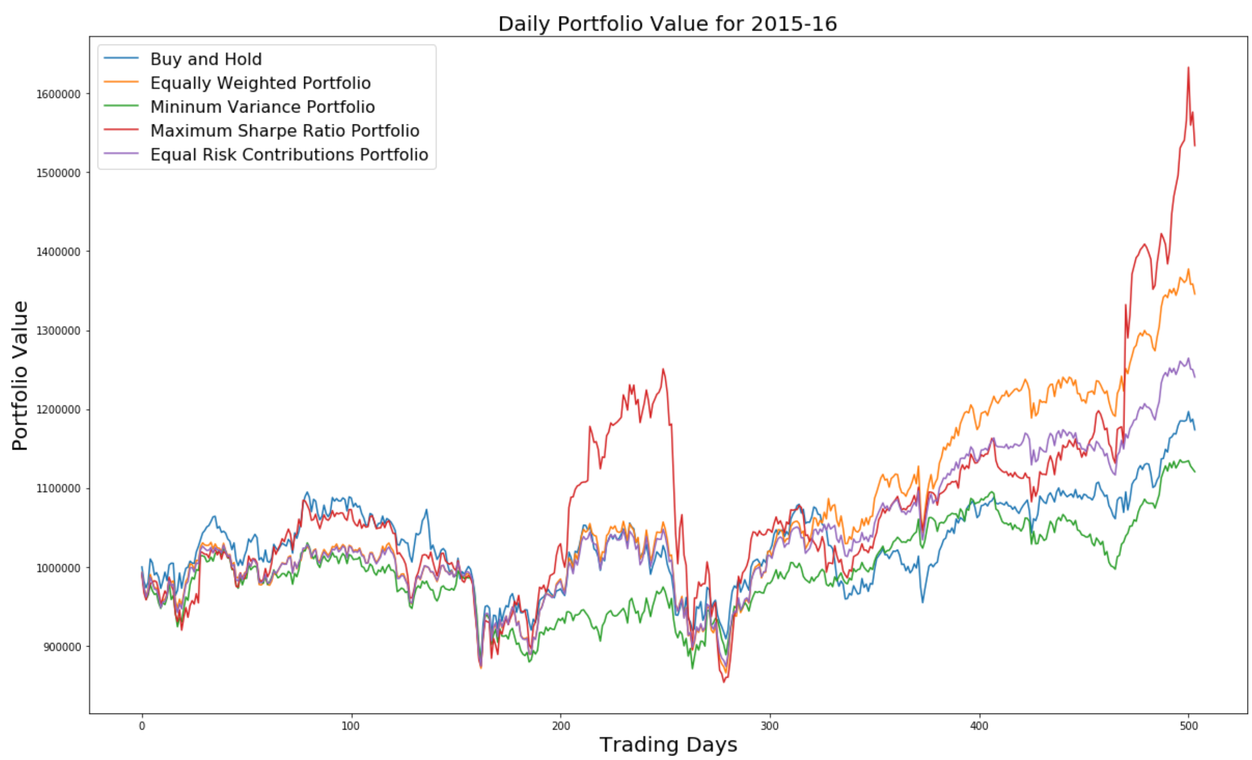
8.3 Comparing Strategies – 2015-2016

8.3.1 Comparing Daily Portfolio Values -5,6,7 Strategies

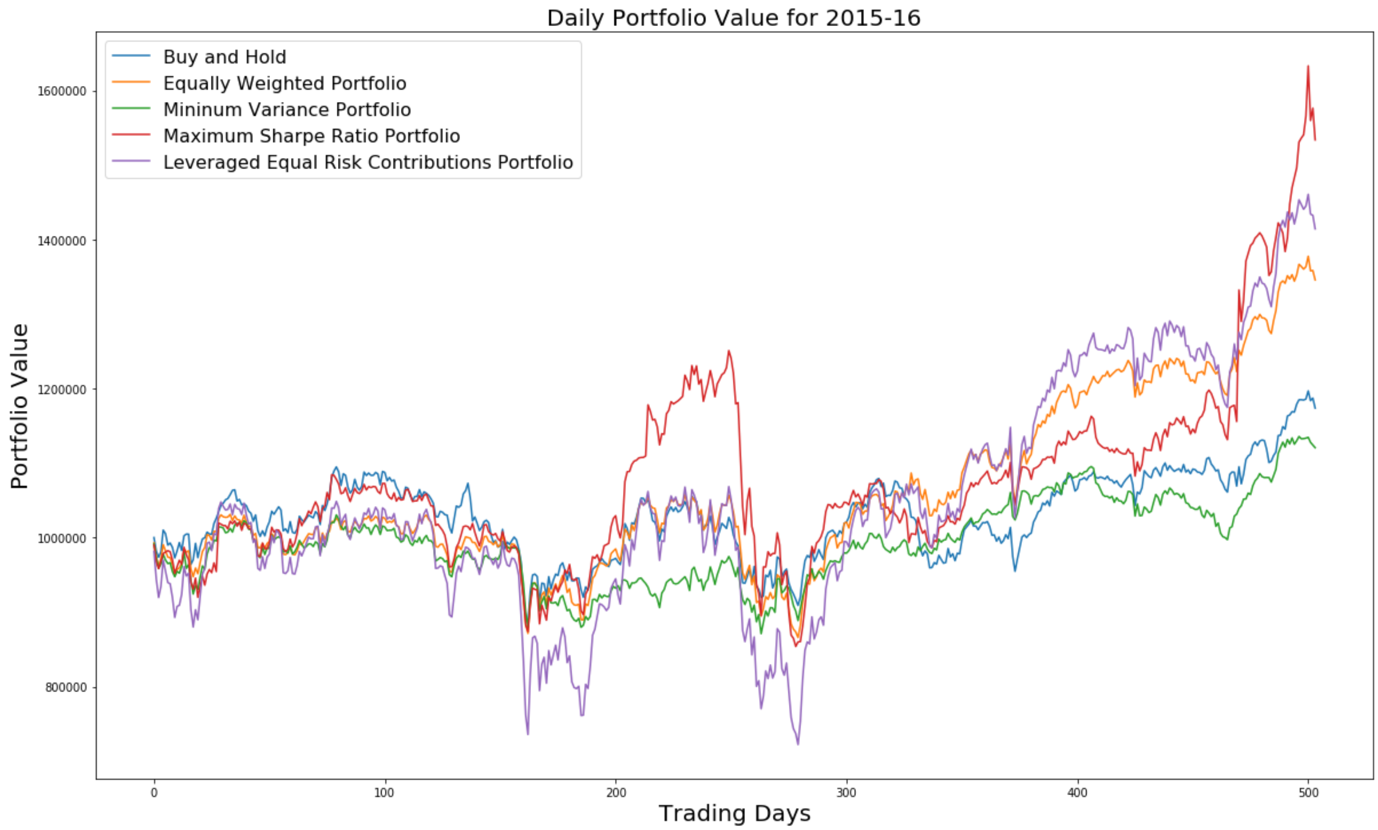


The above plot compares the portfolio value over 12 holding period using the Equal risk contribution, Leverages equal risk contribution and the robust mean variance optimization strategy.

