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Problem Statement

To process income tax forms, the IRS first sends each form through the data preparation (DP) department, where information is coded for computer entry. Then the form is sent to data entry (DE), where it is entered into the computer.

During the next three weeks, the following number of forms will arrive: week 1, 40,000; week 2, 30,000; week 3, 60,000.

The IRS meets the crunch by hiring employees who work 40 hours per week and are paid $200 per week. Data preparation of a form requires 15 minutes, and data entry of a form requires 10 minutes. Each week, an employee is assigned to either data entry or data preparation. The IRS must complete processing of all forms by the end of week 5 and wants to minimize the cost of accomplishing this goal. Formulate an LP that will determine how many workers should be working each week and how the workers should be assigned over the next five weeks.

Verbal Formulation

The problem can be broken down into following sub points:

- IRS send a form first to Data Preparation department and then to Data Entry department. Hence the order is important on any week the number of form whose Data Entry is performed will have to be less than equal to number of forms whose data is processed.
- Data Preparation takes 15 mins per form while Data Entry takes 10 mins per form.
- The demand or number of forms received are for week 1,2 and 3. However extension is given to complete all the task by end of 5 weeks.
- Employees are allocated either Data Preparation task or Data Entry task and not both.
- An Employee can work only for 40 hours (2400 mins) per week. The cost per employee is $200 per week.
- The objective is to minimise the cost of processing all forms within the deadline of 5 weeks and analyse the number of workers required every week per department.

It is to be noted that there can be 2 approach to solving this problem that is considering :

1. **Hiring for employees are done on a weekly basis as per requirement.**
2. **All hiring are done at once and for 5 weeks the employees are distributed in numbers**

APPROACH 1

HIRING OF EMPLOYEES PERFORMED WEEKLY

Mathematical Model

Decision Variables:

- Number of Employees for Data Entry for i-th week : DEEmpi (for i from 1 to 5)
- Number of Employees for Data Processing for i-th week : DPEmpi (for i from 1 to 5)
- Number of Forms processed for Data Entry on i-th week : FormDEi (for i from 1 to 5)
- Number of Forms processed for Data Preparation on i-th week : FormDPi (for i from 1 to 5)
- Number of Forms in Inventory left for Data Entry on i-th week : InvDEi (for i from 1 to 4)
- Number of Forms in Inventory for Data Preparation on i-th week : InvDPi (for i from 1 to 4)

Note: There are no inventory of forms for week 5.

Constraints:

Time Constraints:

- The amount of time taken for data of forms to be entered must not exceed the amount of time employees allocated for data entry have to offer per week.
- The amount of time taken for data of forms to be prepared must not exceed the amount of time employees allocated for data preparation have to offer per week.

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Demand Constraints:

