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Bridge Class

(Fundamentals of Electrical & Electronics Engineering)

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Kirchhoff's Law

- Kirchhoff's Voltage Law (KVL)
- Kirchhoff's Current Law (KCL)

Kirchhoff's Voltage Law

- Kirchhoff's voltage law tells us how to handle voltages in an electric circuit.
- Kirchhoff's voltage law basically states that the algebraic sum of the voltages around any closed path (electric circuit) equals zero.
- There are three ways we can interpret that the algebraic sum of the voltages around a closed path equals zero. This is similar to what we encountered with Kirchhoff's current law.

Kirchhoff's Voltage Law

Consideration 1. We define a voltage drop as positive if we enter the positive terminal and leave the negative terminal.

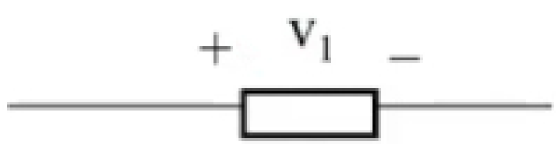


Figure 1.2

The drop moving from left to right above is $+v_1$.

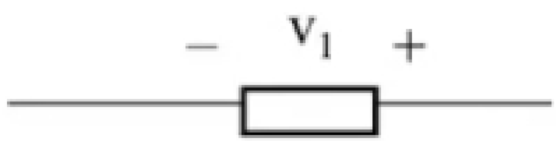


Figure 1.3

The drop moving from left to right above is $-v_1$.

Kirchhoff's Voltage Law

Consider the circuit of Figure 1.4 once again. If we sum the voltage drops in the clockwise direction around the circuit starting at point "a" we write:

$$- v_1 - v_2 + v_4 + v_3 = 0 \qquad \text{drops in CW direction starting at "a"}$$

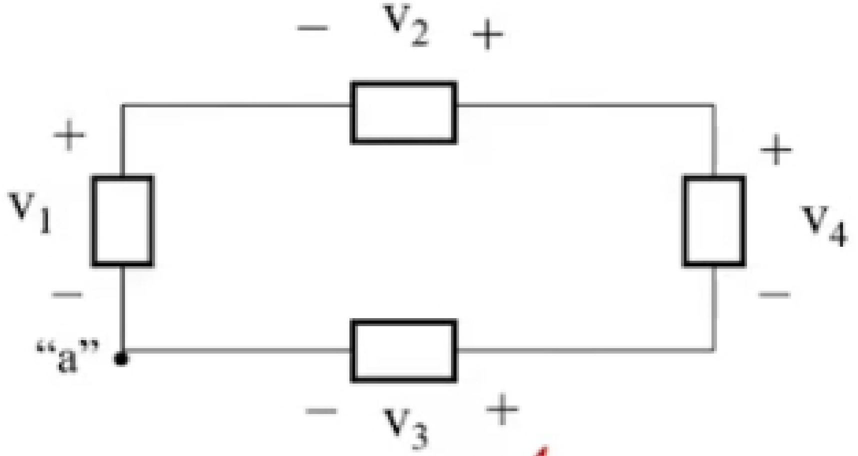


Figure 1.4

$$- v_3 - v_4 + v_2 + v_1 = 0 \qquad \text{drops in CCW direction starting at "a"}$$

Kirchhoff's Voltage Law

Consideration 2: The Sum of the voltages around a circuit is equal to zero. We first define a drop.

We define a voltage rise in the following diagrams:

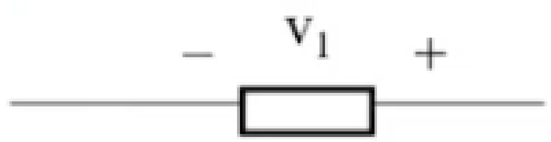


Figure 1.5

The voltage rise in moving from left to right above is $+ v_1$.

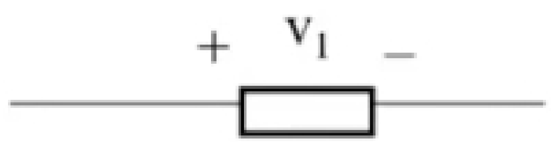


Figure 1.6

The voltage rise in moving from left to right above is $- v_1$.