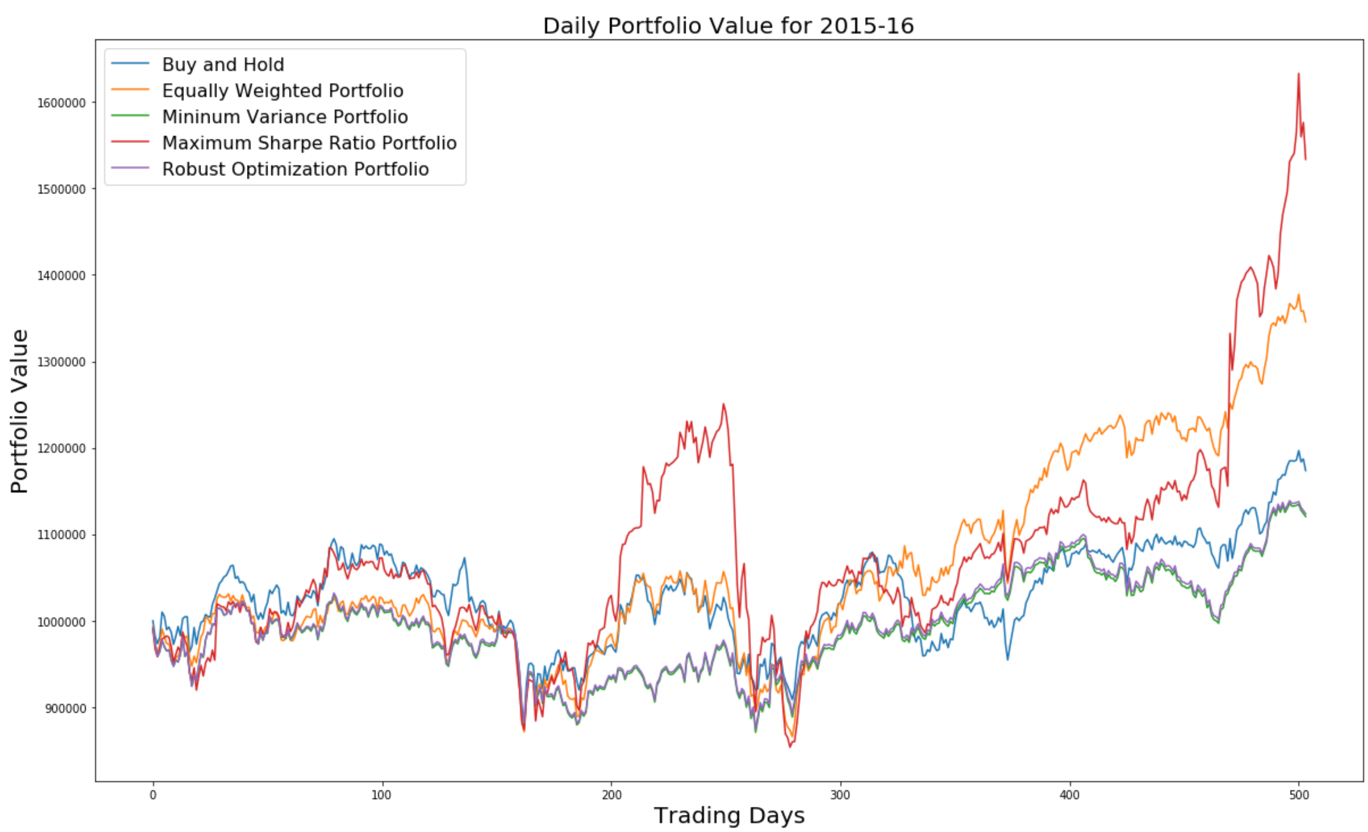
8.3.2 Comparing Daily Portfolio Values - 1,2,3,4,5 Strategies



8.3.3 Comparing Daily Portfolio Values - 1,2,3,4,6 Strategies



8.3.4 Comparing Daily Portfolio Values - 1,2,3,4,7 Strategies

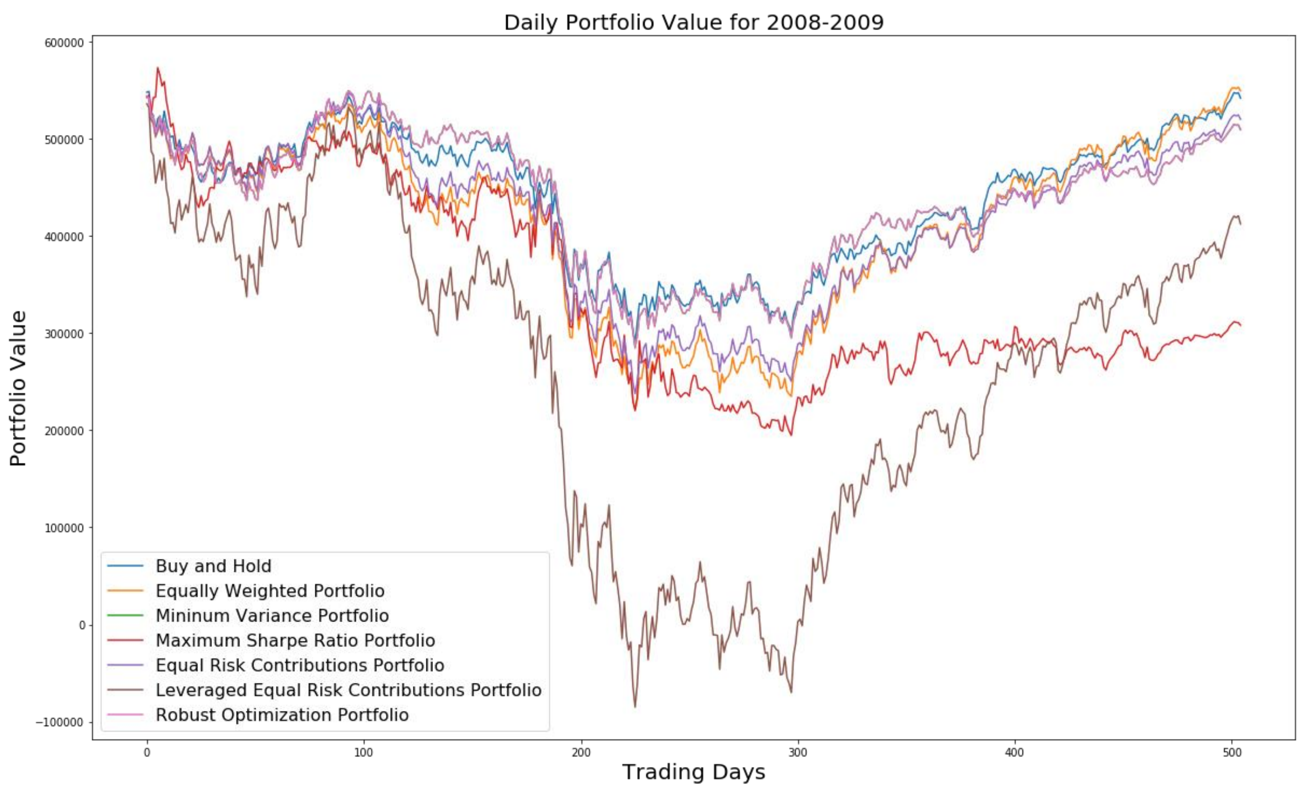


The strategy selected by an investor is dependent on risk-taking ability or willingness. A risk averse investor would select minimum variance strategy since the risk is minimized, however, the expected returns are also less in that case. From my point to view, I have a higher risk-taking ability and not a risk averse person which is common among the investors in their mid-20s and 30s. Moreover, the amount of investment will also be less thus risk-taking capacity is further reinforced. Therefore, keeping this factor in mind I will prioritize higher expected return and go with the leveraged equal risk contribution strategy.

Even though the maximum Sharpe ratio strategy gives higher return when compared with the leveraged equal risk contribution strategy, I would prefer leveraged equal risk contribution strategy over the other strategies because maximum Sharpe ratio strategy does not diversify the portfolio as much as the leveraged equal risk contribution does which is clear from the dynamic weight allocation plot. Furthermore, the maximum Sharpe ratio strategy involves very high level of trading as compared to the equal risk contribution as seen from the dynamic weight allocation. Moreover, the equal risk contribution strategy fixes the estimation error in the Markowitz's mean-variance model by avoiding using the return estimated which deviate for the actual returns. Thus, by selecting the leveraged equal risk contribution strategy, I am not only sure my portfolio is diversified and avoid estimate error but also gain respectable returns by going long 200% and shorting risk-free assets which has lower interest at each period as compared to the returns.

