

Differentiation

Dec 2023

1. If the radius of the circle is increasing at the rate of 0.5cm/s , then the rate of increase of its circumference is.
2. Differentiate $\sec^2(x^2)$ with respect to x^2 .
3. If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$.
4. Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$.
5. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$
6. If $\tan^{-1}\left(\frac{y}{x}\right) = \log \sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.
7. If $y = e^{a \cos^{-1} x}$, $-1 < x < 1$, then show that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0 \quad (1)$$