Kirchhoff's Voltage Law

Consider the circuit of Figure 1.7 once again. If we sum the voltage rises in the clockwise direction around the circuit starting at point "a" we write:

$+ v_1 + v_2 - v_4 - v_3 = 0$ rises in the CW direction starting at "a"

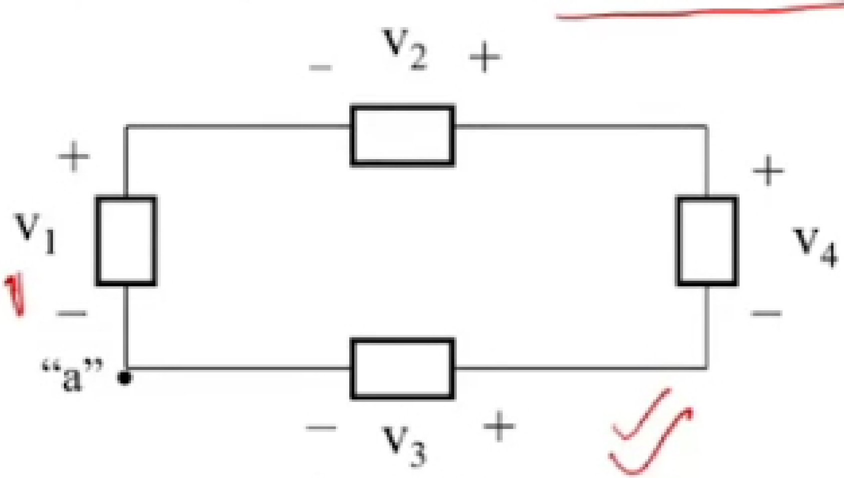


Figure 1.7

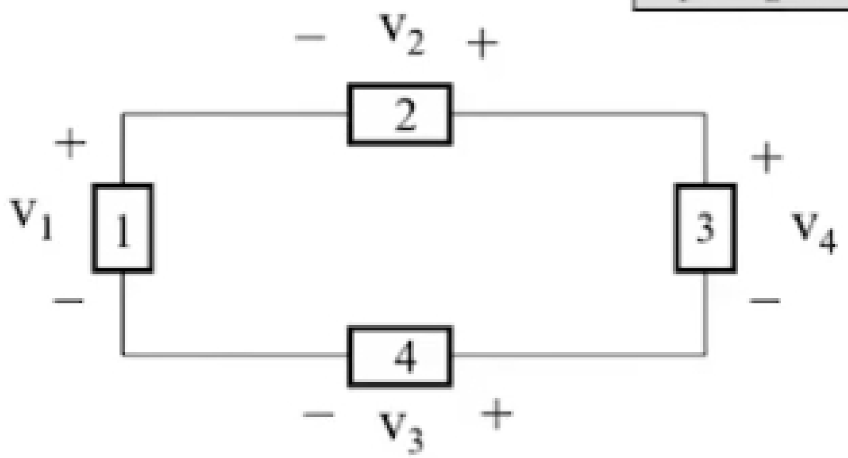
$+ v_3 + v_4 - v_2 - v_1 = 0$ rises in the CCW direction starting at "a"

Kirchhoff's Voltage Law

Consideration 3: The Sum of the voltage rises around a circuit equals the sum of the voltage drops.

Again, consider the circuit of Figure 1.1 in which we start at point "a" and move in the CW direction. As we cross elements 1 & 2, we use voltage rise; as we cross elements 4 & 3, we use voltage drops. This gives the equation,

$$\boxed{v_1 + v_2 = v_4 + v_3}$$



Kirchhoff's Voltage Law

- We note that a positive voltage drop = a negative voltage rise.
- We note that a positive voltage rise = a negative voltage drop.
- We do not need to dwell on the above tongue-twisting statements.
- There are similarities in the way we state Kirchhoff's voltage and Kirchhoff's current laws: algebraic sums …

However, one would <u>*never*</u> say that the sum of the voltages entering a junction point in a circuit is equal to zero.

Likewise, one would *never* say that the sum of the currents around a closed path in an electric circuit equals zero.

Kirchhoff's Voltage Law

For the circuit of Figure 1.8 there are a number of closed paths. Three have been selected for discussion.