9. <u>Portfolio Values and comparison of Strategies – (2008-2009)</u>

9.1 Daily Portfolio Values 2008-2009 - All Strategies



```
Period 12: start date 11/2/2009, end date 12/31/2009
  Strategy "Buy and Hold", value begin = $ 490582.55, value end = $ 542246.05
  Strategy "Equally Weighted Portfolio", value begin = $ 480850.78, value end = $ 549786.82
  Strategy "Mininum Variance Portfolio", value begin = $ 454892.32, value end = $ 509768.27
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 272162.99, value end = $ 308310.92
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 465983.88, value end = $ 520392.21
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 315481.39, value end = $ 412647.80
  Strategy "Robust Optimization Portfolio", value begin = $ 455141.00, value end = $ 509863.08
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

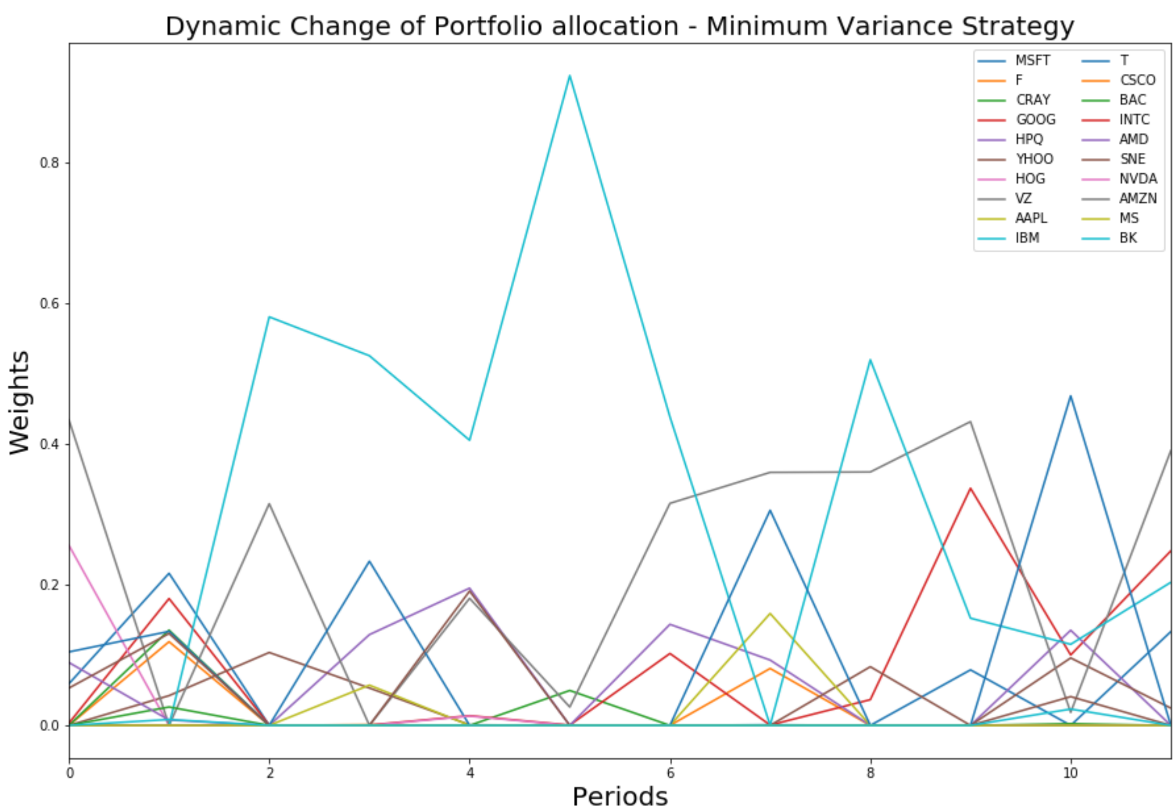
Based on the plot and the portfolio values mentioned above, it is found that the strategies

- Equally weighted
- Buy and Hold
- Equal Risk Contributions
- Robust mean variance Optimization

have performed well even during the crisis period compared to other strategies. Equally weighted and Equal Risk contribution strategies have the weights allotted equally in any condition, this would prevent assigning more weights to the stocks which are risky. While Robust portfolio optimization performs well as our strategy