Useful Formulas in Hilbert spaces
· Parallelogrum rule: 11x14112 + 11x-4112 = 211x112 + 2114112
· Polarization:
If T is a bounded operator. CTx, y) = 44 Etro inc T(x+:hy), x+ihy). So, if we know CTu, u) Yueth, we know To
So, if CTU,U7=0 Guell, then T=0.
BCHD= {T:H->H bounded operator]. (BCHD. HA) is a Panach +-algebra, T*=adjoint of T.
Properties of operator norm
· HTH = CO = T = O = T = O = T T = O = T T I I I
• TS ± TI 1
= 117+11=11711 Self-adjoint
• 1174711=117112
Let's grave last two: Breview HTH= sugnannyment (<tray>)</tray>
1178 THE 1178 111711 . Also, 1178 TH = Support, Higher 16 TRT x , y > 1 = 5 up to x of the york of the then the the then the the the then the the the then the then the then the then the
117112= Sugnation 117x112= Sugnation CT* Tx,x> = 117xTx111411 = 11747(1, 0) 117xT1 = 117112.
11112=117 * THE 11711 11 CH 1741 . Using To glos 11741 E 11711.
Examples of operators
* H=E". Any matrix AEMacED gives T= Ca Cleft mult by AD. TOXD=Ax. Exercise
"H=E". Any matrix AEMacco gives T=Ca Cleft mult by AD. TCx2=Ax. Exercise
· H=2°CM). SH-H left shift, SKx1,x2, 2= C+2,x3,x4,
Note 554 = 545, as 554 = I and 548 ≠ I. \$ ←+ (x1,, xn2, Mpsf2=Cc1+1,, cn xn2.)
· Let Cx,,,, be a mensura space. H=C2Cx,,,), and yel@Cx,,,). Let T=Mp be given by Tcf3=yf Yf6H.

why is I bounded? HITH= HM-Eliz = Sieliz = Heliz Sielz = Heliz HEliz. So, HITH=Heliz. In fact:
Thm: IR Cx, x> 6-finite, then Mell= lello. e-Me, L=(x, x) = OCC(x, x)
e) Imefilzzciitiiz
Proof: 11Mple liplo done. (et occollellos. We create a vector fecerno wi 11Mpfilzzo, 11611z=10 Sign = nces
Try fore CE mus. in x, mces = 03. Ilmelizes Scienzes Scienzes Seigle zermces Seigle zermces. (et ec lxexiterated) a ormcesco
Cenists by 6-finitenesso. So, Seletz 2 Secz = czuceo.
CENTA 10 0 - 4 initen (232 m - 20) 7 E 1 61 - 2 7 E 2 - 2 Merica
Important clusses of operators
CID TEBCHO is self-adjoint if T=T*
CO TECSTO TO SELECTION OF THE TECSTOR OF THE TECSTO
CSD NEOCHD is mormal if Non-WN+
de la companya de la
CODUEBOHO is unitary if U*U=UU*=I. Equivalently, <ux,uy>= <x,y> \frac{1}{2} \f</x,y></ux,uy>
C30 U6BCHD is an isometry if U=U=I. Equivalently, Ux = x Ux6H.
CSD VEDCHO is a partial isometry cp.:.) if VUTU=V. Equivalently, 3+cH closed subspace will Vit isometric and VI+1=0.
Equivalently, U*U, UU* are projections cond U*U= projhs.
(6) PEBCH) is a projection if P=P=P=P=. Equivalently, there is Irall closed subspace of P=proje, meaning
Yx6H, y=Pcx2 affains the infyehlly-x11.