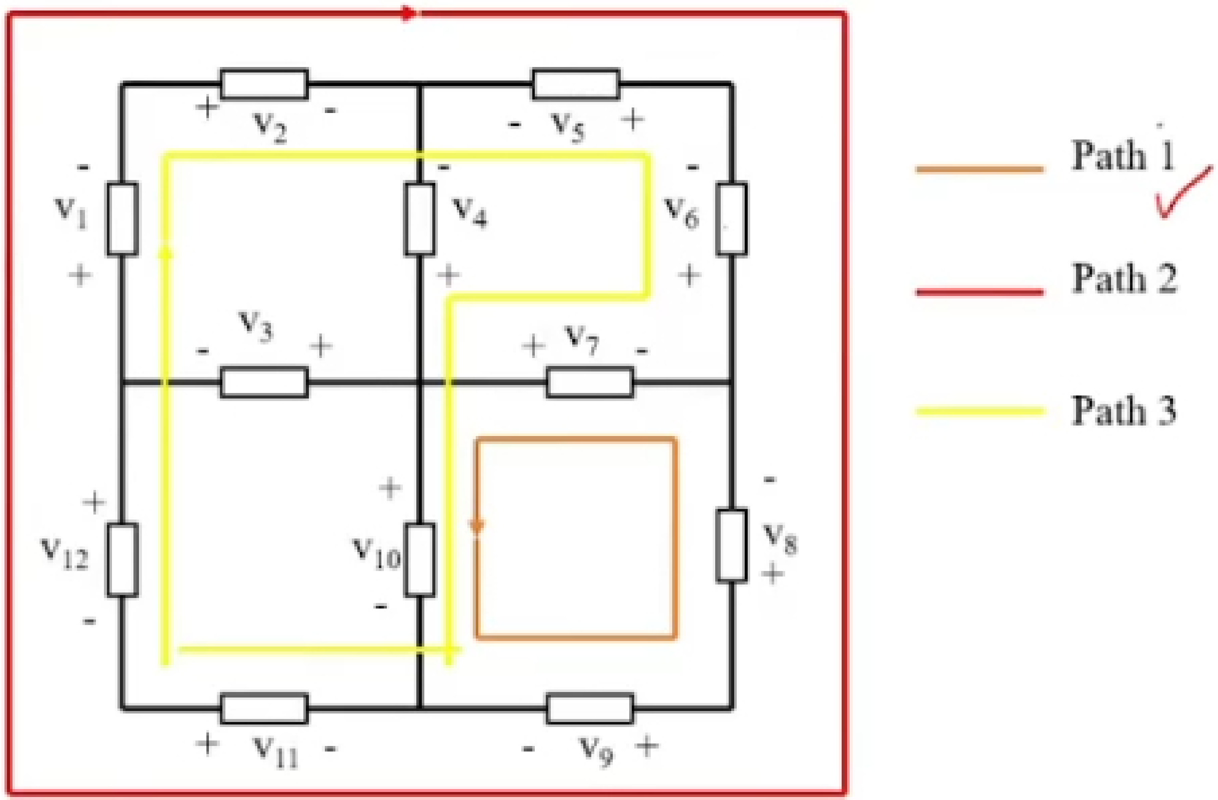


Figure 1.8
Multi-path
Circuit.

Kirchhoff's Voltage Law

Further details.

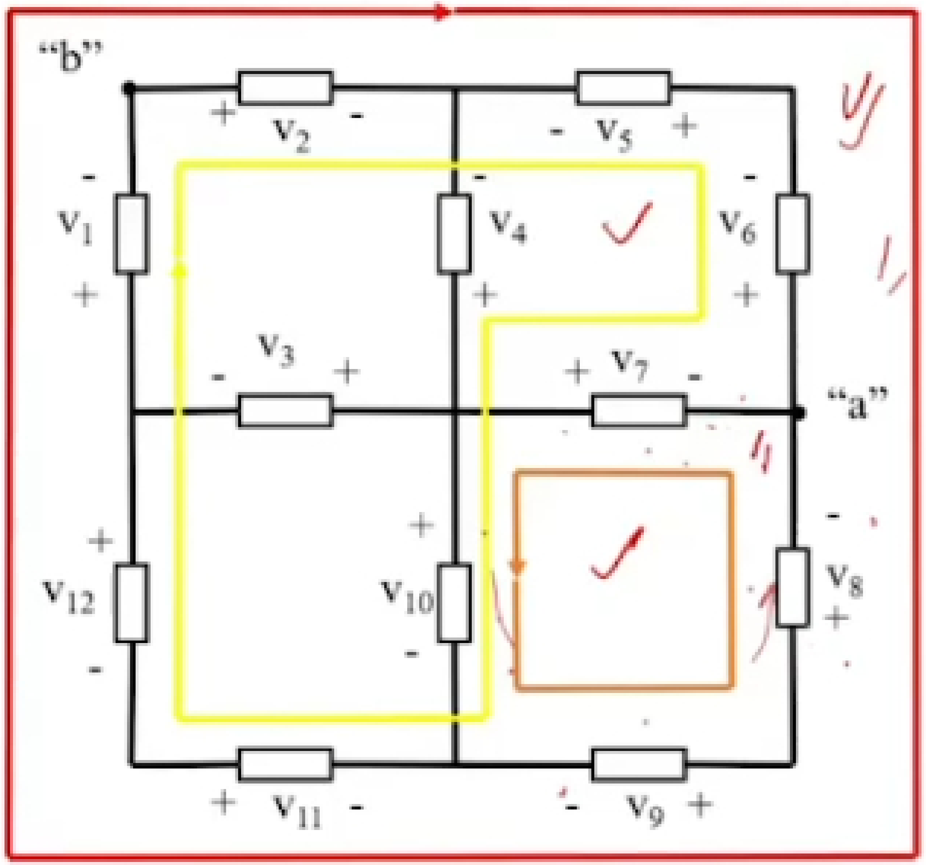
1 For any given circuit, there are a fixed number of closed paths that can be taken in writing Kirchhoff's voltage law and still have linearly independent equations.