considers the values based on minimum variance. On the other hand, Max. Sharpe ratio is the least followed by the leveraged ERC. As Max. Sharpe is returns over risk and in the crisis time, risk impacts much more than the return. As we have borrowed to invest, and we have invested in crisis time, so we cannot get the appropriate returns to be able to pay back the borrowed capital.

9.2 Dynamic Change of Portfolio allocation – (2008-2009)

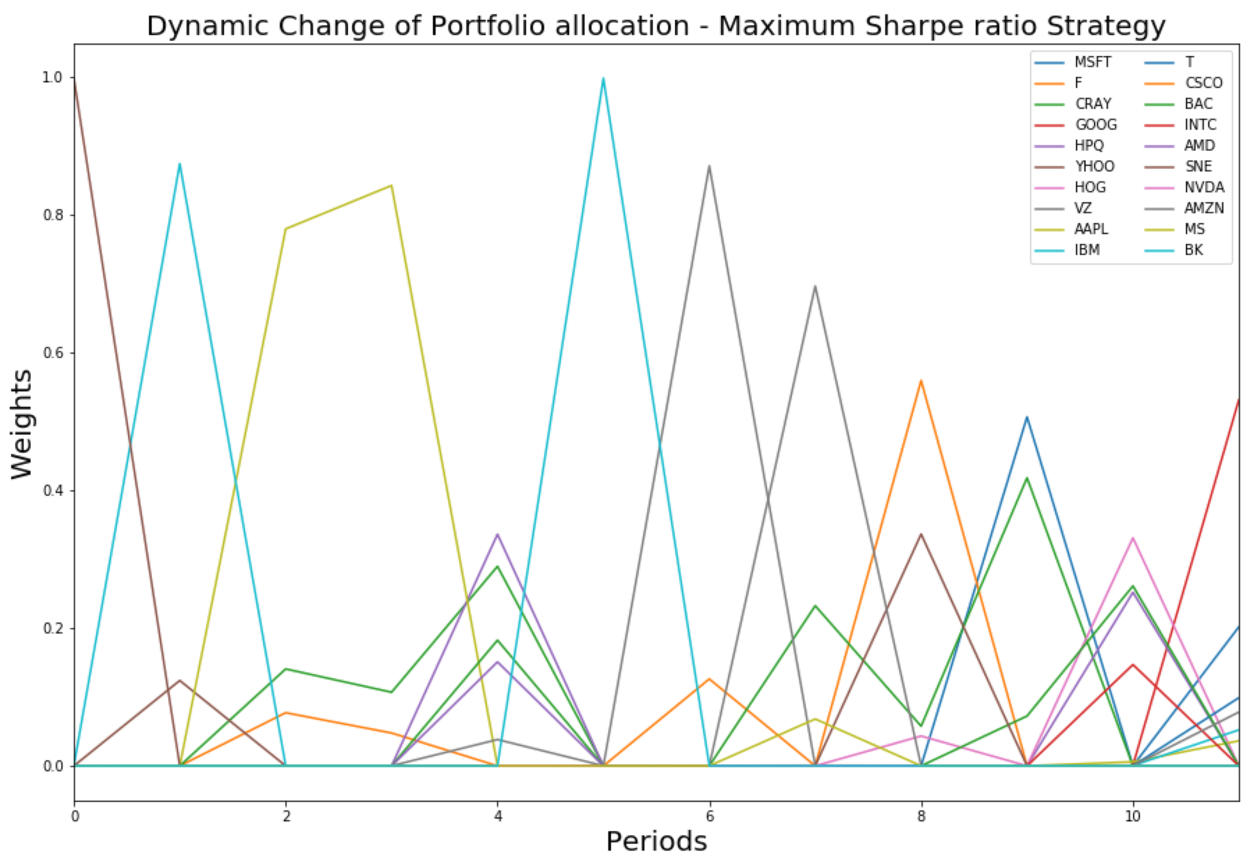
9.2.1 "Minimum Variance Strategy"



The above plot signifies the weight allocation to each stock over 12 holding period using the minimum variance portfolio selection strategy. Thus, the strategy gives higher weights to the stocks with minimum variance.

9.2.2 "Maximum Sharpe Ratio Strategy"

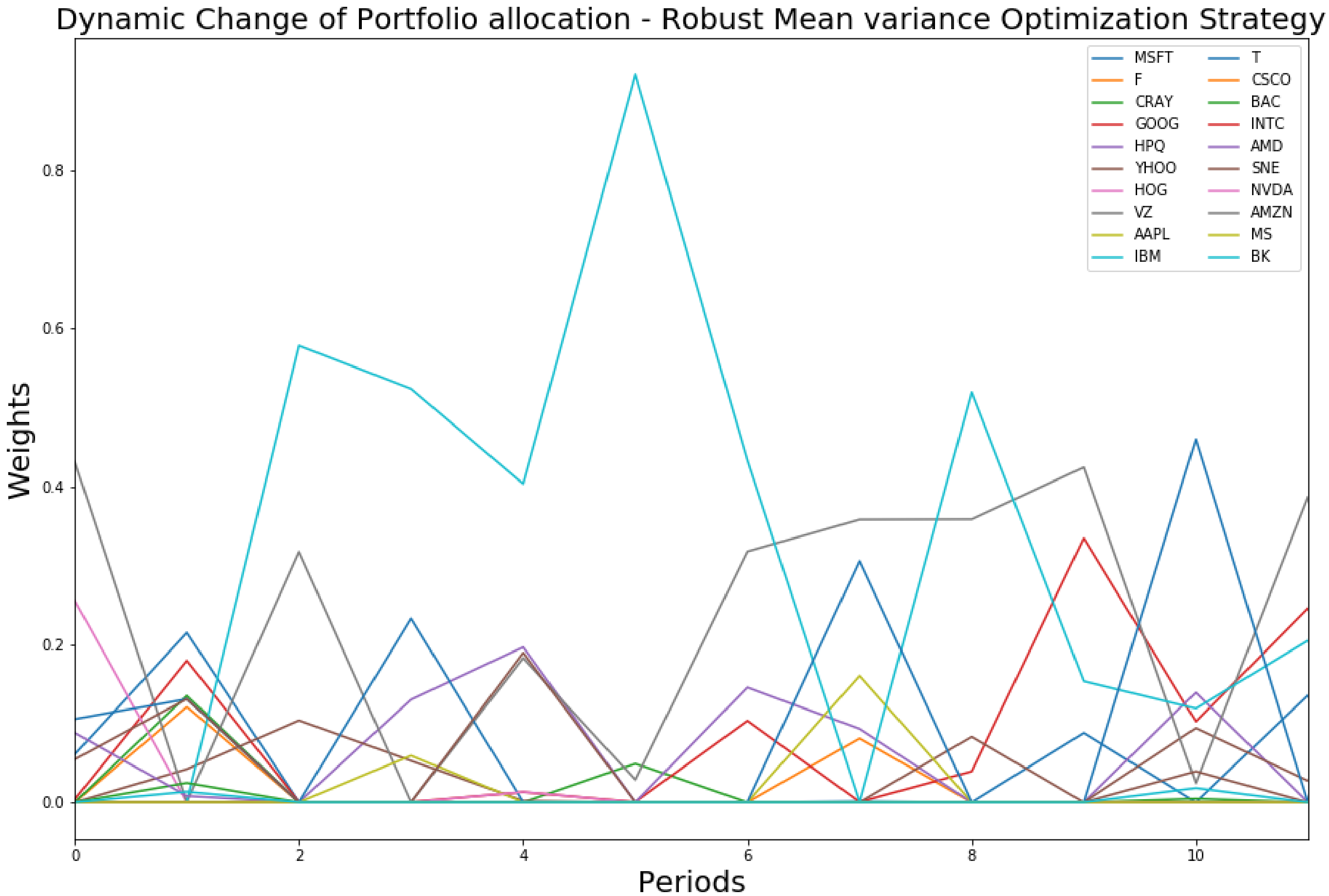
The above plot signifies the weight allocation to each stock over 12 holding period using the maximum Sharpe ratio portfolio selection strategy. In this strategy, the portfolio which has the largest return per unit of variance/ risk is preferred. Thus, it considers risk as well as the return. Unlike the minimum variance portfolio selection strategy where the weight of T's stock in the portfolio was predominantly higher, this strategy has almost distributed the weights diversely based on the ratio for each stock obtained. This led to its highest return as seen from plot 1 as compared to the other strategies.



