Machine Learning and Data Analytics
ME 5013- Fall 2019

Lectures 18

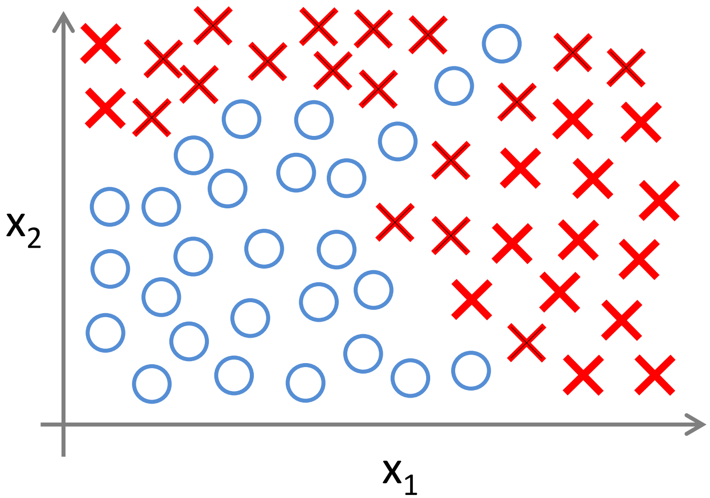
- Neural networks basics

UTSA®
The University of Texas at San Antonio™

Adel Alaeddini, PhD
Associate Professor of Mechanical Engineering
Advanced Data Engineering Lab
adel.alaeddini@utsa.edu

Disclosure: the slides are adopted from Andew Ng online Machine Learning course

Non-linear & High dimensional Classification



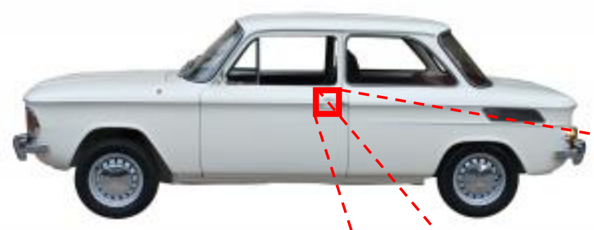
$$g(\theta_0 + \theta_1 x_1 + \theta_2 x_2$$
$$+\theta_3 x_1 x_2 + \theta_4 x_1^2 x_2$$
$$+\theta_5 x_1^3 x_2 + \theta_6 x_1 x_2^2 + \ldots)$$

x_1 = size
x_2 = # bedrooms
x_3 = # floors
x_4 = age
$\ldots$
x_{100}

Example: Computer Vision

What is this?

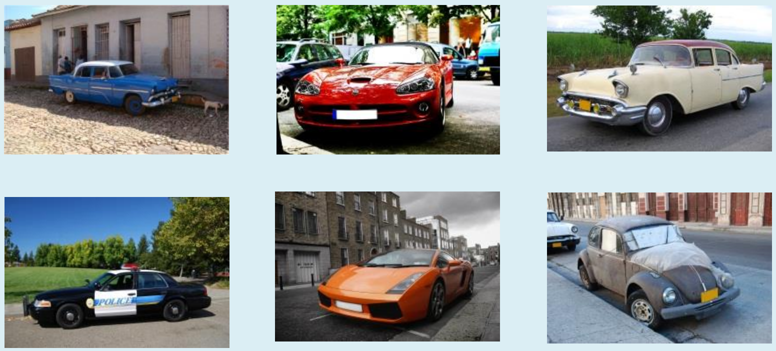
You see this:



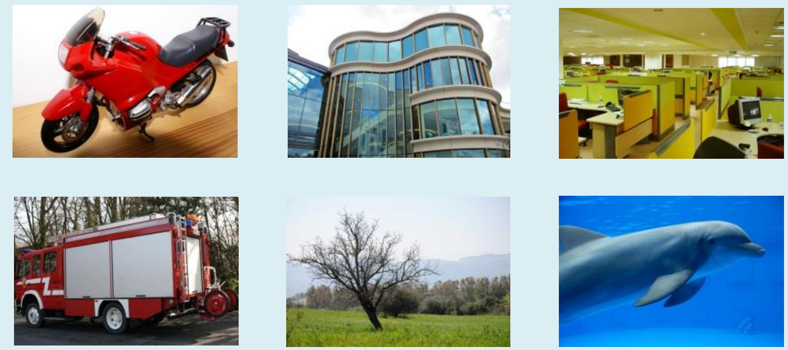
But the camera sees this:

194	210	201	212	199	213	215	195	178	158	182	209
180	189	190	221	209	205	191	167	147	115	129	163
114	126	140	188	176	165	152	140	170	106	78	88
87	103	115	154	143	142	149	153	173	101	57	57
102	112	106	131	122	138	152	147	128	84	58	66
94	95	79	104	105	124	129	113	107	87	69	67
68	71	69	98	89	92	98	95	89	88	76	67
41	56	68	99	63	45	60	82	58	76	75	65
20	43	69	75	56	41	51	73	55	70	63	44
50	50	57	69	75	75	73	74	53	68	59	37
72	59	53	66	84	92	84	74	57	72	63	42
67	61	58	65	75	78	76	73	59	75	69	50

Computer Vision: Car detection



Cars



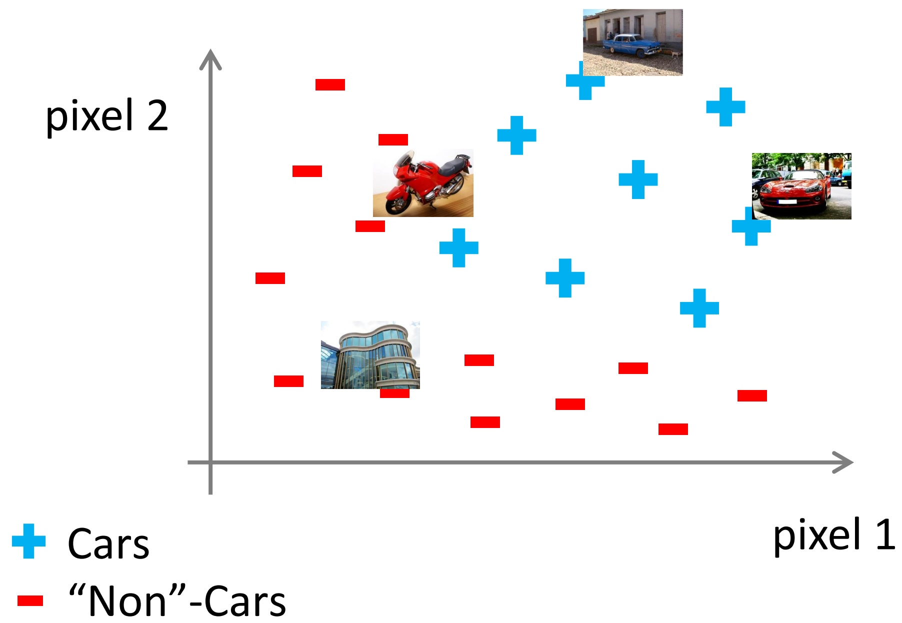
Not a car

Testing: 

What is this?