2 Both the starting point and the direction in which we go around a closed path in a circuit to write Kirchhoff's voltage law are arbitrary. However, one must end the path at the same point from which one started.

Conventionally, in most texts, the sum of the voltage drops is normally equal to zero when applying Kirchhoff's voltage law.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Illustration from Figure 1.8.



Using sum of the drops = 0

Orange path, starting at "a"

$- v_7 + v_{10} - v_9 + v_8 = 0$

Red path, starting at "b"

$+v_2 - v_5 - v_6 - v_8 + v_9 - v_{11} - v_{12} + v_1 = 0$

Yellow path, starting at "b"

$+ v_2 - v_5 - v_6 - v_7 + v_{10} - v_{11} - v_{12} + v_1 = 0$

Kirchhoff’s Voltage Law

Kirchhoff’s Voltage Law: Double subscript notation.

Voltages in circuits are often described using double subscript notation.

Consider the following:

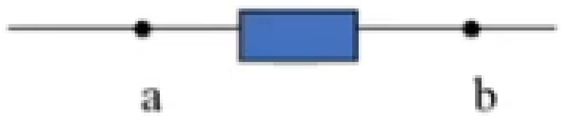


Figure 3.9: Illustrating double subscript notation.

V_{ab} means the potential of point a with respect to point b with point a assumed to be at the highest (+) potential and point b at the lower (-) potential.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Double subscript notation.

Task: Write Kirchhoff's voltage law going in the clockwise direction for the diagram in Figure 3.10.

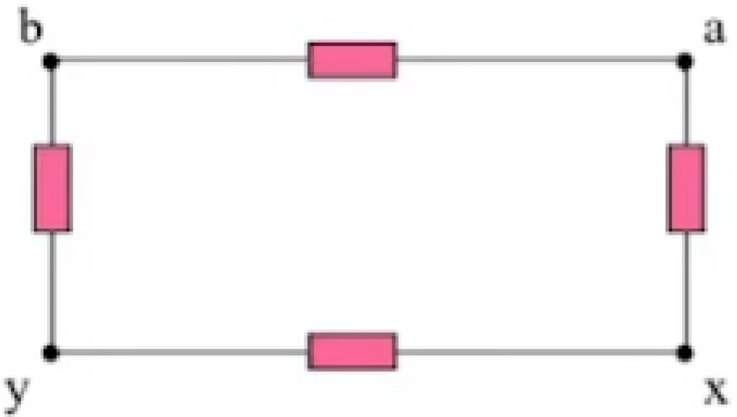


Figure 3.10: Circuit for illustrating double subscript notation.