While implementing the Sharpe ratio for the crisis duration (2008-2009), we cannot find a solution for the 4^{th} period. This happens because the efficient frontier is present below the risk-free rate during this period. This can be computed by multiple ways.

- By selecting the initial portfolio weights if the CPLEX doesn't return a solution. (Implemented)
- By plotting a tangential line to the frontier to compute the weights of the asset giving max. return
- By using the weights from the previous periods to prevent any loss in the returns

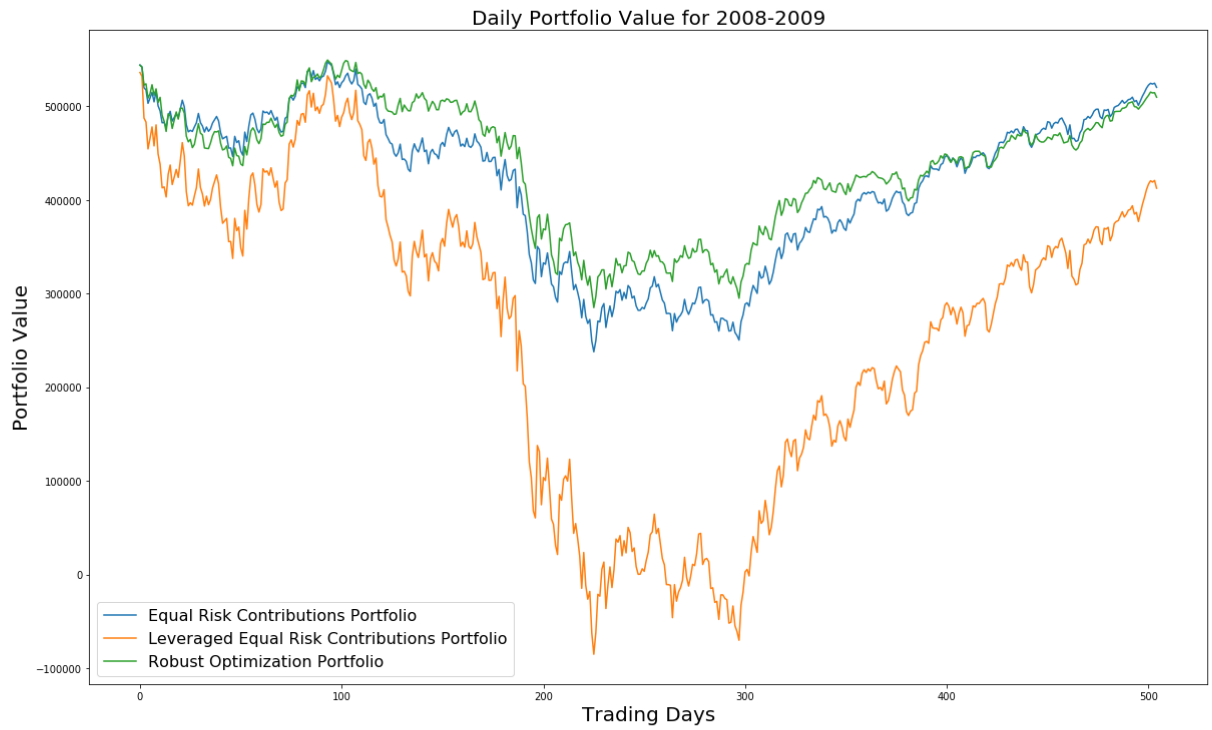
9.2.3 “Robust Mean Variance Optimization Strategy”



The plot below signifies the weight allocation to each stock over 12 holding period using the Robust mean variance portfolio selection strategy. The changes in the weight allocation between the consecutive periods signifies the amount of trading involved in the strategy. Thus, in can be concluded that the robust mean variance optimization strategy reduces the trading drastically as compared to the maximum Sharpe ratio strategy and the minimum variance strategy. Moreover, maximum Sharpe ratio strategy involves very high level of trading as the dynamic changes in the weight allocation between the consecutive periods it too high.