Non-Linear Hypotheses

→ **Learning Algorithm**



50 x 50 pixel images→ 2500 pixels

$n = 2500$ (7500 if RGB)

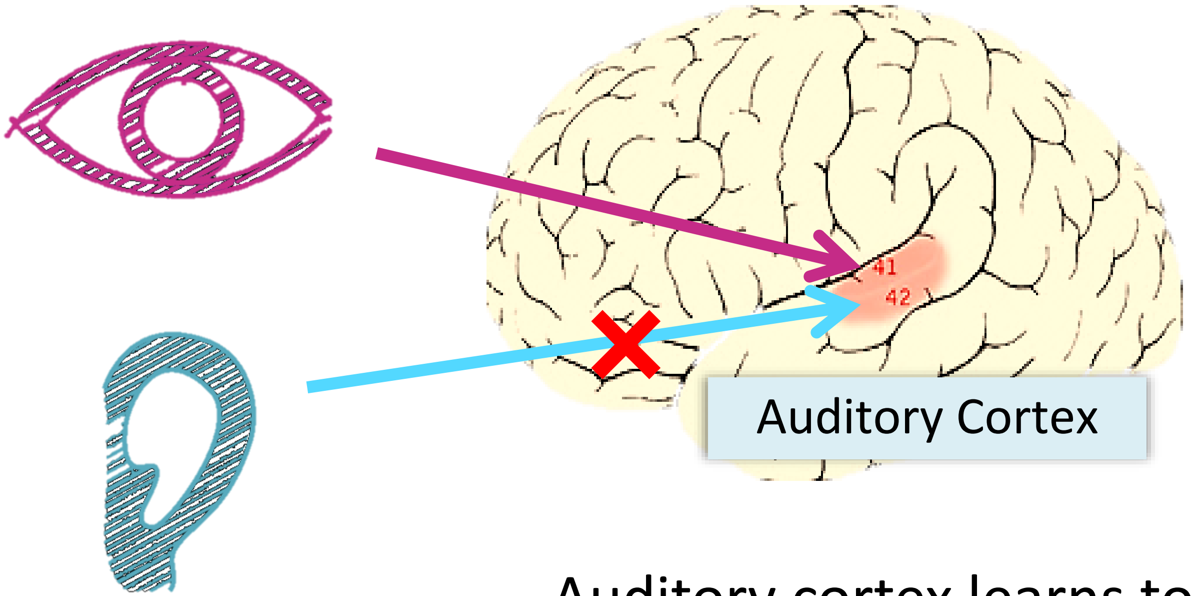
$$x = \begin{bmatrix} \text{pixel 1 intensity} \\ \text{pixel 2 intensity} \\ \vdots \\ \text{pixel 2500 intensity} \end{bmatrix}$$

Quadratic features ($x_i \times x_j$): ≈3 million features

Neural Networks

Origins: Algorithms that try to mimic the brain.
Was very widely used in 80s and early 90s; popularity diminished in late 90s.
Recent resurgence: State-of-the-art technique for many applications

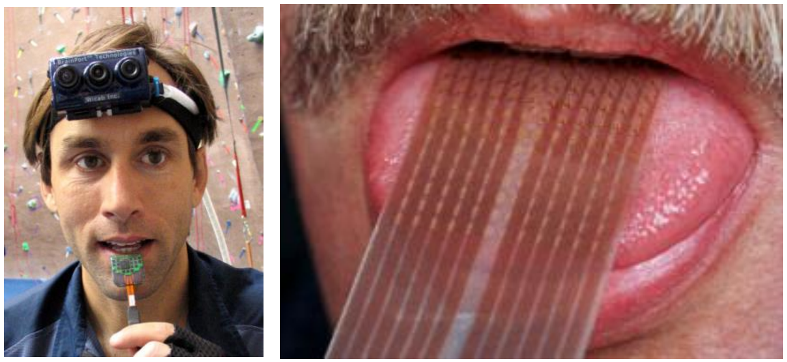
The "one learning algorithm" hypothesis



Auditory cortex learns to see

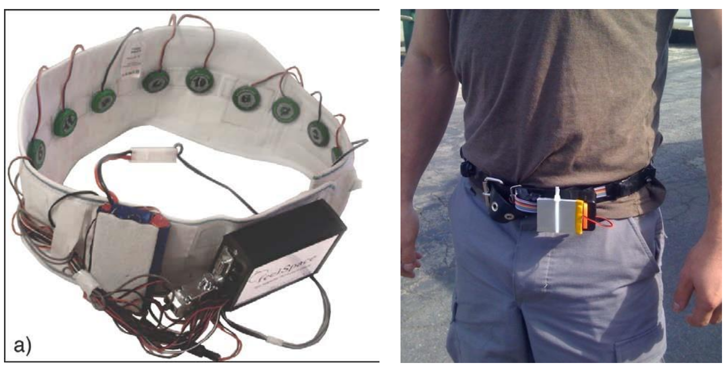
[Roe et al., 1992]

Sensor representations in the brain

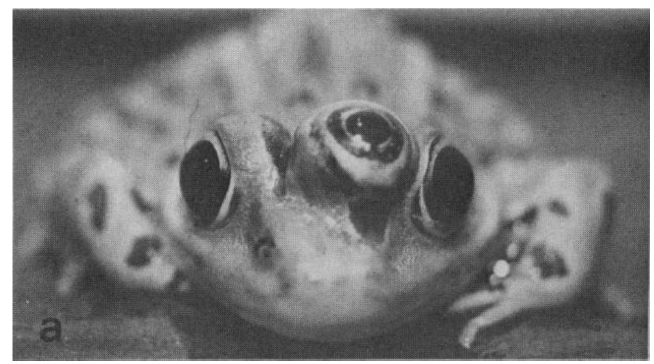


Seeing with your tongue

Human echolocation (sonar)