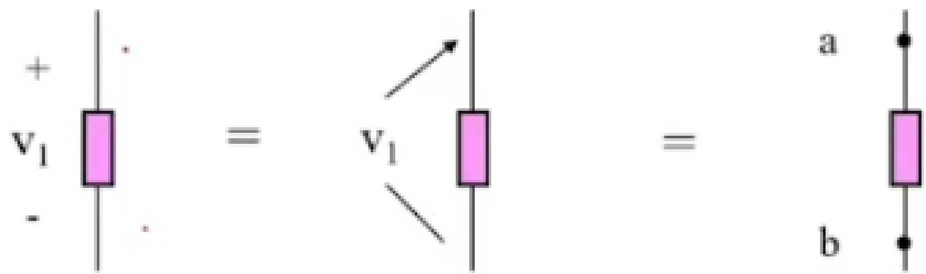
Going in the clockwise direction, starting at "b", using rises;

$$v_{ab} + v_{xa} + v_{yx} + v_{by} = 0$$

Kirchhoff's Voltage Law

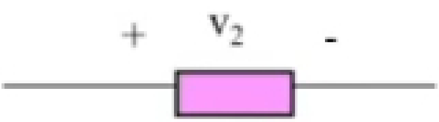
Kirchhoff's Voltage Law: Equivalences in voltage notations

The following are equivalent in denoting polarity.



$v_{ab} = v_1$

Assumes the upper terminal is positive in all 3 cases



$v_2 =$ - 9 volts means the right hand side of the element is actually positive.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Application.

Given the circuit of Figure 1.11. Find V_{ad} and V_{fc}.

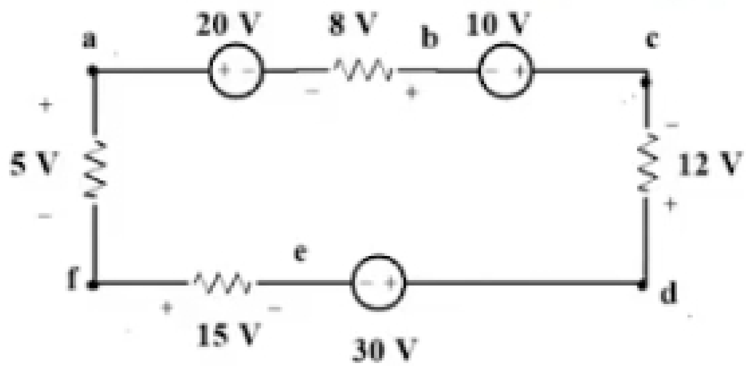


Figure 1.11: Circuit for illustrating KVL.

Using drops = 0; $V_{ad} + 30 - 15 - 5 = 0 \longrightarrow V_{ab} = -10\text{ V}$

$V_{fc} - 12 + 30 - 15 = 0 \longrightarrow V_{fc} = -3\text{ V}$

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Single-loop circuits.

We are now in a position to combine Kirchhoff's voltage and current Laws to the solution of single loop circuits. We start by developing the *Voltage Divider Rule.* Consider the circuit of Figure 1.12.

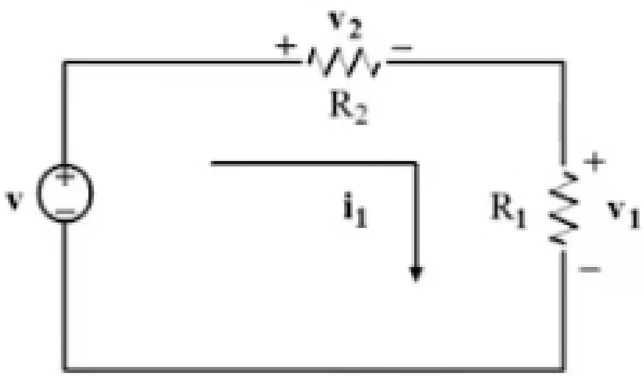


Figure 1.12: Circuit for developing voltage divider rule.

$$v = v_1 + v_2$$

$$v_1 = i_1 R_1, \quad v_2 = i_1 R_2$$

then,

$$v = i_1(R_1 + R_2) \text{ , and } \quad i_1 = \frac{v}{(R_1 + R_2)}$$

so,

$$\boxed{v_1 = \frac{vR_1}{(R_1 + R_2)} \; *}$$

* You will be surprised by how much you use this in circuits.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Single-loop circuits.

Find V_1 in the circuit shown in Figure 1.13.

