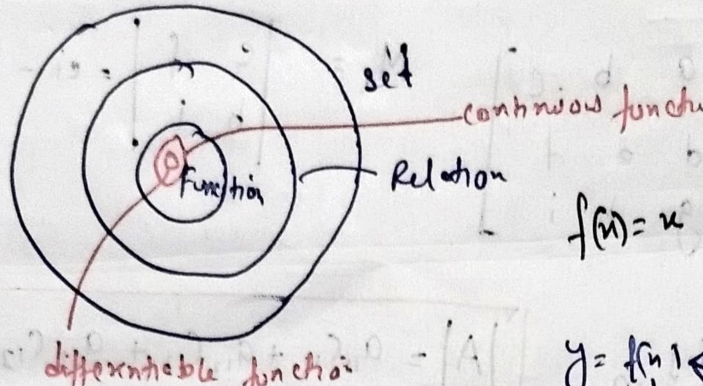
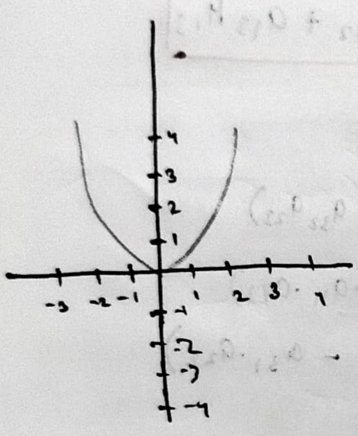
DAY 3

# FUNCTIONS




$f(x) = x$ — $x$ is the independent variable

$y = f(x)$ < $x$, $x^2$, const.



the input is the independent var. & output is dependent

for a given value of '$x$', we try to find $y$ value $\Rightarrow$ $f(x)$

$y = x^2$

$x=0$
$x=3$

| $x$ | 0 | 1 | 2 | 3 | 4 | -1 | -2 | -3 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 0 | 1 | 4 | 9 | 16 | 1 | 4 | 9 |

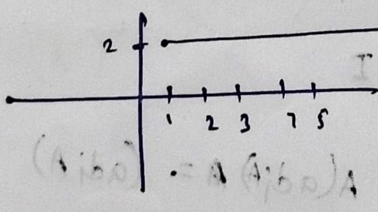
$y = x^2$

## Types of Function

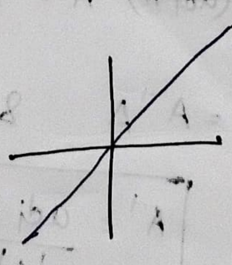
① Const. Func. $\Rightarrow$ $f(x)$. i.e. whatever is the input we get a constant (same) output $\forall x$

$f(x) = 2$, $\forall (x \in N)$ — natural no. set domain

$f(1) = 2$ $f(2) = 2$ $f(3) = 2$



② Identity Function $\Rightarrow$ $f(x) = x$




$(x, f(x)) \equiv (x, y)$

(independent var, $x \rightarrow$ input)

($y \rightarrow$ output var. dependent var.)

# Types of Functions

① Polynomials < linear, Non linear < quad., cubic, higher

$n^{th}$ degree polynomial

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \quad \rightarrow (\text{degree} = n)$$

② Trigonometric

$\sin x$, $\sin 2x$, $\cos x$, $\cos(nx)$, $\tan x$ etc.