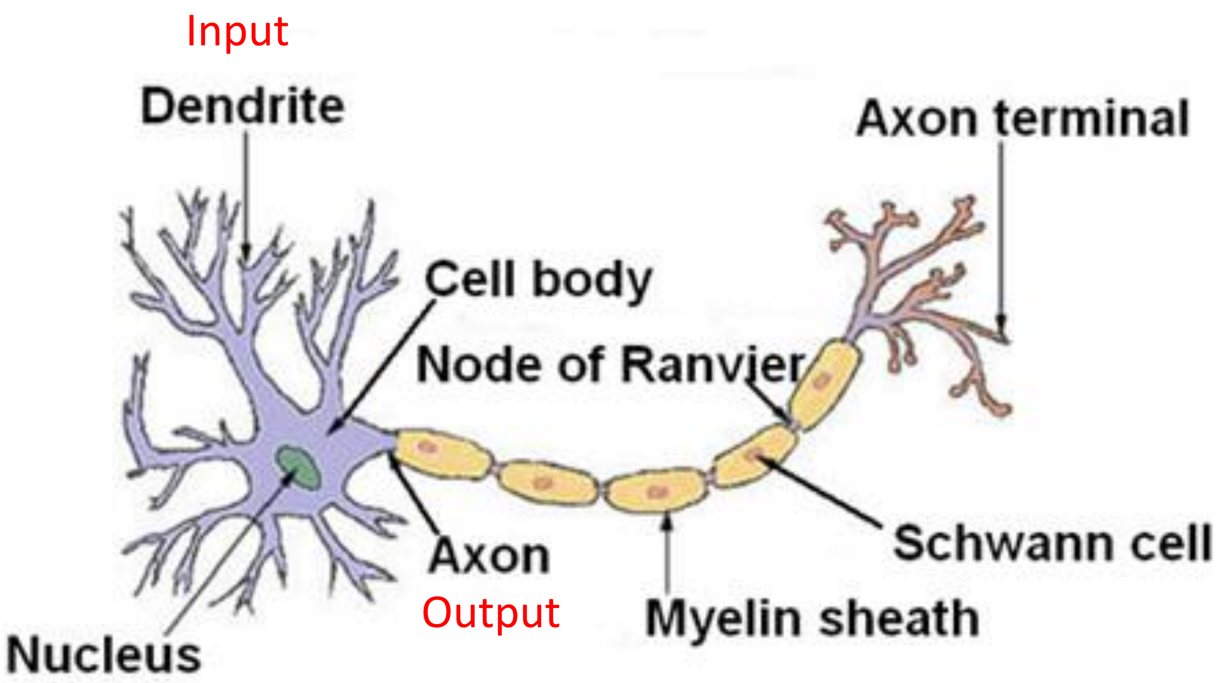
Haptic belt: Direction sense



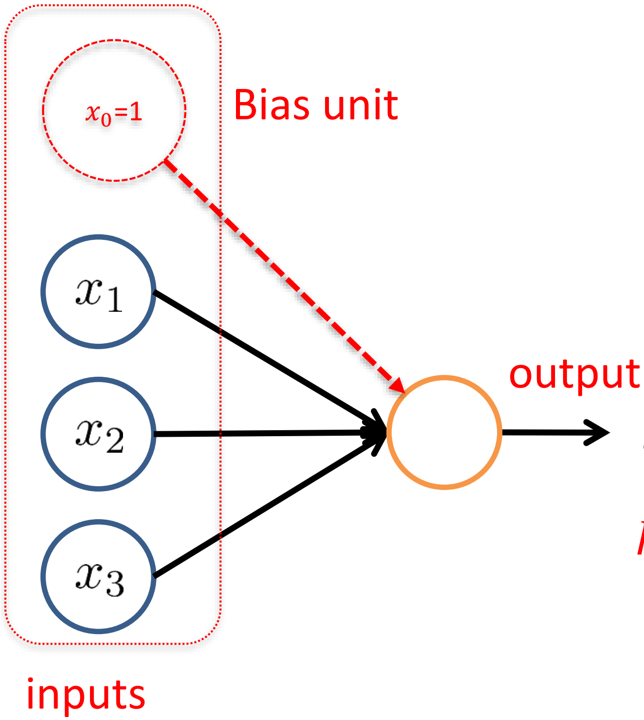
Implanting a 3rd eye

[BrainPort; Welsh & Blasch, 1997; Nagel et al., 2005; Constantine-Paton & Law, 2009]

Neuron in the brain



Neuron model: Logistic unit



$h_\theta(x)$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\theta = [\theta_0, \theta_1, \dots, \theta_3]$$