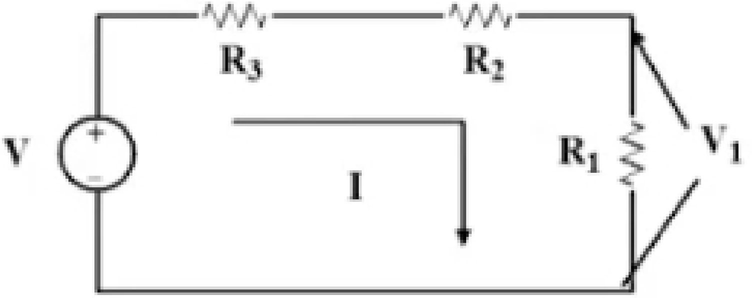


Figure 1.13

$$I = \frac{V}{(R_1 + R_2 + R_3)}$$

$$V_1 = IR_1 \quad ,so, we\ have$$

$$\boxed{V_1 = \frac{VR_1}{(R_1 + R_2 + R_3)}}$$

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Single-loop circuits.

Example 1.1: For the circuit of Figure 1.14, the following is known:

$R_1 = 4$ ohms, $R_2 = 11$ ohms, $V = 50$ volts, $P_1 = 16$ watts

Find R_3.

Solution:

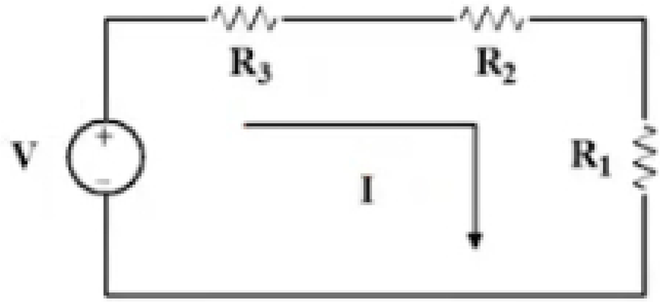


Figure 1.14: Circuit for example 3.1.

$P_1 = 16$ watts $= I^2R_1$, thus,

$I = 2$ amps

$V = I(R_1 + R_2 + R_3)$, giving,

$R_1 + R_2 + R_3 = 25$, then solve for R_3,

$R_3 = 25 - 15 = 10$ ohms

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Single-loop circuits.

Example 1.2: For the circuit in Figure 1.15 find I, V_1, V_2, V_3, V_4 and the power supplied by the 10 volt source.

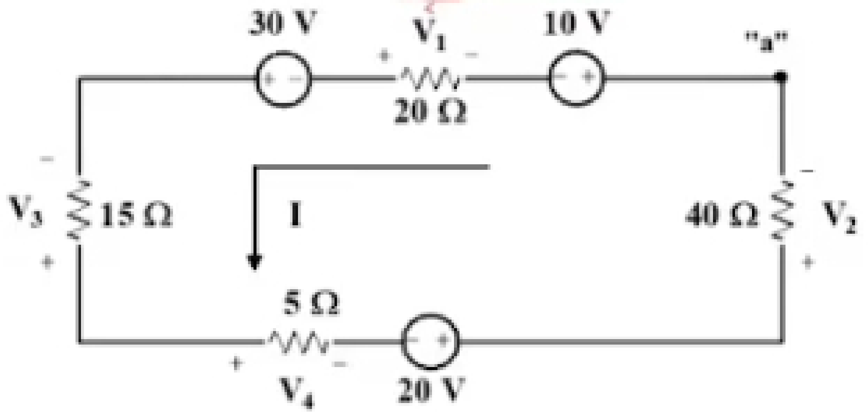


Figure 1.15: Circuit for example 1.2.

For convenience, we start at point "a" and sum voltage drops =0 in the direction of the current I.

$$+10 - V_1 - 30 - V_3 + V_4 - 20 + V_2 = 0 \qquad \text{Eq. 1.1}$$

Kirchhoff's Voltage Law

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits. Ex. 1.2 cont.

We note that: $V_1 = -20I$, $V_2 = 40I$, $V_3 = -15I$, $V_4 = 5I$ Eq. 1.2

We substitute the above into Eq. 1.1 to obtain Eq. 1.3 below.

$$10 + 20I - 30 + 15I + 5I - 20 + 40I = 0 = 0 \quad \text{Eq. 1.3}$$

Solving this equation gives, $I = 0.5$ A.

Using this value of I in Eq. 1.2 gives;

$V_1 = -10$ V $\quad$ $V_3 = -7.5$ V

$V_2 = 20$ V $\quad$ $V_4 = 2.5$ V

$P_{10(supplied)} = -10I = -5$ W

(We use the minus sign in $-10I$ because the current is entering the + terminal)
In this case, power is being absorbed by the 10 volt supply.

Kirchhoff's Voltage Law

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits, Equivalent Resistance.

