Differentiation

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1. If the radius of the circle is increasing at the rate of $0.5cm/s$, then the rate of increase of its circumference is.
2. Differentiate $\sec^2(x^2)$ with respect to x^2.
3. If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$,then find $\frac{dy}{dx}$.
4. Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$.
5. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$,find $f'\left(\frac{\pi}{3}\right)$
6. If $\tan^{-1}\left(\frac{y}{x}\right) = \log \sqrt{x^2 + y^2}$,prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.
7. If $y = e^{a\cos^{-1} x}, -1 < x < 1$,then show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0 \tag{1}$$