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Problem Statement

Figure below shows an illustration of an electromechanical device that contains three components in serial arrangement so that each component must work for the system to function. The reliability of the system can be improved by installing several parallel units in one or more of the components. Table 1 shows the reliability of each component as a function of the number of parallel units.

The reliability of the device is the product of the reliabilities of the three components. The cost of installing one, two, or three parallel units in each component (in thousands of dollars) is given in Table 2.

Because of budget limitations, a maximum of $10,000 can be expended. We need to determine the number of units for each component so that the reliability of the system is maximized.

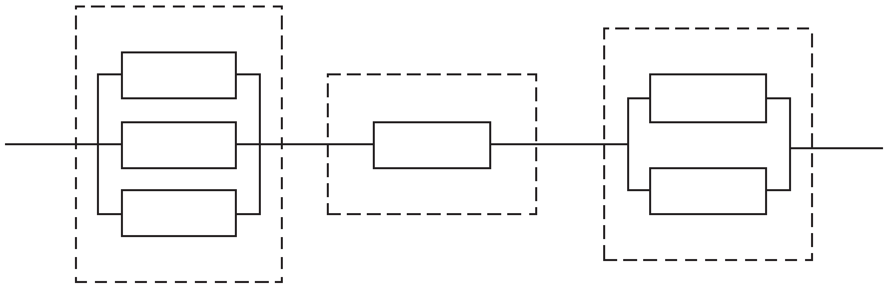


FIGURE Illustration of the electromechanical system.

TABLE 1 Reliability for Each Component

Parallel Units (x_i)	Probability of Functioning $p_i(x_i)$ Component $i=1$	Component $i=2$	Component $i=3$
1	0.6	0.7	0.8
2	0.7	0.8	0.9
3	0.9	0.9	0.95

TABLE 2 Cost of Each Component (in Thousands of Dollars)

Parallel Units (x_i)	Cost of the Component $c_i(x_i)$ Component $i=1$	Component $i=2$	Component $i=3$
1	3	2	1
2	5	4	4
3	6	5	5

Verbal Explanation

We will be decomposing the entire problem into 3 stages as shown in the figure below. Starting from the very last stage and moving across the starting stage or initial stage to obtain the best path possible after back tracking.
At every stage we will be calculating the state value which is the total money left at beginning of the stage for expenditure.
We will hence be calculating the reliability at every stage for each component by adding different number of units, provided the cost of the units are less than the available expense. The objective is to maximise the reliability probability of the system within the scope of the expense limitation

DP formulation for Reliability System Design

Stage n : Each component is considered as a stage. We begin the procedure from the very last component.

State n (sn): The amount of money available at the beginning of each stage.

Decision variable (xn) : Number of units added in the component provided it satisfy the expense condition defined by state sn.

p(sn, xn) : Reliability when a new unit xn is added to the component given the available expense.

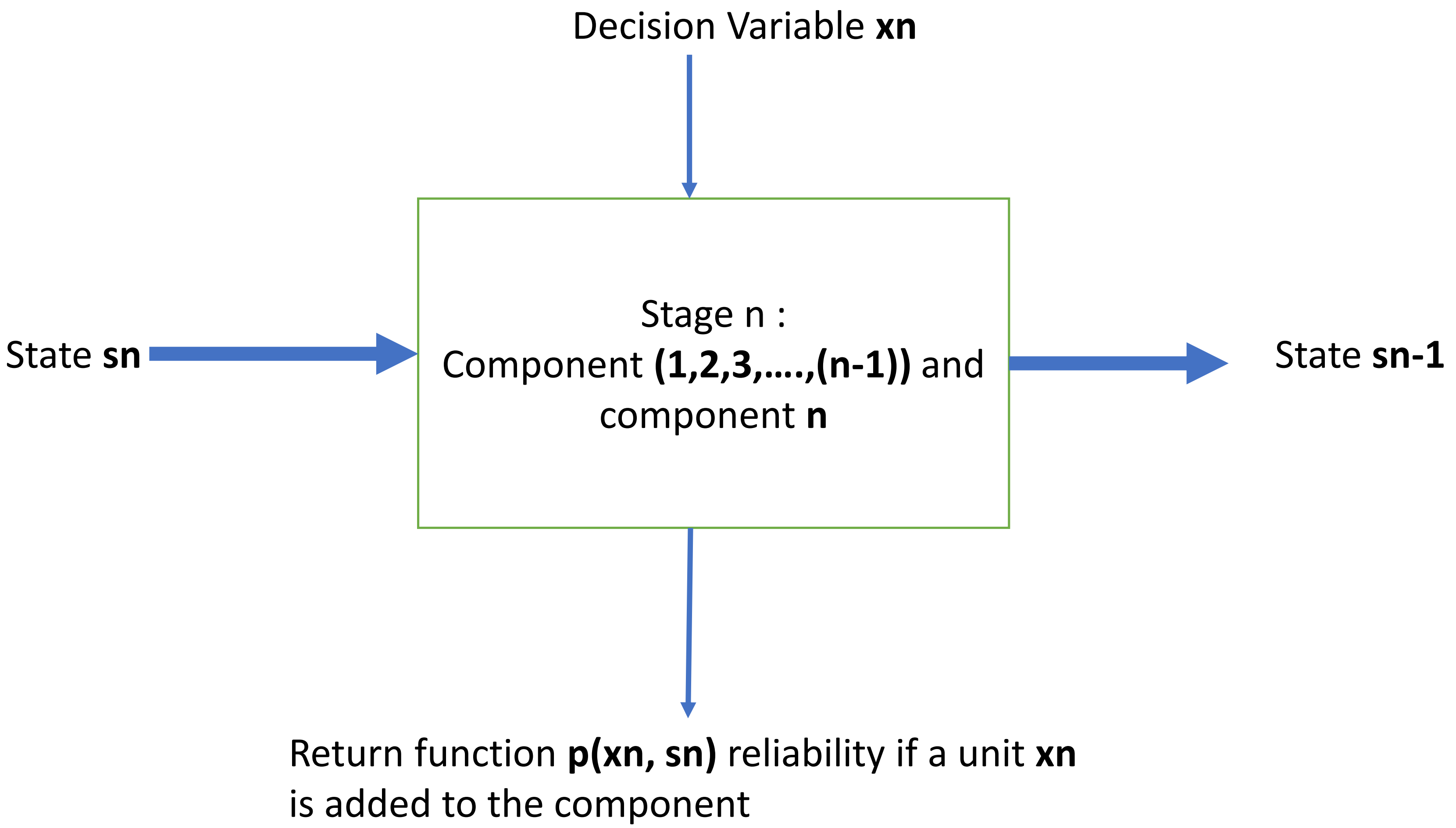
f(sn, xn): Total Reliability of the components till nth component.

