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Problem Statement

Alcoa produces 100-, 200-, and 300-foot-long aluminium ingots for customers. This week's demand for ingots is shown in Table 1.

Alcoa has 4 furnaces in which ingots can be produced. During a week, each furnace can be operated for 50 hours. Because ingots are produced by cutting long strips of aluminium, longer ingots take less time to produce than shorter ingots. If a furnace is devoted completely to producing one type of ingot; the number it can produce in a week is shown in Table 2.

For example, furnace 1 could produce 350 300-foot ingots per week. The material in an ingot costs $10 per foot. If a customer wants a 100- or 200-foot ingot, then she will accept an ingot of that length or longer. How can Alcoa minimize the material costs incurred in meeting required weekly demands.

TABLE 1

Ingot (ft)	Demand
100	700
200	300
300	150

TABLE 2

	Ingot Length		
Furnace	100′	200′	300′
1	230	340	350
2	230	260	280
3	240	300	310
4	200	280	300

Verbal Formulation

The problem can be broken down into following sub points:

- Alcoa company produces ingots for customers of length 100-ft , 200-ft and 300-ft. Whose demands are 700, 300 and 150 respectively.
- There are 4 furnaces that can operate in a week, these furnaces can produces ingots of each type one at a time. The details of production of all three types of ingots per furnace **provided they operate to produce only that type** is given in table 2.
- Material cost of $10 per foot of ingot is the variable cost and needs to be minimised.
- A furnace can operate at maximum of 50 hours only per week. However, the time consumed to manufacture an individual ingot is not know. **It is known that the longer the ingot length, the smaller is the duration to manufacture the same.**
- **If a customer wants a 100- or 200-foot ingot, then she will accept an ingot of that length or longer.**

I have formulated the problem in two ways:
1. Considering that all the furnaces are working on producing only one product through out the week.
2. A furnace may be utilised in variable shifts for producing multiple products without exceeding the 50 hours constraints.

APPROACH 1

EACH FURNACE WORKING TO PRODUCE ONLY ONE PRODUCT ENTIRE WEEK

Mathematical Model

Decision Variables:

- Demand variable: indicating demand of i-th foot ingots **takes real integer values** (d-ith for i = 100,200 and 300.)
- Production variable: indicating number of i-th foot ingots produced, **takes real integer values** (N-ith for i = 100,200 and 300.)
- Furnace variable: indicates i-th furnace producing j-th foot ingots **takes binary values** (fij for i = 1 to 4 and j = 100,200,300)
- compensate variable: indicate number of j-th foot high dimension ingots purchased to compensate for the i-th low dimension ingots. This is to satisfy the customer compensation constraint. **This takes any real value.**

Constraints:

Demand Constraint:

- The demand variable must be equal to the number of ingots demanded.

Production Constraint:

- The number of i-th foot ingot produced must come in accordance to the furnace operation rule and should be produced according to table 2.
- The production is done keeping in mind that each furnace will work in order to produce only one type of ingot entire week.

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Furnace Operation Constraint:
- Each furnace can operate to produce only one type of ingots.
- Since they accept only binary value, the sum per furnace should be equal to 1.

Customer Compensation Constraint:
- **If a customer wants a 100- or 200-foot ingot, then she will accept an ingot of that length or longer.**
- Hence this constraint ensures that the ingots are produced in such a quantity, that it compensates for ingots that are low in dimension. Keeping in mind the production for those type of ingots which are most profitable.

Non-negative Constraints:
- All Decision Variables should be greater than equal to zero

Objective Function:
The objective is to minimise the cost incurred in producing the ingots of required dimension. That is ($10*100 = $1000) for each ingots of 100 ft. Similarly $2000 and $3000 for ingots 200-ft and 300-ft.

```
min=1000*N100 + 2000*N200 + 3000*N300;

!Demand constraint;
d100 = 700;
d200 = 300;
d300 = 150;
!Production constraint;
230*f1100 + 230*f2100 + 240*f3100 + 200*f4100 - N100 = 0;
340*f1200 + 260*f2200 + 300*f3200 + 280*f4200 - N200 = 0;
350*f1300 + 280*f2300 + 310*f3300 + 300*f4300 - N300 = 0;

!Furnace operation per week constraint;
f1100 + f1200 + f1300 = 1;
f2100 + f2200 + f2300 = 1;
f3100 + f3200 + f3300 = 1;
f4100 + f4200 + f4300 = 1;

!Customer compensation purchase ;
d100 = N100*y11 + N200*y21 + N300*y31;
d200 = N200*y22 + N300*y32;
d300 = N300*y33;
y11 = 1;
y21 + y22 = 1;
y31 + y32 + y33 = 1;

!declaring range of variables;
@GIN(d100);
@GIN(d200);
@GIN(d300);
@GIN(N100);
@GIN(N200);
@GIN(N300);
@BIN(f1100);
@BIN(f1200);
@BIN(f1300);
@BIN(f2100);
@BIN(f2200);
@BIN(f2300);
@BIN(f3100);
@BIN(f3200);
@BIN(f3300);
@BIN(f4100);
@BIN(f4200);
@BIN(f4300);
end
```

