Bridge Course: Number System & Binomial Algebra

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Number System

- Natural Numbers ($\mathbb{N}$): 1, 2, 3, ...,

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- Natural Numbers ($\mathbb{N}$): 1, 2, 3, ..., 101, 123, 100032,...
- Whole Numbers ($\mathbb{W}$): 0, 1, 2, 3, ...

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- Natural Numbers ($\mathbb{N}$): 1, 2, 3, ..., 101, 123, 100032,...
- Whole Numbers ($\mathbb{W}$): 0, 1, 2, 3, ...
- Integers ($\mathbb{Z}$):..., -2, -1, 0, 1, 2, ...

Select All

Rational ($\mathbb{Q}$) and Irrational Numbers

- Rational: Can be expressed as $\frac{p}{q}$, where p, q are integers, and $q \neq 0$.
- Examples: $1, 0, \frac{2}{3}, \frac{109}{106}, \frac{-1}{2}, -5, ...$
- Irrational: An irrational number is a number that cannot be represented as a simple fraction $\frac{p}{q}$, where both p and q are integers and $q \neq 0$.
- Example: $\sqrt{2}, \sqrt{3}, 0.2\bar{3}, ...$

0.23333...

Real Numbers ($\mathbb{R}$)

$\mathbb{R}$

All rational and irrational numbers combined form real numbers.

Complex No: $a + ib$

Diff. between Real and Complex No. (Major)

$x, y \in$ Real

$x, y \rightarrow$ Real Numbers

Then

$\boxed{(1)\ x = y \quad (2)\ x > y \quad (3)\ x < y}$

What about Complex Numbers?

Prop 1: If $x > y$ and $a > 0$, then $ax > ay$.

Result:

What about Complex Numbers?

Prop 1: If $\underline{x > y}$ and $\underline{a > 0}$, then $\underline{ax > ay}$.

Result:

$\boxed{ax > ay}$

$$\underline{i} \neq \underline{0}$$

$i = \sqrt{-1}$

$i^2 = -1$

~~1~~ $\underline{i > 0}$ $\quad i > 0$

$i \cdot i > i \cdot 0$

$i^2 > 0$

$\boxed{-1 > 0}$

2 $\boxed{i < 0} \Rightarrow -i > 0$

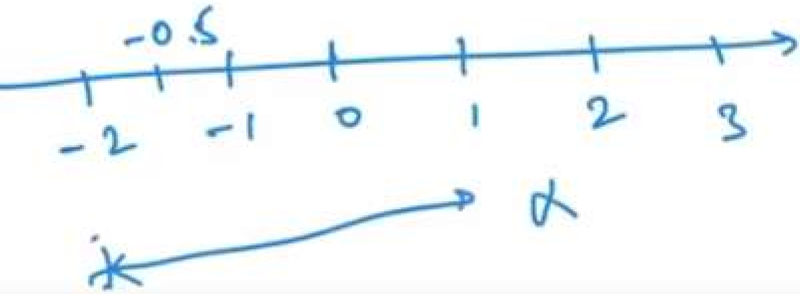
$-i \cdot i < 0 \cdot (-i)$

$-i^2 < 0 \Rightarrow -(-1) < 0 \Rightarrow \underline{1 < 0}$.

The <u>Complex no. are not Comparable.</u>

<u>Real No.</u>

$$\boxed{5-2i} \quad \boxed{\begin{matrix} = \\ > \\ < \end{matrix}} \quad \boxed{5+2i}$$



Properties:

$1-2=-1$

Academic +1

(1) Sum of two natural numbers is a natural no. ✓

(2) Diff. of two natural numbers Cannot decide (Integer)

(3) sum of two integers is an integer no.

(4) Difference of two integer numbers is also an integer no.

(5) sum/Diff. of two rational numbers is a rational number

(6) sum/Diff of two irrational numbers ______

(7) Rational no + Irrational no = Irrational No.

(8) Rational no. × Rational no = Rational No.

(9) Irrational no. × Irrational no =

(10) Irrational no. × Rational no. =

(11) Complex + Complex =

5 11-01-32 (12) Complex no. × Complex No. =

(9) Irrational no. X Irrational no = Cannot decide

$\sqrt{2} \times \sqrt{2} = 2$

(10) Irrational no. X Rational no. = Irrational No.

