CHAPTER-1
RELATIONS AND FUNCTIONS

1.1 Introduction

Recall that the notion of relations and functions, domain, co-domain and range have been introduced in Class XI along with different types of specific real valued functions and their graphs. The concept of the term 'relation' in mathematics has been drawn from the meaning of relation in the English language, according to which two objects or quantities are related if there is a recognisable connection or link between the two objects or quantities. Let A be the set of students of Class XII of a school and B be the set of students of Class XI of the same school. Then some of the examples of relations from A to B are:

- $\{(a, b) \in A \times B : a \text{ is brother of } b\}$,
- $\{(a, b) \in A \times B : a \text{ is sister of } b\}$,
- $\{(a, b) \in A \times B : \text{age of } a \text{ is greater than age of } b\}$,
- $\{(a, b) \in A \times B :$ total marks obtained by a in the final examination is less than the total ma
- $\{(a, b) \in A \times B : a \text{ lives in the same locality as } b\}$.

However, abstracting from this, we define mathematically a relation R from A to B as an arbitrary subset of $A \times B$. If $(a, b) \in R$, we say that a is related to b under the relation R and we write as $a\,R\,b$. In general, if $(a, b) \in R$, we do not bother whether there is a recognisable connection or link between a and b. As seen in Class XI, functions are a special kind of relations.

In this chapter, we will study different types of relations and functions, composition of functions, invertible functions, and binary operations.

1.2 Types of Relations

In this section, we would like to study different types of relations. We know that a relation in a set A is a subset of $A \times A$. Thus, the empty set $\emptyset$ and

$A \times A$ are two extreme relations. For illustration, consider a relation R in the set $A = \{1, 2, 3, 4\}$ given by

$$R = \{(a, b) : a - b = 10\}.$$

This is the empty set, as no pair (a, b) satisfies the condition $a - b = 10$. Similarly, $R' = \{(a, b) : |a - b| \geq 0\}$ is the whole set $A \times A$, as all pairs (a, b) in $A \times A$ satisfy $|a - b| \geq 0$. These two extreme examples lead us to the following definitions:

A relation R in a set A is called **empty relation**, if no element of A is related to any element of A, i.e., $R = \emptyset \subset A \times A$.

A relation R in a set A is called **universal relation**, if each element of A is related to every element of A, i.e., $R = A \times A$.

Both the empty relation and the universal relation are sometimes called trivial relations.

Example 1: Let A be the set of all students of a boys' school. Show that the relation R in A given by $R = \{(a, b) : a \text{ is sister of } b\}$ is the empty relation and $R' = \{(a, b) : \text{the difference between heights of } a \text{ and } b \text{ is less than 3 meters}\}$ is the universal relation.

Solution: Since the school is a boys' school, no student of the school can be a sister of any student of the school. Hence, $R = \emptyset$, showing that R is the empty relation. It is also obvious that the difference between heights of any two students of the school has to be less than 3 meters. This shows that $R' = A \times A$ is the universal relation.

Remark: In Class XI, we have seen two ways of representing a relation, namely the raster method and the set builder method. However, a relation R in the set $\{1, 2, 3, 4\}$ defined by $R = \{(a, b) : b = a + 1\}$ is also expressed as $a\,R\,b$ if and only if $b = a + 1$ by many authors. We may also use this notation, as and when convenient. If $(a, b) \in R$, we say that a is related to b and we denote it as $a\,R\,b$.

One of the most important relations, which plays a significant role in Mathematics, is an equivalence relation. To study equivalence relations, we first consider three types of relations, namely reflexive, symmetric, and transitive.

A relation R in a set A is called:

- **reflexive**, if $(a, a) \in R$ for every $a \in A$,
- **symmetric**, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$,

- **transitive**, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.