Given the circuit of Figure 1.16. We desire to develop an equivalent circuit as shown in Figure 1.17. Find V_s and R_{eq}.

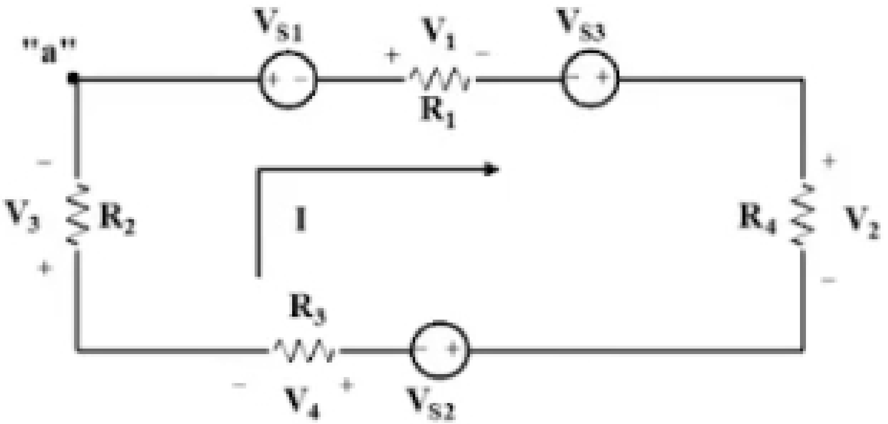


Figure 1.16: Initial circuit for development.

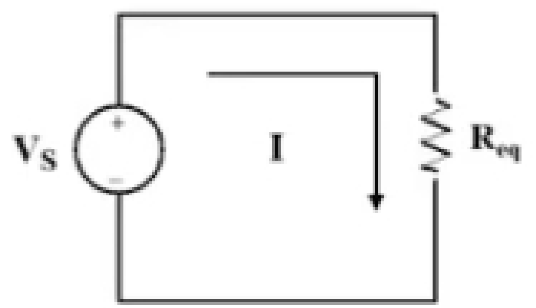


Figure 1.17: Equivalent circuit for Figure 3.16

Kirchhoff's Voltage Law

<u>Kirchhoff's Voltage Law</u>: Single-loop circuits, Equivalent Resistance.

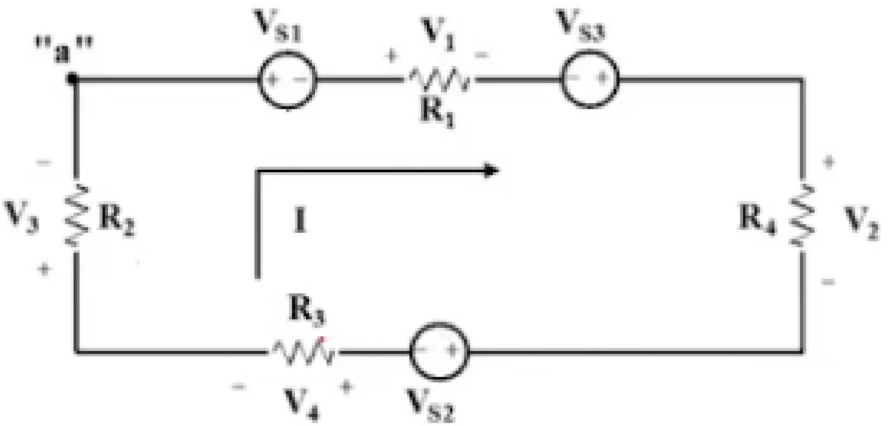


Figure 1.16: Initial circuit.

Starting at point "a", apply KVL going clockwise, using drops = 0, we have

$$V_{S1} + V_1 - V_{S3} + V_2 + V_{S2} + V_4 + V_3 = 0$$

or

$$-V_{S1} - V_{S2} + V_{S3} = I(R_1 + R_2 + R_3 + R_4) \qquad \text{Eq. 1.4}$$

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law: Single-loop circuits, Equivalent Resistance.

Consider again, the circuit of Figure 1.17.

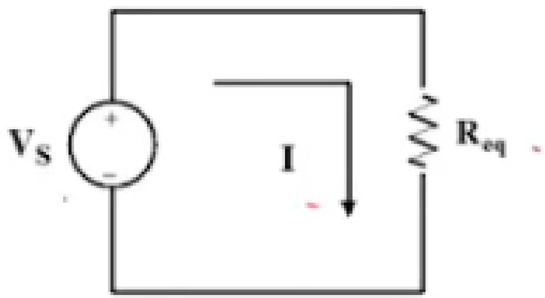


Figure 1.17: Equivalent circuit of Figure 1.16.

