# Relations and Functions

## 1 Introduction

Recall that the notion of relations and functions, domain, co-domain, and range have been introduced earlier.

1. $(a, b) \in A \times B : a$ is brother of $b$
2. $(a, b) \in A \times B : a$ is sister of $b$
3. $(a, b) \in A \times B :$ age of $a$ is greater than age of $b$
4. $(a, b) \in A \times B :$ total marks obtained by $a$ in the final examination is less than the total ma
5. $(a, b) \in A \times B : a$ lives in the same locality as $b$.

## 2 Types of Relations

In this chapter, we will study different types of relations and functions, composition of functions, invertible functions, and binary operations.

1. Definition 1: A relation $R$ in a set $A$ is called an **empty relation** if no element of $A$ is related to any element of $A$, i.e., $R = \emptyset \subseteq A \times A$.
2. Definition 2: A relation $R$ in a set $A$ is called a **universal relation** if each element of $A$ is related to every element of $A$, i.e., $R = A \times A$.

Both the empty relation and the universal relation are sometimes called trivial relations.

### 2.1 Examples

1. **Example 1:** Let $A$ be the set of all students of a boys' school. Show that the relation $R$ in $A$ given by:

$$R = \{(a, b) : a \text{ is sister of } b\}$$

is the empty relation, and

$$R' = \{(a,b) : \text{the difference between heights of } a \text{ and } b \text{ is less than 3 meters}\}$$

is the universal relation.

**Solution:** Since the school is a boys' school, no student can be a sister of any other student. Hence, $R = \emptyset$, showing that $R$ is the empty relation.

It is also obvious that the difference between the heights of any two students in the school must be less than 3 meters. This shows that $R' = A \times A$, which is the universal relation.

# 3 Equivalence Relations

To study equivalence relations, we first consider three important types of relations:

1. **Reflexive:** A relation $R$ in a set $A$ is reflexive if $(a,a) \in R$ for every $a \in A$.

2. **Symmetric:** A relation $R$ in a set $A$ is symmetric if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$ for all $a_1, a_2 \in A$.

3. **Transitive:** A relation $R$ in a set $A$ is transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$.