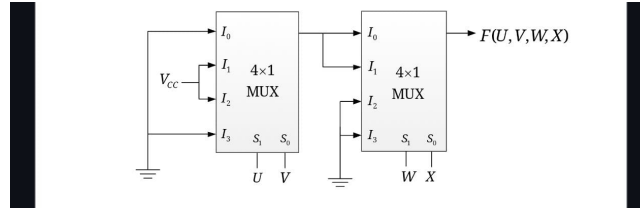


GATE Question Paper 2018, EC Question Number 31

Problem Statement

A four-variable Boolean function is realized using 4×1 multiplexers as shown in the figure.



The minimized expression for $F(U, V, W, X)$ is:

Choose the correct minimized expression for $F(U, V, W, X)$:

- (A) $(UV + \overline{UV})\overline{W}(UV + \overline{UV})(\overline{WX} + \overline{WX})$
(B) $(U\overline{V} + \overline{UV})\overline{W}(U\overline{V} + \overline{UV})(\overline{WX} + \overline{WX})$

Solution

Step 1: First 4×1 MUX Analysis

The first multiplexer has:

(D) Select lines: U and V

- Inputs: I_0, I_1, I_2, I_3

The output equation is:

$$F_1 = I_0\overline{U}\overline{V} + I_1\overline{U}V + I_2U\overline{V} + I_3UV$$

From the diagram, the inputs are:

$$I_0 = 0, \quad I_1 = 1, \quad I_2 = 1, \quad I_3 = 0$$

Substituting these values:

$$F_1 = 0 \cdot \overline{U}\overline{V} + 1 \cdot \overline{U}V + 1 \cdot U\overline{V} + 0 \cdot UV$$

$$F_1 = \overline{U}V + U\overline{V}$$

Step 2: Second 4×1 MUX Analysis

The second MUX has:

- Select lines: W and X
- Inputs: $I_0 = F_1, I_1 = F_1, I_2 = 0, I_3 = 1$
The output equation is:

$$F = I_0 \bar{W} \bar{X} + I_1 \bar{W} X + I_2 W \bar{X} + I_3 W X$$

Substituting values:

$$F = (\bar{U}V + U\bar{V})\bar{W} \bar{X} + (\bar{U}V + U\bar{V})\bar{W} X + 0 \cdot W \bar{X} + 1 \cdot W X$$

Factorizing:

$$F = (\bar{U}V + U\bar{V})(\bar{W} \bar{X} + \bar{W} X) + W X$$

Since $\bar{W} \bar{X} + \bar{W} X = \bar{W}$, we simplify:

$$F = (\bar{U}V + U\bar{V})\bar{W} + W X$$

Final Answer

The correct minimized expression for F(U, V, W, X) is:

$$\boxed{(U\bar{V} + \bar{U}V)(\bar{W} \bar{X} + \bar{W} X)}$$

Thus, the correct option is *(D)*.