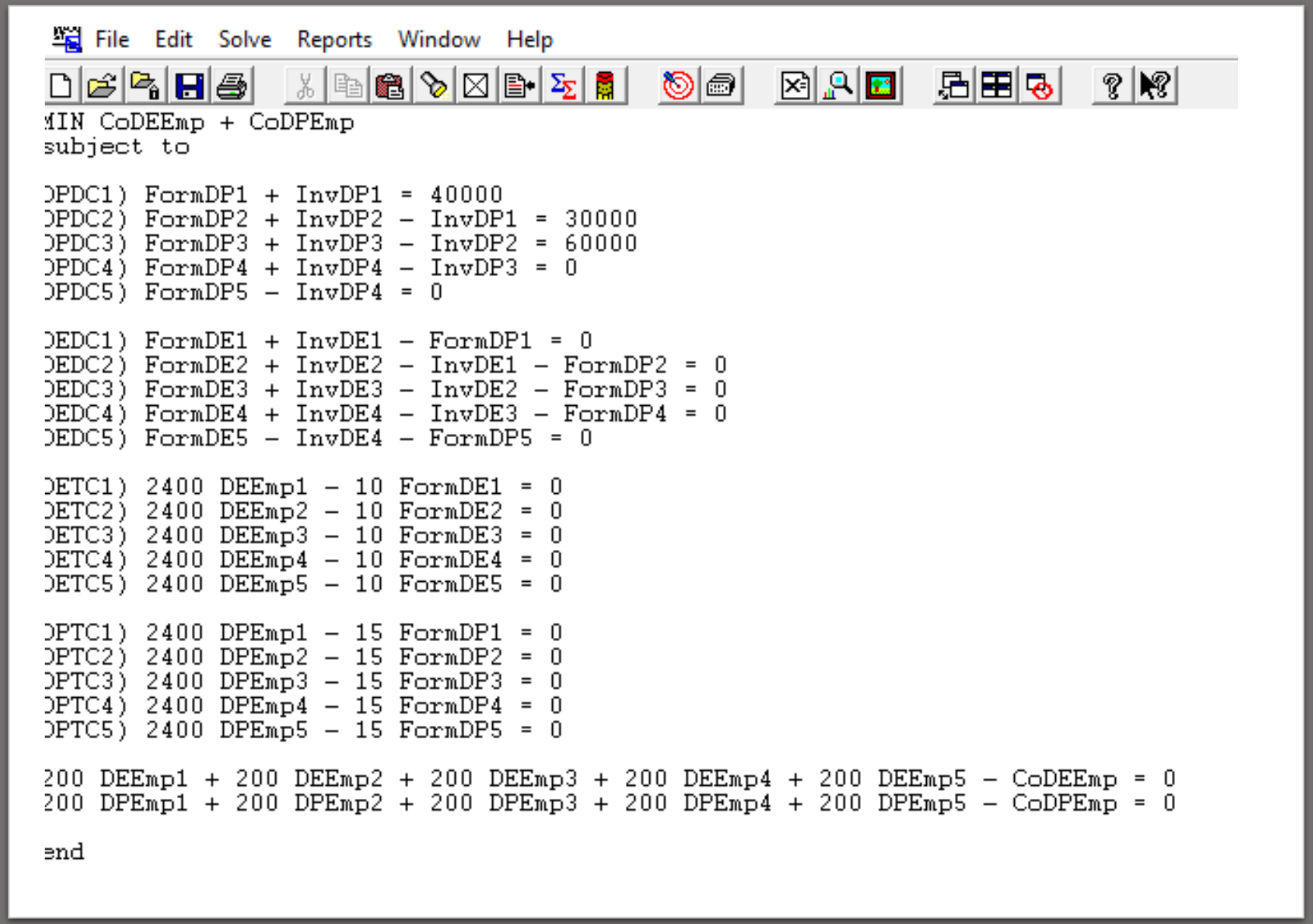
- The number of forms whose data is processed must not increase the number of freshly arrived forms and the inventory left from previous week.
- The number of forms whose data is entered must not increase the **number of forms whose data is processed that week** and the inventory left from previous week.

Non-negative Constraints:

- All Decision Variables should be greater than equal to zero

Objective Function:

The objective is to minimise the cost incurred on the employees. That is every employee is paid $200 per week.
Hence objective function is to minimise (Cost of Salary for Employee allocated for Data Entry + Cost of Salary for Employee allocated for Data Preparation)



LINDO based problem

LINDO Solution

LP OPTIMUM FOUND AT STEP 7

OBJECTIVE FUNCTION VALUE

1) 270833.3

VARIABLE	VALUE	REDUCED COST
CODEEMP	108333.335938	0.000000
CODPEMP	162500.000000	0.000000
FORMDP1	40000.000000	0.000000
INVDP1	0.000000	0.000000
FORMDP2	30000.000000	0.000000
INVDP2	0.000000	0.000000
FORMDP3	0.000000	0.000000
INVDP3	60000.000000	0.000000
FORMDP4	0.000000	0.000000
INVDP4	60000.000000	0.000000
FORMDP5	60000.000000	0.000000
FORMDE1	40000.000000	0.000000
INVDE1	0.000000	0.000000
FORMDE2	30000.000000	0.000000
INVDE2	0.000000	0.000000
FORMDE3	0.000000	0.000000
INVDE3	0.000000	0.000000
FORMDE4	0.000000	0.000000
INVDE4	0.000000	0.000000
FORMDE5	60000.000000	0.000000
DEEMP1	166.666672	0.000000
DEEMP2	125.000000	0.000000
DEEMP3	0.000000	0.000000
DEEMP4	0.000000	0.000000
DEEMP5	250.000000	0.000000
DPEMP1	250.000000	0.000000
DPEMP2	187.500000	0.000000
DPEMP3	0.000000	0.000000
DPEMP4	0.000000	0.000000
DPEMP5	375.000000	0.000000