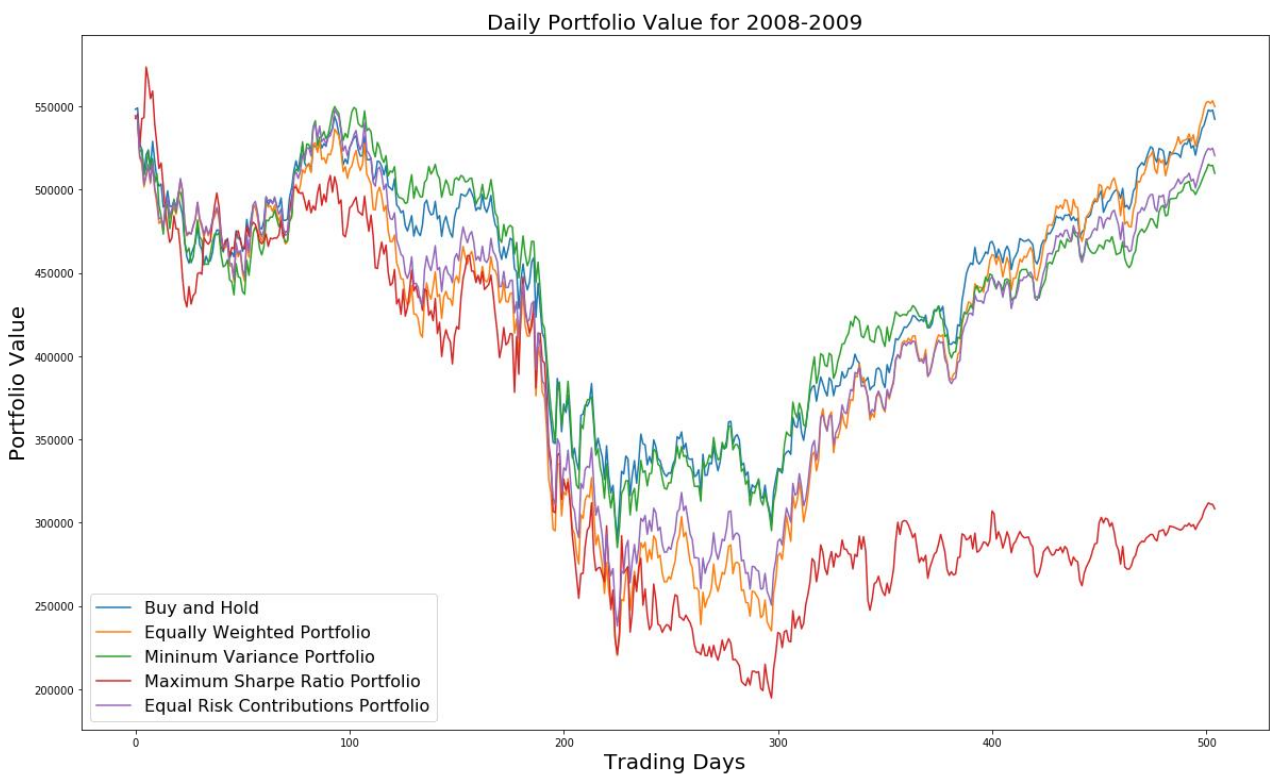
9.3 Comparing Strategies – 2008-2009

9.3.1 Comparing Daily Portfolio Values -5,6,7 Strategies

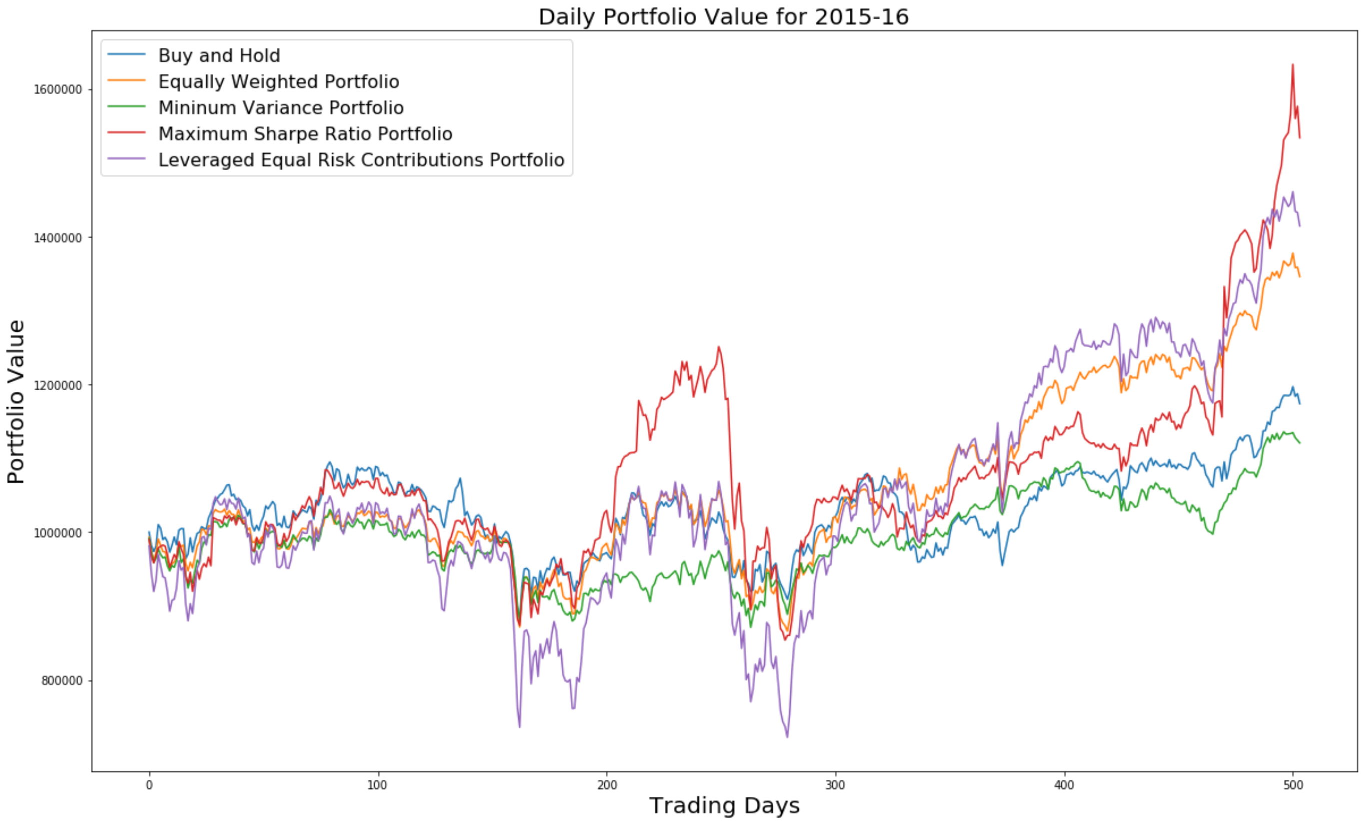


The above plot compares the portfolio value over 12 holding period using the Equal risk contribution, Leverages equal risk contribution and the robust mean variance optimization strategy.