$(5-i) + (5+i) = 10$

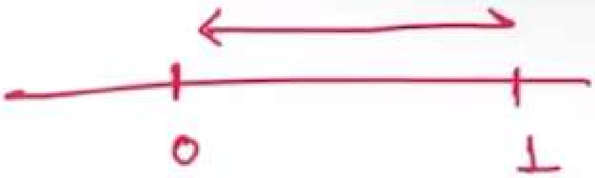
(11) Complex + Complex = Complex no.

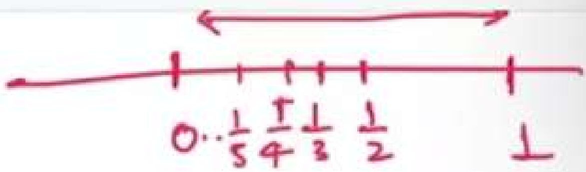
$10 + 0i$

(12) Complex no. X Complex No. = Complex no.

Every Real no. is a Complex no.

Q. How Many Rational No. are there in between 0 and 1?





$\frac{1}{2}$ $\quad 1 > \frac{1}{6}, \frac{1}{7} \ldots$

for all n - natural no. $\quad > 0$

$0 < \frac{1}{n} < 1$

Irrational No.

$0 < \frac{\sqrt{2}}{n} < 1$

n → natural no. $n \geq 2$

$\underline{\underline{n=1}}$

$1 < \sqrt{2}$

(5) Archimedean property:

for any Real no x

we can find a natural no. n s.t.

$$n > x$$

(*) Archimedean property:

Academic +1

① for any Real no x

we can find a natural no. n s.t.

$$\boxed{n > x}$$

② for any Real no $x > 0$,

we can find a natural no n s.t. $0 < \frac{1}{n} < x$.

Sets

Def: A well defined Collection of objects.

Eg:

$\mathbb{N} = \{1, 2, 3 \ldots\}$

$\mathbb{Z}$

$\mathbb{W}$

$\mathbb{Q}$

Elements of a set : member of a set is called element of that set

$1 \in \mathbb{N}$

$2 \in \mathbb{N}$

Elements of a set : member of a set is called element of that set

$1 \in \mathbb{N}$ $0 \in \mathbb{Q}$

$2 \in \mathbb{N}$

$0 \notin \mathbb{N}$

Reperesentation of a set:

(a) Set Builder form:

$N = \{1, 2, 3, \dots\}$

$A = \{1, 2, 3, 4\}$

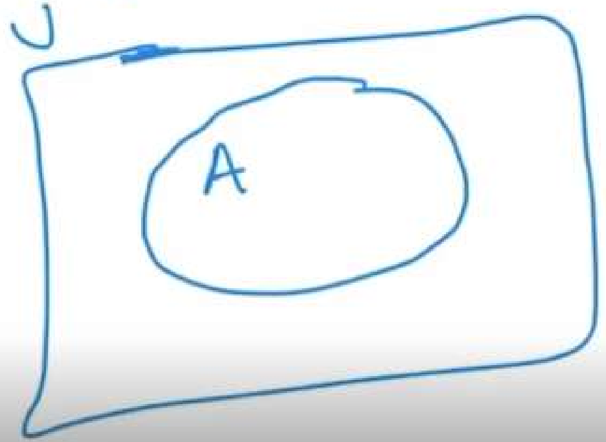
(b) Roaster form:

$A = \{x : x$ is Natural no' and $x \leq 4\}$

Empty set:

ϕ = The set which does not contain any element.

$x \in U$, then $x \notin \phi$



Finite and Infinite set.

$A = \{1, 2, 3, 4\}$

finite set

no. of elements $= 4$

$\parallel$

Cardinality of A (

$|A| = 4$ $\quad |A|$

Otherwise, it will be an infinite set

$\mathbb{N} = \{1, 2, 3, \ldots\}$

Equal sets, Subset, Super set, proper subset :

$$\underbrace{A = B}_{\text{same elements}}$$

$A = \{1, 2, 3\} \neq B = \{1, 2, 4\}$

$A = \{1, 2, 3\} = B = \{1, 1, 2, 3\}$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad \| $

$\qquad\qquad\qquad\qquad\qquad\quad B = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

If $x \in A$, then $x \in B$

$\boxed{A \subseteq B}$

If $A = B$, then

$A \subseteq B$ and $B \subseteq A$

~~$A \subseteq B$~~ If $\underline{x \in A}$, then $\underline{x \in B}$

But there is $\underline{b \in B}$

, $\underline{b \notin A}$

$A \neq B$

$\boxed{A \subsetneq B}$ $\boxed{\text{proper subset}}$

A is a subset of B

B is a proper superset of A.