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ROW	SLACK OR SURPLUS	DUAL PRICES
DPDC1)	0.000000	-2.083333
DPDC2)	0.000000	-2.083333
DPDC3)	0.000000	-2.083333
DPDC4)	0.000000	-2.083333
DPDC5)	0.000000	-2.083333
DEDC1)	0.000000	-0.833333
DEDC2)	0.000000	-0.833333
DEDC3)	0.000000	-0.833333
DEDC4)	0.000000	-0.833333
DEDC5)	0.000000	-0.833333
DETC1)	0.000000	-0.083333
DETC2)	0.000000	-0.083333
DETC3)	0.000000	-0.083333
DETC4)	0.000000	-0.083333
DETC5)	0.000000	-0.083333
DPTC1)	0.000000	-0.083333
DPTC2)	0.000000	-0.083333
DPTC3)	0.000000	-0.083333
DPTC4)	0.000000	-0.083333
DPTC5)	0.000000	-0.083333
22)	0.000000	1.000000
23)	0.000000	1.000000

NO. ITERATIONS= 7

Report

WEEK	NUMBER OF NEW FORMS	PREVIOUS WEEK INVENTORY FORM FOR DATA PREPARATION	NUMBER OF FORM FOR WHICH DATA PREPARED	NUMBER OF EMPLOYEES HIRED FOR DATA PREPARATIO N	PREVIOUS WEEK INVENTORY FORM FOR DATA ENTRY	NUMBER OF FORM FOR WHICH DATA ENTERED	NUMBER OF EMPLOYEES HIRED FOR DATA ENTRY	COST INCURRED AS SALARY ($)
1	40,000	0	40,000	250	0	40,000	166.66	83,332
2	30,000	0	30,000	187.5	0	30,000	125	62,500
3	60,000	0	0	0	0	0	0	0
4	0	60,000	0	0	60,000	0	0	0
5	0	60,000	60,000	375	60,000	60,000	250	1,25,000
TOTAL	1,30,000		1,30,000	812.5		1,30,000	541.66	2,70,832

Approach 2

Hiring all the employees at once and then distributing them for task every week

Mathematical Model

Decision Variables:

- Total Number of Employees hired : TotEmp
- Number of Employees for Data Entry for i-th week : DEEmpi (for i from 1 to 5)
- Number of Employees for Data Processing for i-th week : DPEmpi (for i from 1 to 5)
- Number of Forms processed for Data Entry on i-th week : FormDEi (for i from 1 to 5)
- Number of Forms processed for Data Preparation on i-th week : FormDPi (for i from 1 to 5)
- Number of Forms in Inventory left for Data Entry on i-th week : InvDEi (for i from 1 to 4)
- Number of Forms in Inventory for Data Preparation on i-th week : InvDPi (for i from 1 to 4)

Note: There are no inventory of forms for week 5.
For every week i the sum of employee assigned to data entry and prepration must be equal to total employee
(DEEmpi + DPEmpi = TotEmp)

Constraints:

Time Constraints:

- The amount of time taken for data of forms to be entered must not exceed the amount of time employees allocated for data entry have to offer per week.
- The amount of time taken for data of forms to be prepared must not exceed the amount of time employees allocated for data preparation have to offer per week.

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Demand Constraints:

- The number of forms whose data is processed must not increase the number of freshly arrived forms and the inventory left from previous week.
- The number of forms whose data is entered must not increase the **number of forms whose data is processed that week** and the inventory left from previous week.