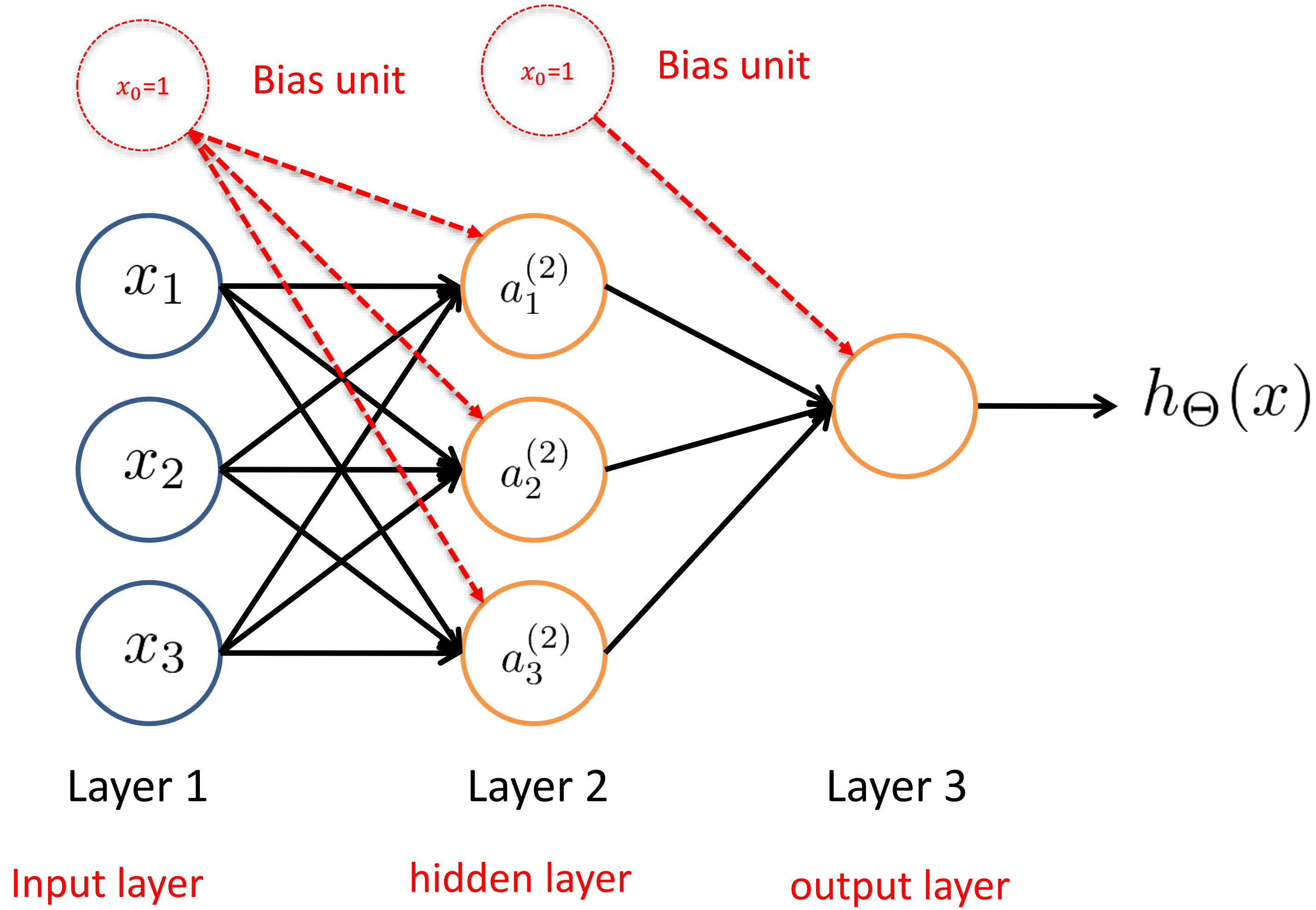
Weights/ parameters

$$h_\theta(x) = \frac{1}{1+e^{-\theta x}}$$

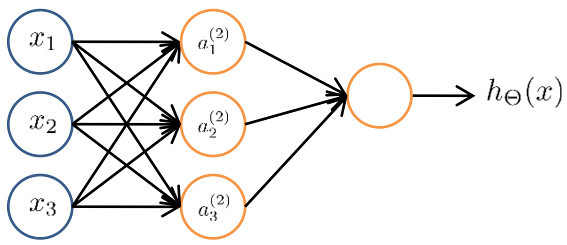
Sigmoid (logistic) activation function.

$$g(z) = \frac{1}{1+e^{-z}}$$

Neural Network



Neural Network



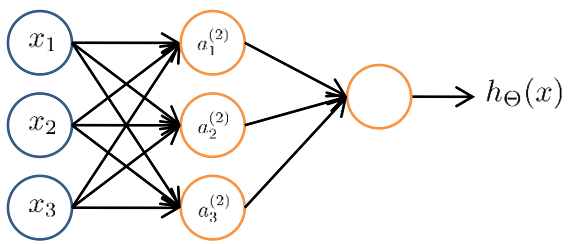
$a_i^{(j)} =$ "activation" of unit i in layer j

$\Theta^{(j)} =$ matrix of weights controlling function mapping from layer j to layer $j+1$

$$
\begin{aligned}
a_1^{(2)} &= g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3) \\
a_2^{(2)} &= g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3) \\
a_3^{(2)} &= g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3) \\
h_\Theta(x) = a_1^{(3)} &= g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})
\end{aligned}
$$

If network has s_j units in layer j, s_{j+1} units in layer $j+1$, then $\Theta^{(j)}$ will be of dimension $s_{j+1} \times (s_j + 1)$.

Forward propagation: Vectorized implementation



$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$
$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$
$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

$$h_\Theta(x) = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$

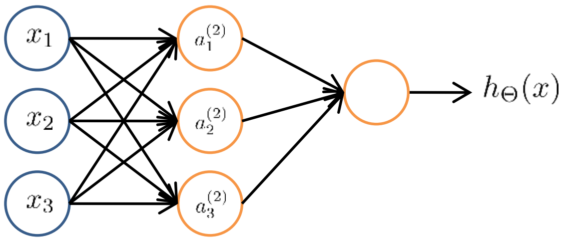
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)}x$$
$$a^{(2)} = g(z^{(2)})$$

Add $a_0^{(2)} = 1$.

$$z^{(3)} = \Theta^{(2)}a^{(2)}$$
$$h_\Theta(x) = a^{(3)} = g(z^{(3)})$$

Forward propagation: Vectorized implementation



$$a_1^{(2)} = g(\Theta_{10}^{(1)}x_0 + \Theta_{11}^{(1)}x_1 + \Theta_{12}^{(1)}x_2 + \Theta_{13}^{(1)}x_3)$$
$$a_2^{(2)} = g(\Theta_{20}^{(1)}x_0 + \Theta_{21}^{(1)}x_1 + \Theta_{22}^{(1)}x_2 + \Theta_{23}^{(1)}x_3)$$
$$a_3^{(2)} = g(\Theta_{30}^{(1)}x_0 + \Theta_{31}^{(1)}x_1 + \Theta_{32}^{(1)}x_2 + \Theta_{33}^{(1)}x_3)$$

$$h_\Theta(x) = g(\Theta_{10}^{(2)}a_0^{(2)} + \Theta_{11}^{(2)}a_1^{(2)} + \Theta_{12}^{(2)}a_2^{(2)} + \Theta_{13}^{(2)}a_3^{(2)})$$