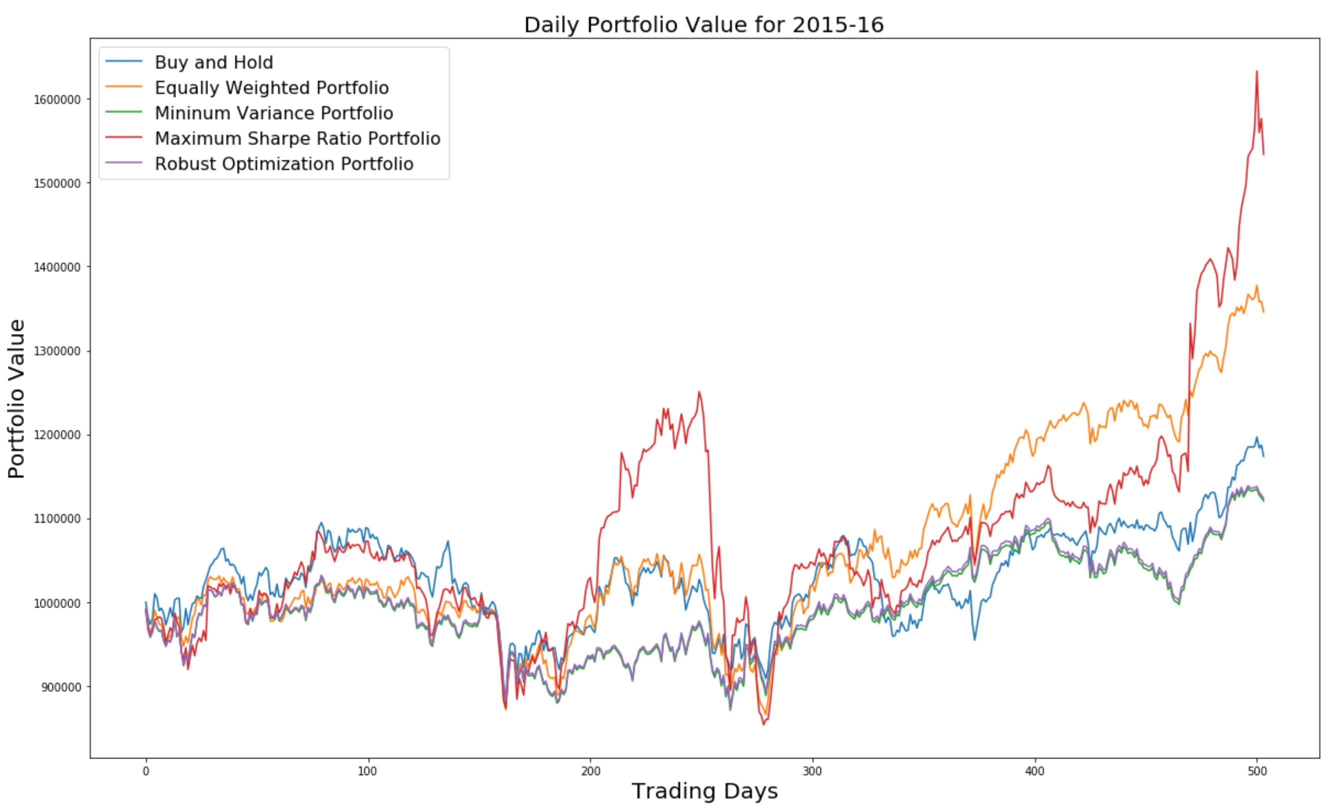
9.3.2 Comparing Daily Portfolio Values - 1,2,3,4,5 Strategies



9.3.3 Comparing Daily Portfolio Values - 1,2,3,4,6 Strategies



9.3.4 Comparing Daily Portfolio Values - 1,2,3,4,7 Strategies



As we know that there was a drop in the prices and returns of almost all the stocks, I would choose to select a strategy with minimum risk. Even though equally weighted performs the best, it will also have the equal chance of the risk. So, I would choose minimum variance or the robust mean variance optimization strategy which offers me a decent amount in the crisis period and assures me of the least risk in any condition.

References:

1. https://www.investopedia.com/terms/b/buyandhold.asp
2. https://www.investopedia.com/terms/e/equalweight.asp
3. MIE1622- Lecture 4- Equity portfolio selection in practice
4. MIE1622- Lecture 6- Robust Portfolio Selection and Factor Models
5. " Momentum and Markowitz: A Golden Combination " by Kelly W J, Budd A, Kipah