Employee Constraints:

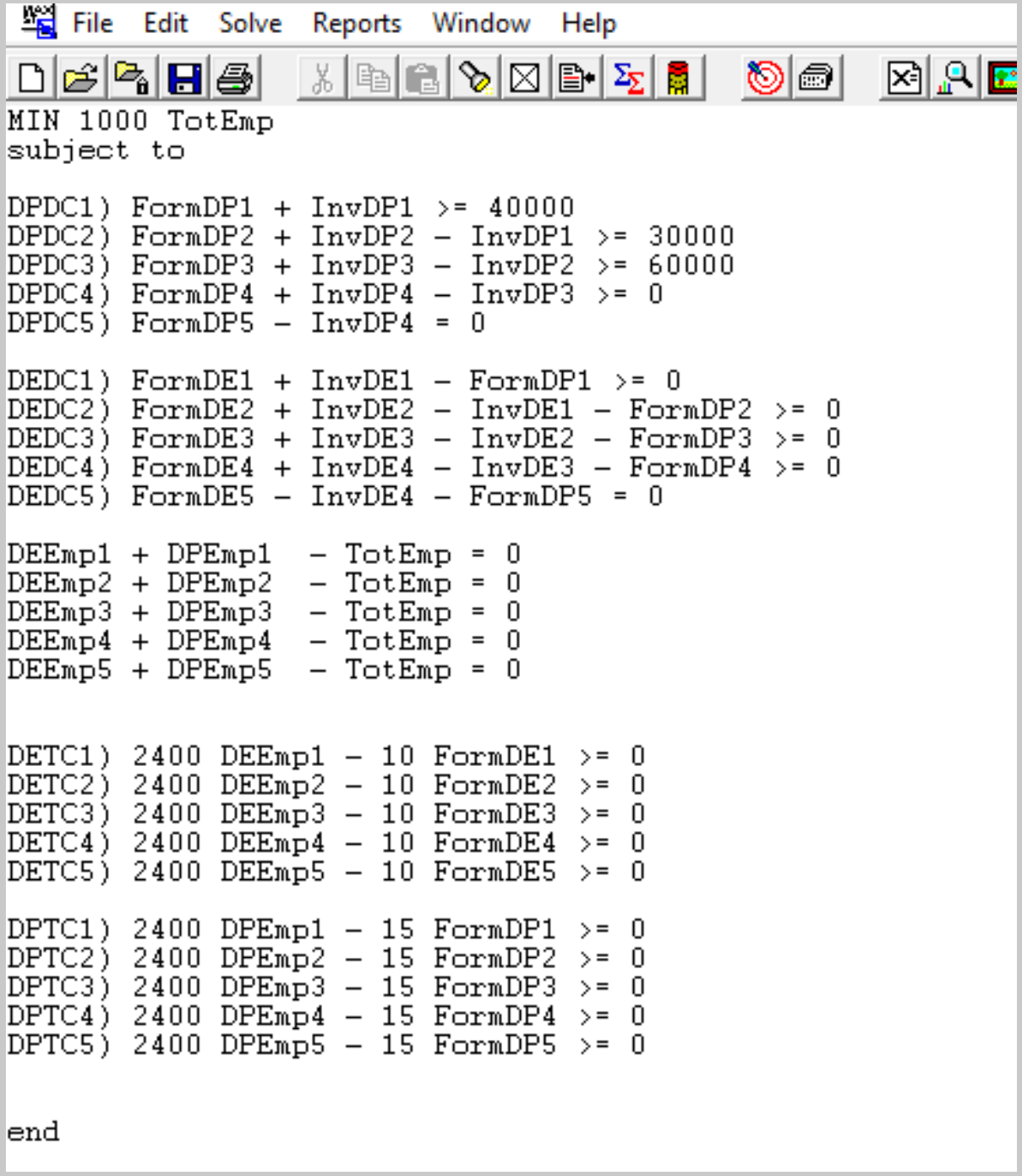
- For every week i the sum of employee assigned to data entry and prepration must be equal to total employee **(DEEmpi + DPEmpi = TotEmp)**

Non-negative Constraints:

- All Decision Variables should be greater than equal to zero

Objective Function:

The objective is to minimise the cost incurred on the employees salary. That is every employee is paid $200 per week. Hence objective function is to minimise the number of employee hired and hence their salary that is $1000 for 5 weeks per employee.