$\tan x = \dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{\cos x}{\sin x}$

③ Hyperbolic → $\sinh(x)$, $\cosh(x)$

④ exponential − $e^{ax}$, $a^x$

⑤ Logarithmic − $\log x$, $\ln x$

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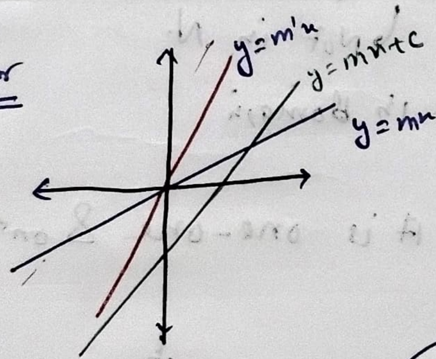
⇒ Polynomial function
- Linear : $y = mx + c$
- Non linear < Parabolic (quad), cubic

* Linear

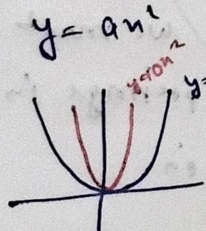



$y = mx + \boxed{c}$ ↳ not passing thru the origin

* quadratic function

$$y = \underbrace{a}x^2 + \underbrace{b}x + \underbrace{c}$$ — coefficients

$y = ax^2$ $\quad b = c = 0$




$y = x^2 \rightarrow a = 1$

$y = 10x^2 \rightarrow a = 10$

$\boxed{\text{a → contraction / expansion parameter}}$

c → up/down translation

## → One-One & Onto Function

**one-one** $f : D_1 \to D_2$ $D_1, D_2 \in R$

$f(1)=1$
$f(-1)=1$ } for 2 input same output

$f(x) = x^2$ ∀

not one-one

* ~~A function is Bijective if it is one-one & onto~~.

$f(x)=x^2$ $f(1) = 1^2 = 1$
$f(-1) = 1^2 = 1$ Not one-one

**onto** $f : \overset{N}{D_1} \to \overset{R}{D_2}$ $f(x)=$ identity function

∀ N. that are having images in $D_2$,
but not all element in $D_2 (R)$ have pre images in $D_1(N)$

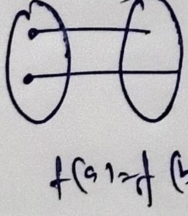
e.g. $\frac{1}{5}$ in $D_2$ → $f(x)=x$
$f(\frac{1}{5}) = (\frac{1}{5})$
↳ Not in N.

not onto

∵ you cannot find pre image in Domain

* A function is bijective if it is one-one & onto

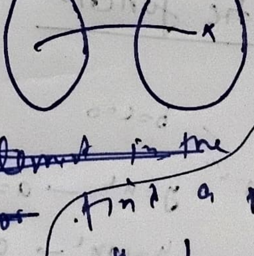
$f$



$f(a)=f(b) \Rightarrow \boxed{a=b}$

~~onto~~ one-one

$f$

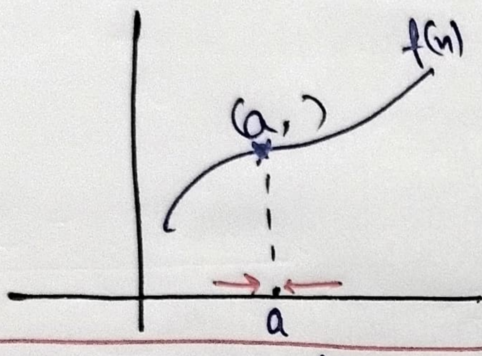


for each ele. in co-domain we must ~~element in the~~ ~~for~~ find a preimage in the domain.

* Only from the data you can construct the function (Temp: graph)

## → Limit of a Function (about a Point)

↳ after finding every location in the domain we are generalizing over this domain




$\lim_{x\to a} f(x) = ?$

$\lim_{x\to a^-} f(x) = LHL$

$\lim_{x\to a^+} f(x) = RHL$

$\boxed{\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = \text{finite}}$

**Existance of limit**

$\Downarrow$

$LHL = RHL = \text{finite}$

$f(x)\Big|_{x=a} = f(a)$

if the concept is valid on entire domain

$\Downarrow$

Limit exist on entire domain

$\boxed{\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)}$ → $f(x)\Big|_{x=a}$

↳ Function is Continuous

↳ if the same is true on entire domain → func$^n$ is continuous in entire domain