Writing KVL for this circuit gives;

$V_S = IR_{eq}$ compared to $-V_{S1} - V_{S2} + V_{S3} = I(R_1 + R_2 + R_3 + R_4)$

Therefore;

$$\boxed{V_S = -V_{S1} - V_{S2} + V_{S3}\,;} \qquad \boxed{R_{eq} = R_1 + R_2 + R_3 + R_4} \qquad \text{Eq. 1.5}$$

Kirchhoff’s Voltage Law

Kirchhoff’s Voltage Law: Single-loop circuits.

Example 1.3: Find the current I in the circuit of Figure 1.18.

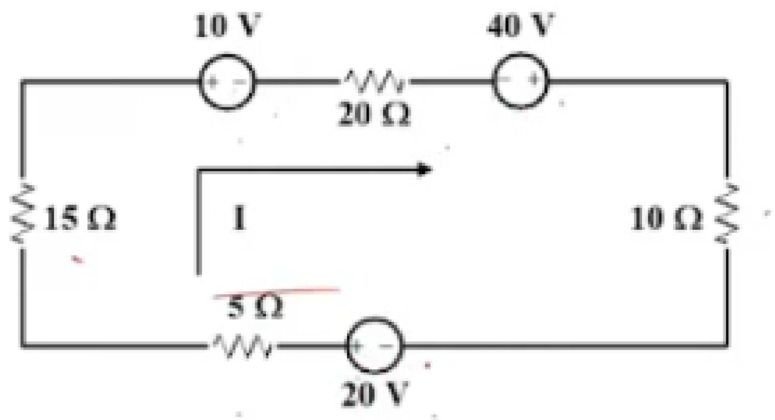
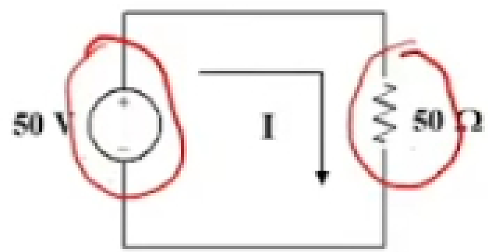


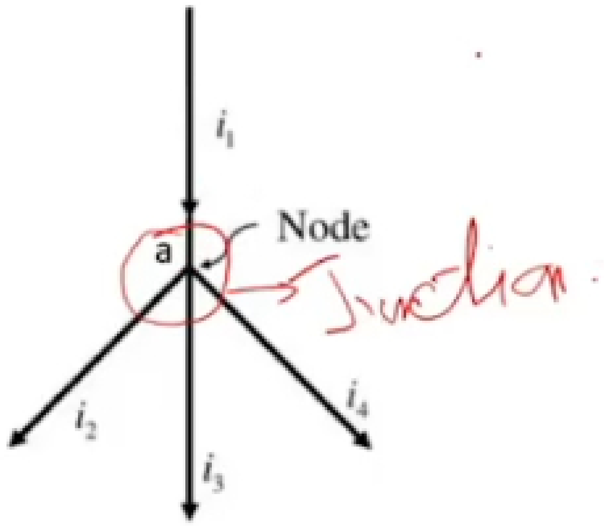
Figure 1.18: Circuit for example 1.3.

From the previous discussion we have the following circuit.



Therefore, I = 1 A

Kirchhoff's Current Law



Kirchhoff's current law (or Kirchhoff's current rule) results from the conservation of charge. It applies to a junction or node in a circuit (a point in the circuit where charge has several possible paths to travel).

In the figure, we see that i_1 is the only current flowing *into* the node. However, there are three paths for current to leave the node, and these currents are represented by i_2, i_3, and i_4.

Once charge has entered into the node, it has no place to go except to leave: this is why we speak about conservation of charge. The total charge flowing *into* a node must be the same as the the total charge flowing *out of* the node. So,

$$i_1 = i_2 + i_3 + i_4$$

Bringing everything to the left side of the above equation, we get

$$i_1 - i_2 - i_3 - i_4 = 0$$

Then, the sum of all the currents is zero. This can be generalized as follows $\sum i_{in} - \sum i_{out} = 0$