LINDO based problem

LINDO Solution

LP OPTIMUM FOUND AT STEP 22

OBJECTIVE FUNCTION VALUE

1) 270833.3

VARIABLE	VALUE	REDUCED COST
TOTEMP	270.833344	0.000000
FORMDP1	40000.000000	0.000000
INVDP1	0.000000	0.000000
FORMDP2	30000.000000	0.000000
INVDP2	0.000000	0.000000
FORMDP3	8000.000000	0.000000
INVDP3	52000.000000	0.000000
FORMDP4	26000.000000	0.000000
INVDP4	26000.000000	0.000000
FORMDP5	26000.000000	0.000000
FORMDE1	5000.000000	0.000000
INVDE1	35000.000000	0.000000
FORMDE2	20000.000000	0.000000
INVDE2	45000.000000	0.000000
FORMDE3	53000.000000	0.000000
INVDE3	0.000000	0.000000
FORMDE4	26000.000000	0.000000
INVDE4	0.000000	0.000000
FORMDE5	26000.000000	0.000000
DEEMP1	20.833334	0.000000
DPEMP1	250.000000	0.000000
DEEMP2	83.333336	0.000000
DPEMP2	187.500000	0.000000
DEEMP3	220.833328	0.000000
DPEMP3	50.000000	0.000000
DEEMP4	108.333336	0.000000
DPEMP4	162.500000	0.000000
DEEMP5	108.333336	0.000000
DPEMP5	162.500000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
DPDC1)	0.000000	-2.083333
DPDC2)	0.000000	-2.083333
DPDC3)	0.000000	-2.083333
DPDC4)	0.000000	-2.083333
DPDC5)	0.000000	-2.083333
DEDC1)	0.000000	-0.833333
DEDC2)	0.000000	-0.833333
DEDC3)	0.000000	-0.833333
DEDC4)	0.000000	-0.833333
DEDC5)	0.000000	-0.833333
12)	0.000000	200.000000
13)	0.000000	200.000000
14)	0.000000	200.000000
15)	0.000000	200.000000
16)	0.000000	200.000000
DETC1)	0.000000	-0.083333
DETC2)	0.000000	-0.083333
DETC3)	0.000000	-0.083333
DETC4)	0.000000	-0.083333
DETC5)	0.000000	-0.083333
DPTC1)	0.000000	-0.083333
DPTC2)	0.000000	-0.083333
DPTC3)	0.000000	-0.083333
DPTC4)	0.000000	-0.083333
DPTC5)	0.000000	-0.083333

NO. ITERATIONS= 22

Report

WEEK	NUMBER OF NEW FORMS	PREVIOUS WEEK INVENTORY FORM FOR DATA PREPARATION	NUMBER OF FORM FOR WHICH DATA PREPARED	NUMBER OF EMPLOYEES FOR DATA PREPARATION	PREVIOUS WEEK INVENTORY FORM FOR DATA ENTRY	NUMBER OF FORM FOR WHICH DATA ENTERED	NUMBER OF EMPLOYEES FOR DATA ENTRY	COST INCURRED AS SALARY ($)
1	40,000	0	40,000	250	0	4800	20	54,166
2	30,000	0	29,920	187	35200	19920	83	54,166
3	60,000	80	8000	50	45,280	53,200	221.66	54,166
4	0	52080	26000	162.5	52,080	25920	108	54,166
5	0	26080	26080	163	26160	26160	109	54,166
TOTAL	1,30,000		1,30,000			1,30,000		2,70,830

Conclusion

It was observed that both the approaches resulted in the same minimise output for the objective function. However for a managerial point of view approach 2 seems more appropriate as it may reduce the recruitment cost (if any) for hiring the suitable candidates.