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}$

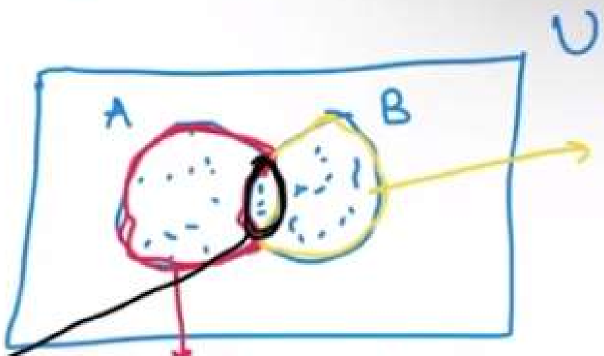
$\boxed{A \subsetneq B}$

$A \neq B$

A is a proper subset of B

Venn Diagram:

A   B   U



$x \in B$
$x \notin A$

$x \in A,\ x \notin B$

$x \in A$ and $x \in B$

$x \in A,\ x \notin B$

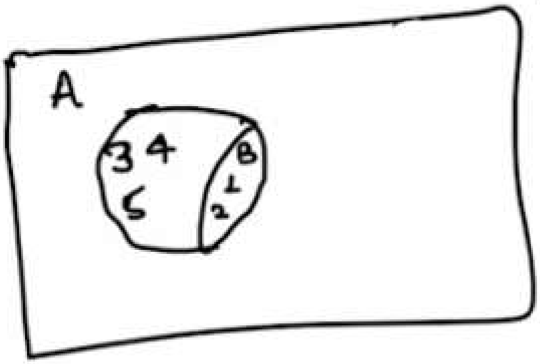
$U = \mathbb{N}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2\}$

$x \in A$ and $x \in B$

$U = \mathbb{N}$



Operations on sets : A B

① $A \cup B = \{ x \in A \text{ or } x \in B \}$

↓
Union

$A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

② $A \cap B = \{ x \in A \text{ and } x \in B \}$

↓
intersection

$A \cap B = \{1, 2, 3\}$

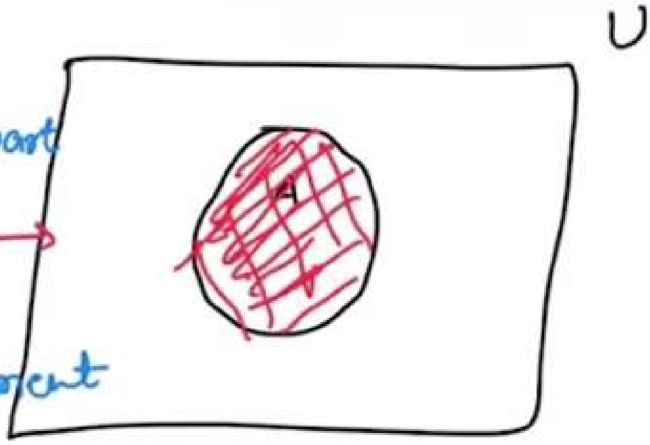
Complement of a set: A' A^c

Remaining part is

~~Comp~~ called

the Complement

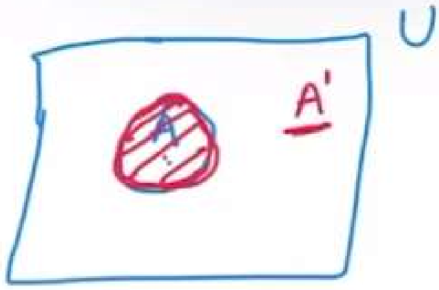
c



Properties:

(1) $A \cup A' = U$

$A \cap A' = \phi$



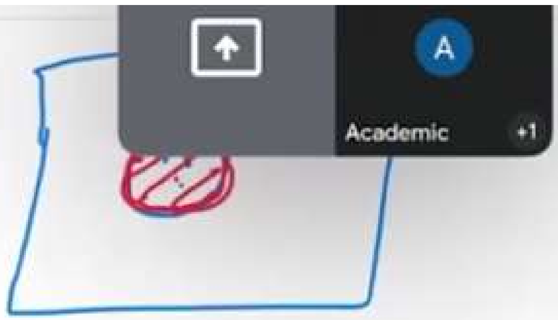
(2) $(A \cup B)' =$

$(A \cap B)' =$

Properties:

(1) $A \cup A' = U$

$A \cap A' = \phi$



(2) $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

(3) $\underline{\cap}' = \underline{\cup}$

$\underline{\cup}' = \underline{\cap}$