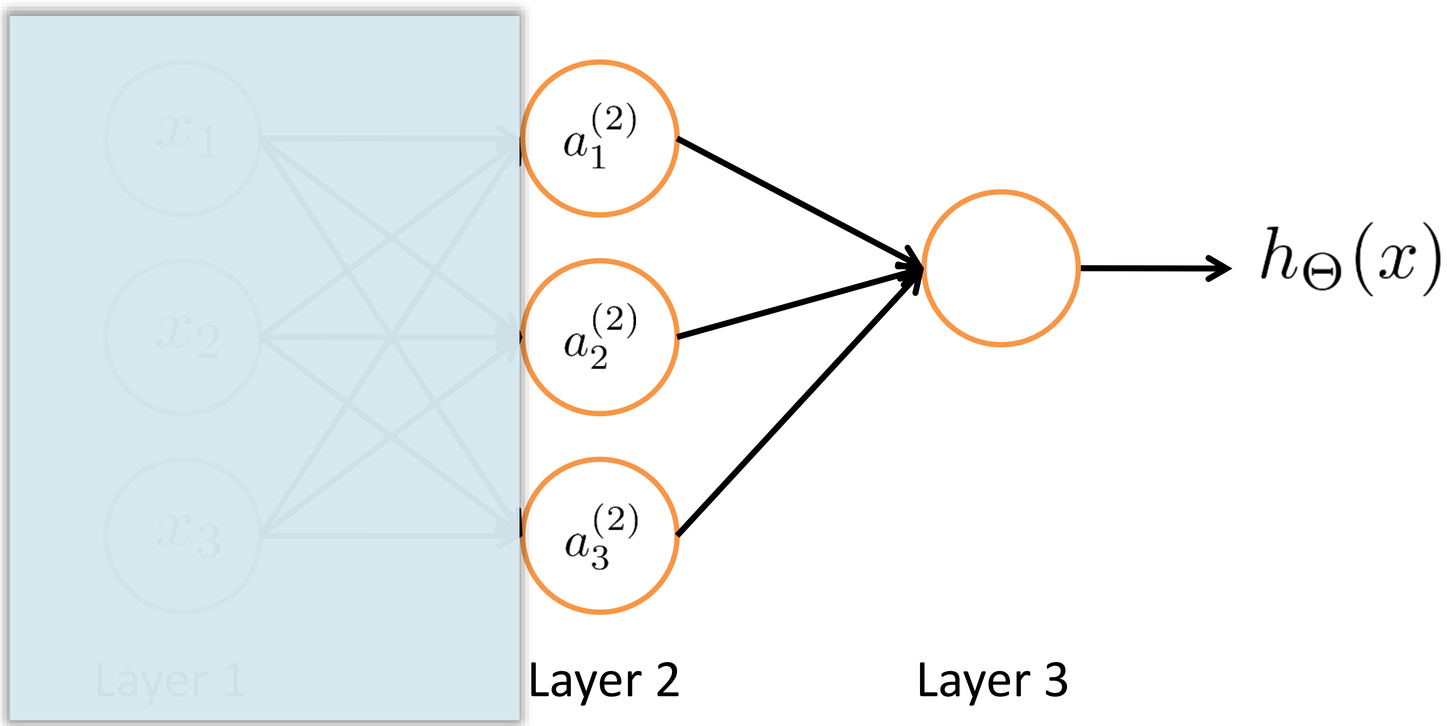
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad z^{(2)} = \begin{bmatrix} z_1^{(2)} \\ z_2^{(2)} \\ z_3^{(2)} \end{bmatrix}$$

$$z^{(2)} = \Theta^{(1)}x$$
$$a^{(2)} = g(z^{(2)})$$

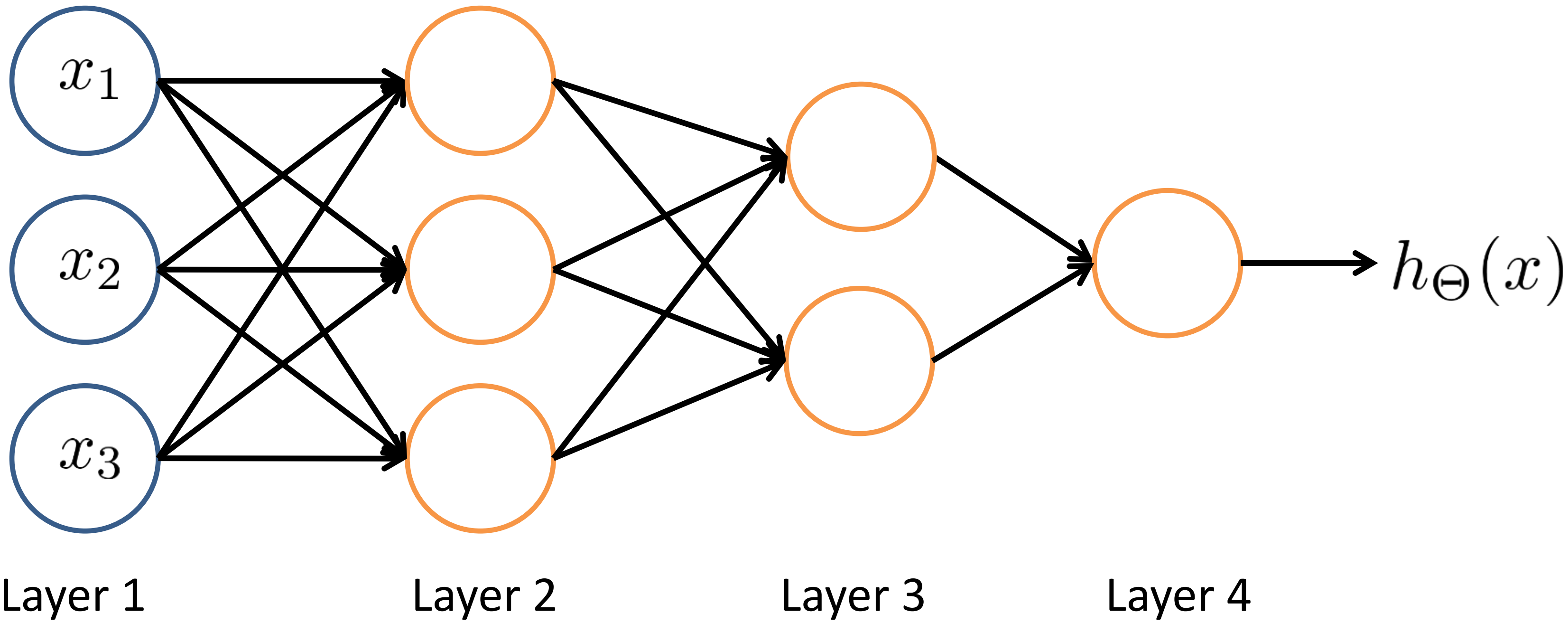
Add $a_0^{(2)} = 1$.