f*(sn): Maximum reliability of the components till nth component.

xn*: Optimised number of units in the component for maximum reliability.

C(xn): Cost of the xn units in that component.

Single stage structure



Stage 3

Current State sn (in thousands)	**Minimum value in Next state Sn+1** (in thousands)	Reliability associated with the component **f(sn, xn) = p(xn, sn) * f*(sn-C(xn))**			Maximum reliability **f*(sn)**	Number of optimised unit **xn***
		xn = 1	**xn=2**	**xn=3**		
1	4	0.8	-	-	0.8	1
2	3	0.8	-	-	0.8	1
3	2	0.8	-	-	0.8	1
4	1	-	0.9	-	0.9	2
5	0	-	-	0.95	0.95	3

n = 1

The range of feasible states will lie between 1000 to 5000, this is because a minimum of 5000 dollars will be spent on component 1 and 2 at stage 1 and 2. This will leave a maximum of 5000 to be spent on stage 3, since minimum cost of component is 1000 at stage 3 and maximum is 5000 the state value will range from 1 to 5

Stage 2

Current State sn (in thousands)	**Minimum value in Next state Sn+1** (in thousands)	Reliability associated with the component **f(sn, xn) = p(xn, sn) * f*(sn-C(xn))**			Maximum reliability **f*(sn)**	Number of optimised unit **xn***
		xn = 1	**xn=2**	**xn=3**		
3	1	0.7 * 0.8 = **0.56**	-	-	0.56	1
4	2	0.7 * 0.8 = **0.56**	-	-	0.56	1
5	1	0.7 * 0.8 = **0.56**	0.8 * 0.8 = **0.64**	-	0.64	1
6	1	0.7 * 0.9 = **0.63**	0.8 * 0.9 = **0.72**	0.9 * 0.8 = **0.72**	0.72	2 or 3
7	2	0.7 * 0.95 = **0.665**	0.8 * 0.8 = **0.64**	0.9 * 0.8 = **0.72**	0.72	3

n = 2

The range of feasible states will lie between 3000 to 7000, this is because a minimum of 3000 dollars will be spent on component at stage 1 . This will leave a maximum of 7000 to be spent on stage 2, since minimum cost of component is 2000 and at stage 3 is 1000, this will leave us spending at least 3000 and maximum of 7000 in stage 2.

Stage 1

Current State sn (in thousands)	**Minimum value in Next state Sn+1** (in thousands)	Reliability associated with the component **f(sn, xn) = p(xn, sn) * f*(sn-C(xn))**			Maximum reliability **f*(sn)**	Number of optimised unit **xn***
		xn = 1	**xn=2**	**xn=3**		
10	4	0.6 * 0.72 = **0.432**	0.7 * 0.64 = **0.448**	0.9 * 0.56 = **0.504**	0.504	3

The range of feasible states will lie 10000, since this is the very first stage, and n = 3.

Observation and Results

Optimal Solution:

$s1 = 10,$ $x1^* = 3$ units of component 1 are used

$s2 = 10 - 6 = 4$ $x2^* = 1$ unit of component 2 is used

$s3 = 4 - 2 = 2$ $x3^* = 1$ unit of component 3 is used

Optimal solution at 0.504 system reliability.

Recursive formula:

If n = 1, Then

f*(sn) = p(sn, xn) for all xn = 1,2,3.

Else

f*(sn) = Max { f(sn, xn)} = Max { p(sn, xn) * f*(sn – C(xn)) } over all xn = 1,2,3; for all sn >= C(xn)

Endif