DAY-1

NUMBER SYSTEM & BINOMIAL ALGEBRA

Dr. Akshay Kumar

- N: Natural Numbers : 1, 2, 3 ...
- W: Whole Numbers : 0, 1, 2, ...
- Z: Integers : ... -3, -2, -1, 0, 1, 2, 3 ...
- Rational nos (Q) $\frac{p}{q}$, $p, q \in Z$ & $q \neq 0$ 1, -5, 0, $\frac{101}{202}$, ...
- Irrational $Q^* \rightarrow \times\left(\frac{p}{q}, p, q \in Z \;\&\; q \neq 0\right) \times$ $\sqrt{2}, \sqrt{3}, 0.2\bar{3}$ ...
- All rational + irrational = Real Nos. (R).
- Complex Number : $a+ib$, $a, b \in R$ $i = \sqrt{-1} \Rightarrow i^2 = -1$
  $i^3 = -i$
  $i^4 = 1$

Diff. b/w Real & complex nos. (Major)

x, y — Real

Then

(I) $x > y$
(II) $x = y$
(III) $x < y$

exactly any one of these three is true for x, y Real nos. → We can Compare Real numbers.

What about complex nos.?? → Not comparable

Prop. 1 : if $x > y$ and $a > 0$, then $\boxed{ax > ay}$.

Result :

$i \neq 0$ $i = \sqrt{-1}$
$i^2 = -1$

① $i > 0$

$i \cdot i > i \cdot 0$

$i^2 > 0$

$-1 > 0$ × false

×

② $i < 0 \Rightarrow -i > 0$ we can multiply a positive no. on both side

$\cancel{i \cdot i < i \cdot 0}$ $-i \cdot i < 0 \cdot (-i)$

$-i \cdot i < 0 \cdot (-i)$

$-i^2 < 0$

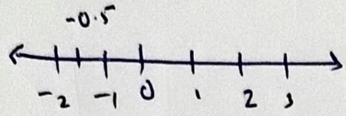
$-(-1) < 0$

$1 < 0$ × false

×

complex nos. are not comparable

Real Number



we cannot represent complex no. on number line.

Properties:

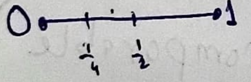
Every Real Number is a Complex no.

⇓

$10 \rightarrow 10 + 0i$

1. Sum of 2 nat. nos = natural
2. diff of 2 nat. nos = Cannot decide (int)
3. integer + integer = integer
4. integer − integer = integer
5. Rational ± Rational = Rational
6. Irrational ± Irrational = cannot decide
7. Rational + Irrational = Irrational
8. Rational x Rational = Rational
9. Irration x Irrational = Cannot decide
10. Irration x Rational = Irrational
11. Complex + Compl = Complex $(5+i)+(5-i) = 10+0i$
12. complex no x Complex no. = Complex no.

How many Rational numbers are theire b/w 0 & 1?



$1 > \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{2}, \frac{1}{3} \ldots > 0$

∀ n-natural no.

$\boxed{0 < \frac{1}{n} < 1}$

Irrational No.

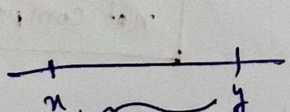
$0 < \frac{\sqrt{2}}{n} < 1$

→ natural no $n \geq 2$

∴ b/w 0 & 1 we can find infinitely many ~~natu~~ rational & irration nos.

$x, y \rightarrow$ Real no

$x \neq y$, $x < y$



we can find infinitely many rational or Irrational nos.

ARCHIMEDEAN PROPERTY

① For any Real no. x we can find a natural number n. s.t. $n > x$

② For any Real no. $x > 0$, we can find a natural no. n, s.t. $0 < \frac{1}{n} < x$

SETS — Well defined Collection of Objects

E.g. → N, Z, W, Q,

C = {Collection of all good persons} → X not a set.

* Element of set — members of set ($1 \in N$) ($2 \in N$) ($0 \notin N$) ($0 \in Q$)

* Representation of a set:
  * Set Builder Form
    $N = \{1, 2, 3, \ldots\}$
    $A = \{1, 2, 3, 4\}$
  * Roster form : $A = \{x : x$ is a Natural no & $x \leq 4\}$

B = {a, e, i, o, u}

B = {vowels of eng alphabet}

* Empty set : ϕ = No element.

* Finite & Infinite set : $A = \{1, 2, 3, 4\}$ — $n(A) = 4$

or cardinality of $A = 4$

$|A|$ = (Finite)

$N = \{1, 2, 3, \ldots\}$ → $n(A) = ND(\infty)$

cardinality of $(N) = |N| = \infty$

(infinite set)

* Equal sets, Subsets, superset

proper subset

$A = B$

$\{1,2,3\} = \{1,1,2,3\}$

⇓

$A \subseteq B$ & $B \subseteq A$

$A \subseteq B$

A is a subset of B

[All elements of A are present in B]

(A may be equal to B)

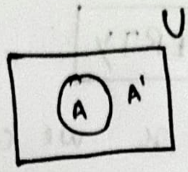
B is a superset of A

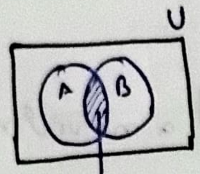
$A \subset B$ → A is a subset of B but not equal to B → B is a proper superset of A

[Every set is a subset of itself]

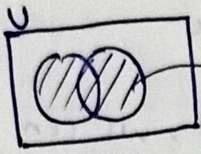
*but not proper subset

Venn Diagram





A∩B



AUB

Operation on Sets : A B

① $A \cup B = \{x \in A \text{ or } x \in B\}$

$A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

② $A \cap B = \{x \in A \text{ and } x \in B\}$

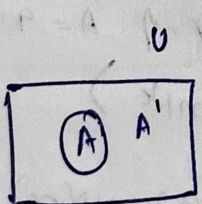
$A \cap B = \{1, 2, 3\}$

$A \times B = \{(a, b) : a \in A \text{ \& } b \in B\}$

$A|_B = A - B = \{x \in A \text{ but } x \notin B\}$

Complement of set : $\boxed{A' = U - A}$

$A' = \{x \notin A\}$



Properties

① $A \cup A' = U$

$A \cap A' = \emptyset$

② $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

③ $\emptyset' = U$

$U' = \emptyset$