$$z^{(3)} = \Theta^{(2)}a^{(2)}$$
$$h_\Theta(x) = a^{(3)} = g(z^{(3)})$$

Neural Network learning its own features

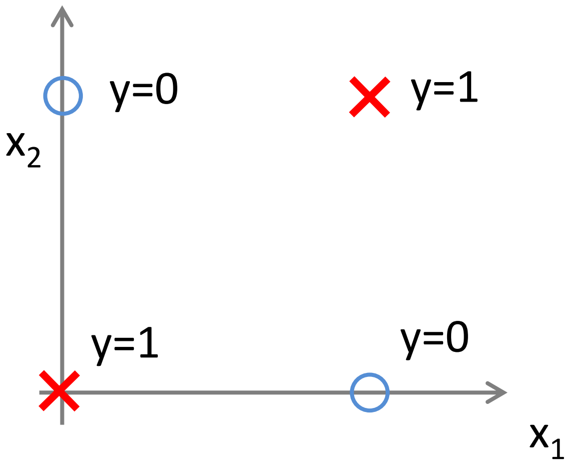


Other network architectures

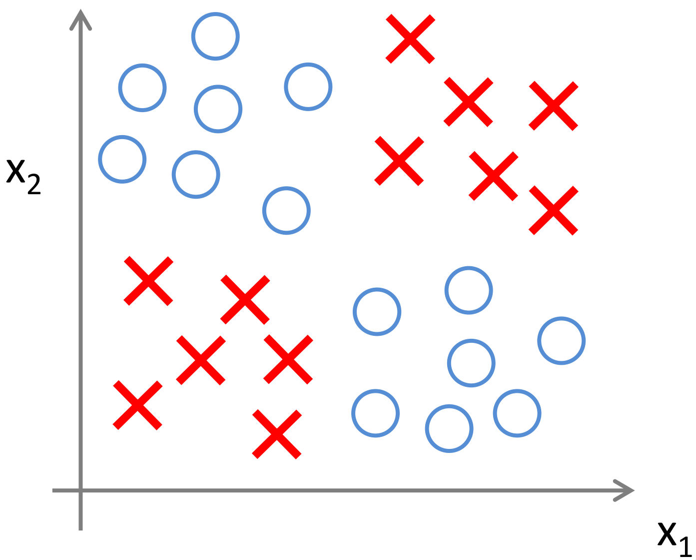


Non-linear classification example: XOR/XNOR

$x_1 \quad x_2$ are binary (0 or 1)



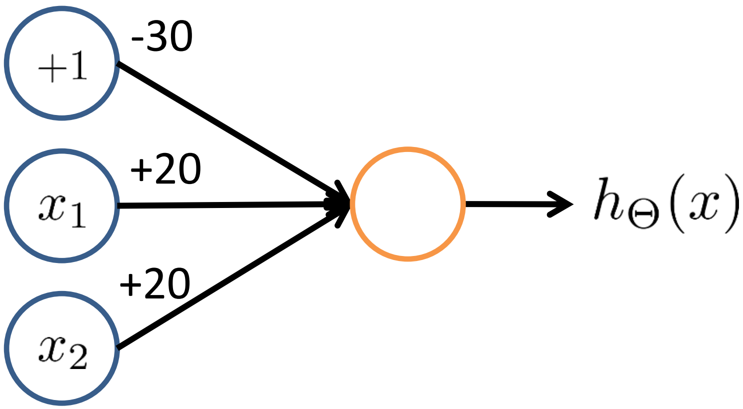
$$y = x_1 \text{ XOR } x_2$$
$$x_1 \text{ XNOR } x_2$$
$$\text{NOT } (x_1 \text{ XOR } x_2)$$

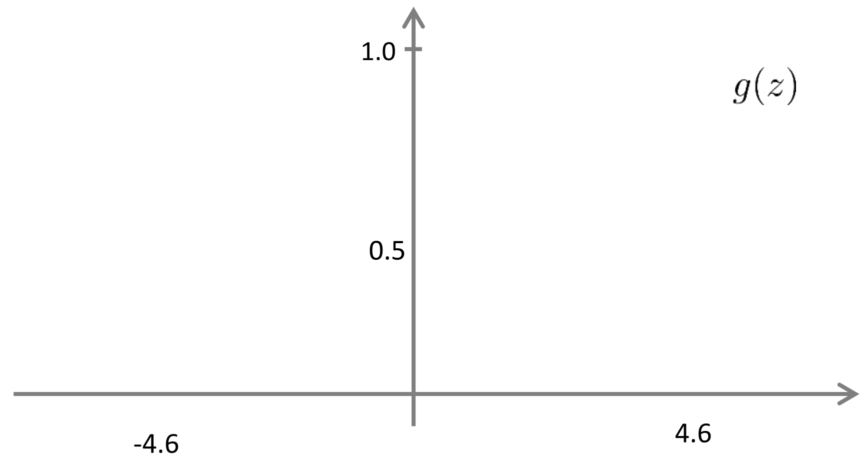


Simple example: AND

$x_1, x_2 \in \{0, 1\}$

$y = x_1 \text{ AND } x_2$



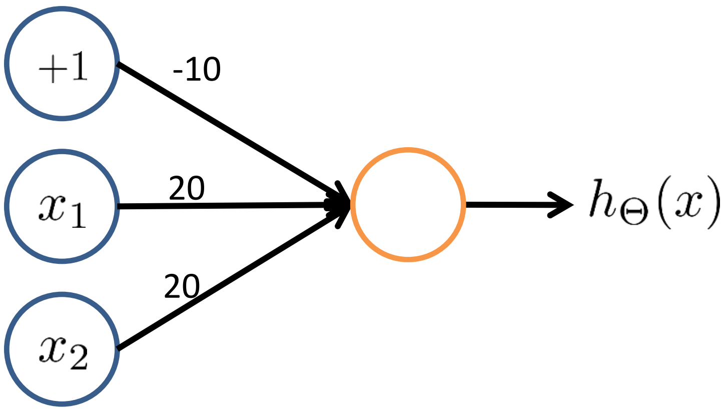


x_1	x_2	$h_\Theta(x)$
0	0	
0	1	
1	0	
1	1	

$$h_\theta(x) = g(-30 + 20\, x_1 + 20\, x_2)$$

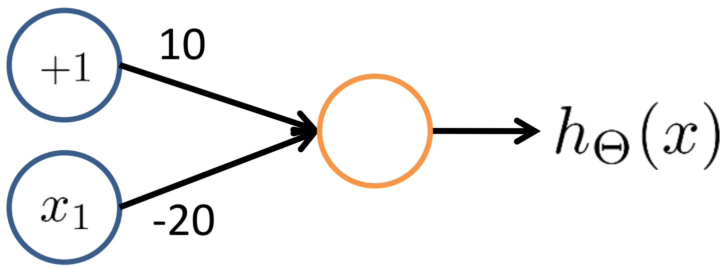
$\theta_{10}^{(1)}$ $\theta_{11}^{(1)}$ $\theta_{12}^{(1)}$

Example: OR function



x_1	x_2	$h_\Theta(x)$
0	0	
0	1	
1	0	
1	1	

Negation



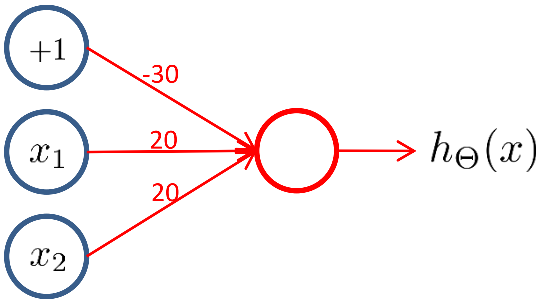
x_1	$h_\Theta(x)$
0	
1	

$$h_\Theta(x) = g(10 - 20x_1)$$

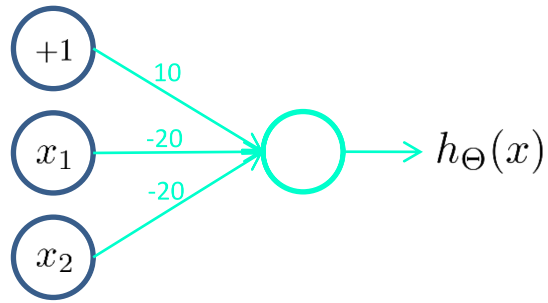
$$(\text{NOT } x_1) \text{ AND } (\text{NOT } x_2)$$

?