LINGO based problem

LINGO OUTPUT

```
Local optimal solution found.
Objective value:                              2690000.
Objective bound:                              2690000.
Infeasibilities:                              0.9205191E-04
Extended solver steps:                              59
Total solver iterations:                           853
Elapsed runtime seconds:                          0.27

Model Class:                                      MIQP

Total variables:                    20
Nonlinear variables:                 7
Integer variables:                  15

Total constraints:                  13
Nonlinear constraints:               3

Total nonzeros:                     46
Nonlinear nonzeros:                  5
```

Variable	Value	Reduced Cost
N100	240.0000	1000.000
N200	280.0000	2000.000
N300	630.0000	3000.000
D100	700.0000	0.000000
D200	300.0000	0.000000
D300	150.0000	0.000000
F1100	0.000000	0.000000
F2100	0.000000	0.000000
F3100	1.000000	0.000000
F4100	0.000000	0.000000
F1200	0.000000	0.000000
F2200	0.000000	0.000000
F3200	0.000000	0.000000
F4200	1.000000	0.000000
F1300	1.000000	0.000000
F2300	1.000000	0.000000
F3300	0.000000	0.000000
F4300	0.000000	0.000000
Y11	1.000000	0.000000
Y21	0.4829718	0.000000
Y31	0.5155048	0.000000
Y22	0.5170282	0.000000
Y32	0.2464000	0.000000
Y33	0.2380952	0.000000

Row	Slack or Surplus	Dual Price
1	2690000.	-1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000
10	0.000000	0.000000
11	0.000000	0.000000
12	-0.8890302E-04	0.000000
13	-0.9205191E-04	0.000000
14	-0.3148892E-05	0.000000
15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	0.000000

Report

	Demand in units	Supply in units			Cost in 1000's $
		100-ft Ingot	200-ft Ingot	300-ft Ingot	
100-ft Ingot	700	240	135	325	**1485**
200-ft Ingot	300	0	145	155	**755**
300-ft Ingot	150	0	0	150	**450**
Total		**240**	**280**	**630**	**2690**

Supply Demand and Cost Report

	Quantity Produced per week		
Furnace	100-ft Ingot	200-ft Ingot	300-ft Ingot
1	0	0	350
2	0	0	280
3	240	0	0
4	0	280	0
Total	**240**	**280**	**630**

Furnace Operation Report

Approach 2

Furnace producing multiple types of ingot per week.

Mathematical Model

Decision Variables:

- Demand variable: indicating demand of i-th foot ingots **takes real integer values** (d-ith for i = 100,200 and 300.)
- Production variable: indicating number of i-th foot ingots produced, **takes real integer values** (N-ith for i = 100,200 and 300.)
- Furnace variable: indicates i-th furnace producing j-th foot ingots **takes binary values** (fij for i = 1 to 4 and j = 100,200,300)
- Time variable: indicates the time for which i-th furnace operate to produce ingots of type j, **takes any real value** (tij for i = 1 to 4 and j = 1 to 3)
- compensate variable: indicate number of j-th foot high dimension ingots purchased to compensate for the i-th low dimension ingots. This is to satisfy the customer compensation constraint. **This takes any real value.**

Constraints:

Demand Constraint:

- The demand variable must be equal to the number of ingots demanded.

Production Constraint:

- The number of i-th foot ingot produced must come in accordance to the furnace operation rule and should be produced according to table 2.
- The production is done keeping in mind that each furnace will work in order to produce only one type of ingot entire week.

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Furnace Operation time Constraint:
- Each furnace can operate to produce multiple type of ingots, however if they are on they will only operate for tij hours.
- In total a furnace can operate for less than 50 hours only.
- **In order to ensure that all furnace are utilised sum of fij will be >= 1, incase there are operating at more than one shift fij being a binary variable, its sum will be more than 1.**

Production time constraint:
- **Will work on the principle that for higher dimension ingots, it will take shorter duration to produce them.**

Customer Compensation Constraint:
- **If a customer wants a 100- or 200-foot ingot, then she will accept an ingot of that length or longer.**
- Hence this constraint ensures that the ingots are produced in such a quantity, that it compensates for ingots that are low in dimension. Keeping in mind the production for those type of ingots which are most profitable.