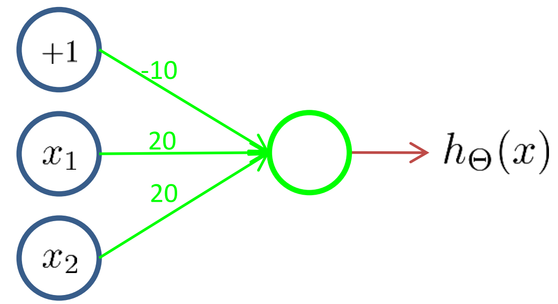
Putting it together: x_1 XNOR x_2



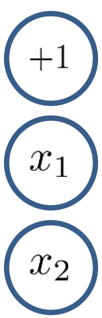
x_1 AND x_2



(NOT x_1) AND (NOT x_2)

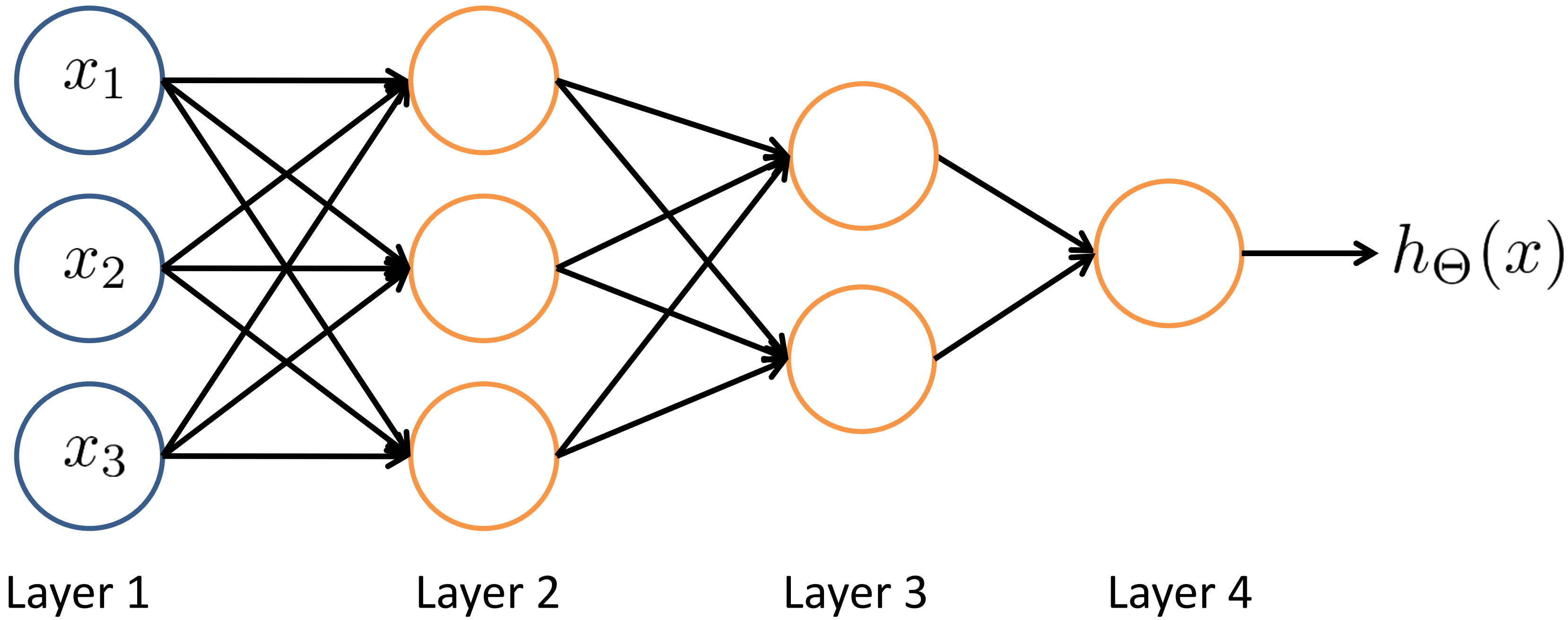


x_1 OR x_2

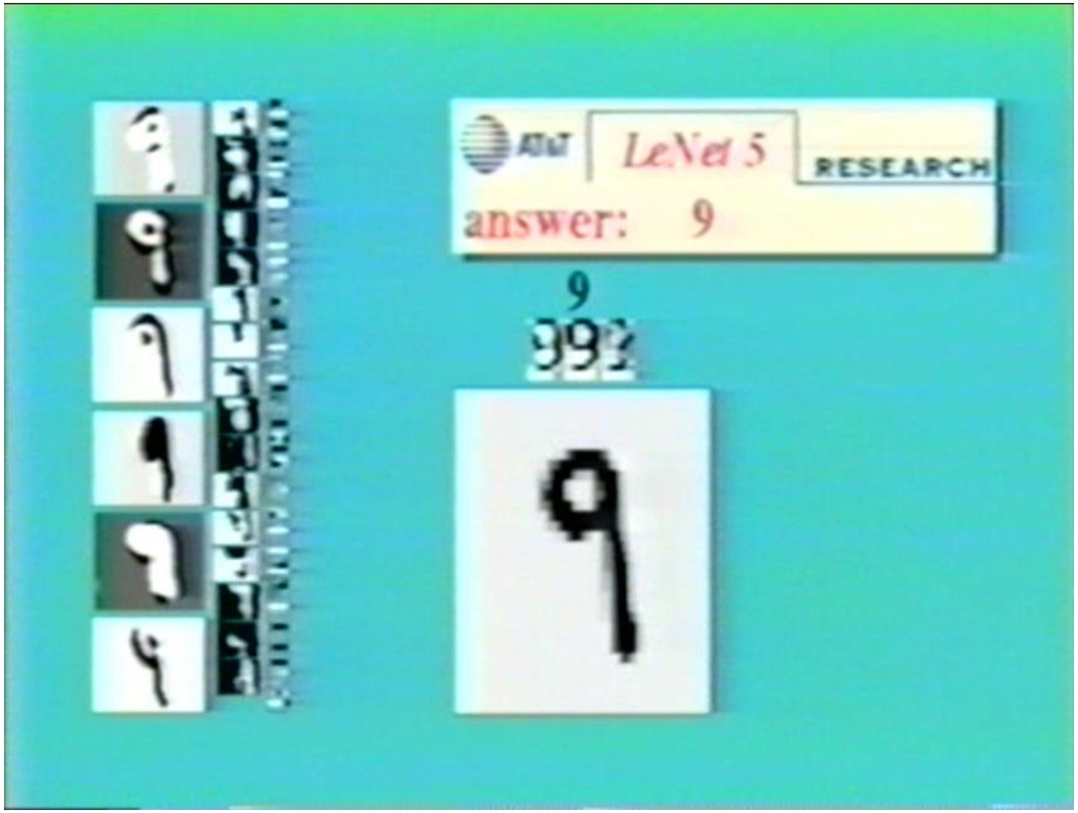


x_1	x_2	$a_1^{(2)}$	$a_2^{(2)}$	$h_\Theta(x)$
0	0			
0	1			
1	0			
1	1			

Neural Network intuition

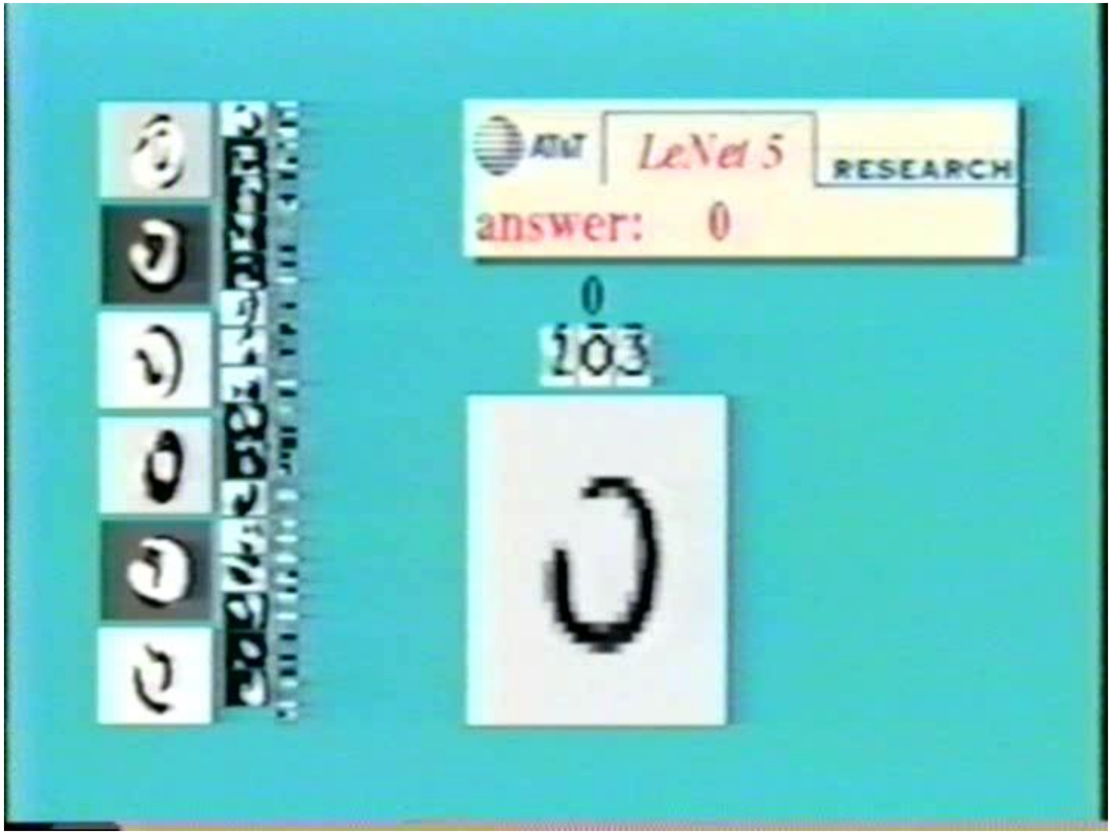


Handwritten digit classification



[Courtesy of Yann LeCun]

Handwritten digit classification



[Courtesy of Yann LeCun]

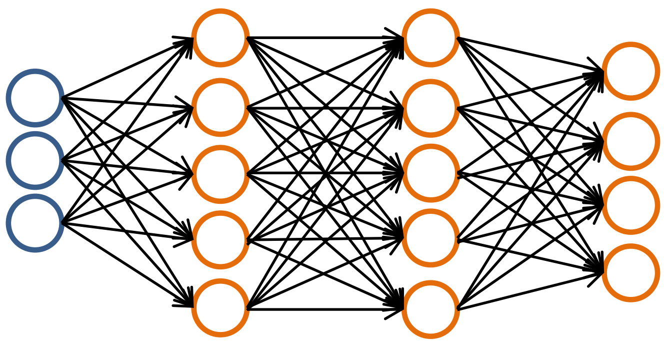
Multiple output units: One-vs-all.

Pedestrian

Car

Motorcycle

Truck

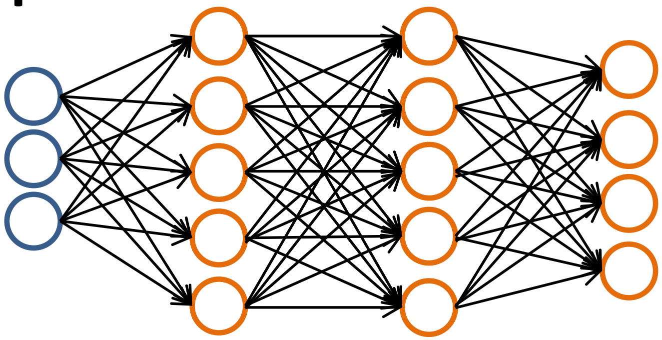


$$h_\Theta(x) \in \mathbb{R}^4$$

Want $h_\Theta(x) \approx \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $h_\Theta(x) \approx \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $h_\Theta(x) \approx \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, etc.

when pedestrian when car when motorcycle

Multiple output units: One-vs-all.



$$h_\Theta(x) \in \mathbb{R}^4$$

Want $h_\Theta(x) \approx \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $h_\Theta(x) \approx \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$, $h_\Theta(x) \approx \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$, etc.

when pedestrian　when car　when motorcycle

Training set: $(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})$

$y^{(i)}$ one of $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $\begin{bmatrix}0\\1\\0\\0\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$, $\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

pedestrian　car　motorcycle　truck