Non-negative Constraints:
- All Decision Variables should be greater than equal to zero

Objective Function:
The objective is to minimise the cost incurred in producing the ingots of required dimension. That is ($10*100 = $1000) for each ingots of 100 ft. Similarly $2000 and $3000 for ingots 200-ft and 300-ft.

```
min=1000*N100 + 2000*N200 + 3000*N300;
! Demand Constraint;
d100 = 700;
d200 = 300;
d300 = 150;

!Production Constraint;
230*f1100 + 230*f2100 + 240*f3100 + 200*f4100 - N100 = 0
340*f1200 + 260*f2200 + 300*f3200 + 280*f4200 - N200 = 0
350*f1300 + 280*f2300 + 310*f3300 + 300*f4300 - N300 = 0

!Time constraint per furnace operation;
t11*f1100 + t12*f1200 + t13*f1300 = 50;
t21*f2100 + t22*f2200 + t23*f2300 = 50;
t31*f3100 + t32*f3200 + t33*f3300 = 50;
t41*f4100 + t42*f4200 + t43*f4300 = 50;

f1100 + f1200 + f1300 >= 1;
f2100 + f2200 + f2300 >= 1;
f3100 + f3200 + f3300 >= 1;
f4100 + f4200 + f4300 >= 1;

!Ingots production time constraint;
t13 < t12;
t12 < t11;
t23 < t22;
t22 < t21;
t33 < t32;
t32 < t31;
t43 < t42;
t42 < t41;

!Customer purchase constraint;
d100 = N100*y11 + N200*y21 + N300*y31;
d200 = N200*y22 + N300*y32;
d300 = N300*y33;
y11 = 1;
y21 + y22 = 1;
y31 + y32 + y33 = 1;
```

```
@GIN(d100);
@GIN(d200);
@GIN(d300);
@GIN(N100);
@GIN(N200);
@GIN(N300);
@BIN(f1100);
@BIN(f1200);
@BIN(f1300);
@BIN(f2100);
@BIN(f2200);
@BIN(f2300);
@BIN(f3100);
@BIN(f3200);
@BIN(f3300);
@BIN(f4100);
@BIN(f4200);
@BIN(f4300);
end
```

LINGO based problem

LINGO SOLUTION

Local optimal solution found.	
Objective value:	2690000.
Objective bound:	2690000.
Infeasibilities:	0.4768380E-04
Extended solver steps:	135
Total solver iterations:	2289
Elapsed runtime seconds:	0.22
Model Class:	MIQP
Total variables:	32
Nonlinear variables:	31
Integer variables:	15
Total constraints:	25
Nonlinear constraints:	7
Total nonzeros:	86
Nonlinear nonzeros:	17

Variable	Value
N100	240.0000
N200	280.0000
N300	630.0000
D100	700.0000
D200	300.0000
D300	150.0000
F1100	0.000000
F2100	0.000000
F3100	1.000000
F4100	0.000000
F1200	0.000000
F2200	0.000000
F3200	0.000000
F4200	1.000000
F1300	1.000000
F2300	1.000000
F3300	0.000000
F4300	0.000000
T11	50.00000
T12	50.00000
T13	50.00000
T21	49.99995
T22	49.99995
T23	49.99995
T31	50.00000
T32	50.00000
T33	50.00000
T41	50.00000
T42	50.00000
T43	50.00000
Y11	1.000000
Y21	0.000000
Y31	0.7301587
Y22	1.000000
Y32	0.3174603E-01
Y33	0.2380952

Row	Slack or Surplus
1	2690000.
2	0.000000
3	0.000000
4	0.000000
5	0.000000
6	0.000000
7	0.000000
8	0.000000
9	-0.4768380E-04
10	0.000000
11	0.000000
12	0.000000
13	0.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000
25	0.000000
26	0.000000
27	0.000000
28	0.000000
29	0.000000

Report

	Demand in units	Supply in units			Cost in 1000's $
		100-ft Ingot	200-ft Ingot	300-ft Ingot	
100-ft Ingot	700	240	0	460	**1620**
200-ft Ingot	300	0	280	20	**620**
300-ft Ingot	150	0	0	150	**450**
Total		**240**	**280**	**630**	**2690**

Supply Demand and Cost Report

	Quantity Produced per week		
Furnace	**100-ft Ingot**	**200-ft Ingot**	**300-ft Ingot**
1	0	0	350
2	0	0	280
3	240	0	0
4	0	280	0
Total	**240**	**280**	**630**

Furnace Operation Report

Conclusion

It was observed that both the approaches resulted in the same minimise output for the objective function.
Although the freedom was given for a furnace to produce multiple type of ingot, producing one type of ingot every week seems more optimised.