Differential Privacy in Applications

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PACK

1. **Plan**
   - Frequent Pattern Mining with heterogeneous data types, i.e. string, numeric
   - Federated Analytics with relative small sample size, i.e. $n = 10^3, 10^4$
2. **Achievement**
   - Familiar with differential privacy
   - Study different secure computation schemes
   - Improve TrieHH infeasible problem by reasonably increasing privacy tolerance.
   - Compare different prefix tree structures to implement TrieHH algorithm.
3. **Challenge**
   - TrieHH achieves (ε, δ)-DP with large privacy budget
4. **Knowledge**
   - Sampling and threshold: A simple sample-and-threshold approach provides an (ε, δ)-DP guarantee for histograms
   - Prefix Tree (Trie): predict words or bit strings.

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Differential Privacy: Definition

Definition

A function $\mathcal{M}$ is often called a mechanism satisfying (approximate) differential privacy, if for all neighboring datasets $\mathcal{D}$ and $\mathcal{D}'$, and all possible outputs $\mathcal{S} \subseteq \mathcal{M}(\cdot)$:

$$\mathcal{P}[\mathcal{M}(\mathcal{D}) \in \mathcal{S}] \leq e^{\varepsilon} \times \mathcal{P}[\mathcal{M}(\mathcal{D}') \in \mathcal{S}](+\delta)$$

where ε is the privacy budget, and δ represents a "failure probability"

- Key Idea: With given ε, δ, we can add noise to the output to preserve privacy, e.g Laplace and Gaussian Mechanisms.

Differential Privacy: Properties

Sequential Composition

Let $\mathcal{M}_i$ each provides ε_i-differential privacy. The sequence of $\mathcal{M}_i(\mathcal{D})$: $\mathcal{M}(\mathcal{D}) = (\mathcal{M}_1, \mathcal{M}_2, ...)$ provides $\sum_i \varepsilon_i - DP$

Parallel Composition

If $\mathcal{M}$ satisfies $\varepsilon - DP$, and split dataset $\mathcal{D}$ into k disjoint chunks $d_1 \cup d_2 \cup ... \cup d_k = \mathcal{D}$. Then $\mathcal{M}(d_i)$ achieves $\varepsilon - DP$

Post-processing

If $\mathcal{M}(\mathcal{D})$ achieves $\varepsilon - DP$, then for any function g, $g(\mathcal{M}(\mathcal{D}))$ achieves $\varepsilon - DP$.

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Secure Computation Schemes

1. Secret Sharing: Key Idea

Secret sharing schemes split a secret into multiple shares that are meaningless unless τ(threshold) of them are collected and the secret is reconstructed.

2. Homomorphic Encryption: Key Idea

Homomorphic encryption schemes allow users' data to be protected anytime it is sent to the cloud, because it can allow operations and functions to be preformed over encrypted data.

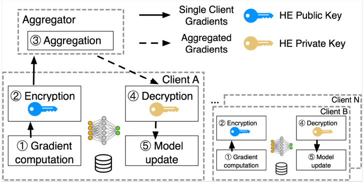

Figure: Homomorphic Encryption applied in Federated Learning

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TrieHH Overview[1]

Research Question: How to develop an interactive heavy hitters discovery algorithm that achieves central DP while minimizing the data collected from users?

Model:

- Following the Federated Learning protocol: (1) Sampling m clients per round. (2) Execute the TrieHH algorithm and the sever aggregates for multiple rounds; and (3) Server broadcasts the result.
- Achieving DP through sampling and mechanism which outputs a Trie.

Contributions

- Achieve central DP without centralizing raw data and adding noise
- Obtain excellent utility compared with local DP

[1] Wennan Zhu et al. "Federated Heavy Hitters Discovery with Differential Privacy". In: *CoRR* abs/1902.08534 (2019). arXiv: 1902.08534. URL: http://arxiv.org/abs/1902.08534.

TrieHH Insights and Improvements

Insights

The author used Corollary 1 to take out experiments.

Corollary 1

To achieve (ε, δ)-differential privacy, set $\gamma = \left(e^{\frac{\varepsilon}{L}} - 1\right)\sqrt{n}/\left(\theta e^{\frac{\varepsilon}{L}}\right)$ and $\theta = \max\left\{10, \left\lceil e^{W}\left(C_\delta\right) + 1 - \frac{1}{2}\right\rceil, \left\lceil e^{\frac{\varepsilon}{L}} - 1\right\rceil\right\}$, where W is the Lambert W function and $C_\delta = e^{-1}\ln\left(\frac{8}{7\sqrt{2\pi}}\delta^{-1}\right)$. Further, when $n \geq 10^4$, choosing $\theta = \lceil \log_{10} n + 6 \rceil$ ensures that Algorithm 1 is $(\varepsilon, \frac{1}{300n})$-differential private .

- The Corollary 1 is derived from the Theorem 1 (using approximation techniques)
  $\Rightarrow$ Corollary 1 results in larger failure probability.
  $\Rightarrow$ Using Theorem 1 to compute θ, γ

TrieHH Insights and Improvements

Theorem 1

When $4 \leq \theta \leq \sqrt{n}$ (threshold) and $1 \leq \gamma \leq \frac{\sqrt{n}}{\theta+1}$ (sampling size $m = \gamma\sqrt{n}$, where n is the total client size),
TrieHH algorithm is $(Lln(1 + \frac{1}{\frac{\sqrt{n}}{\gamma\theta}-1}), \frac{\theta-2}{(\theta-3)\theta!})$-differential private.

Goal: given $\varepsilon, \delta = 1/n^2, n$, calculate γ, θ. By DP's definition:

$$Lln(1 + \frac{1}{\frac{\sqrt{n}}{\gamma\theta} - 1}) \leq \varepsilon \quad (1)$$

$$\frac{\theta - 2}{(\theta - 3)\theta!} \leq \delta \quad (2)$$

TrieHH Insights and Improvements

- *Observation 1*: By Eq.2, $\frac{(\theta-3)\theta!}{\theta-2} > 1/\delta = n^2$, where LHS is incremental function.
- *Observation 2*: By Eq. 1, $\gamma \leq \frac{e^{\varepsilon/L}-1}{\theta * e^{\varepsilon/L}}\sqrt{n}$, and by Theorem 1, $\gamma \in [1, \frac{\sqrt{n}}{\theta+1}]$, to obtain a feasible γ:

$$1 \leq \frac{e^{\varepsilon/L}-1}{\theta * e^{\varepsilon/L}}\sqrt{n}$$

$$\Rightarrow \theta \leq \frac{e^{\varepsilon/L}-1}{e^{\varepsilon/L}}\sqrt{n} \leq \sqrt{n} \; (\text{THETACEIL: } \frac{e^{\varepsilon/L}-1}{e^{\varepsilon/L}}\sqrt{n})$$

Compute θ, γ

1. Initially set $\theta = 4$
   - If $\theta >$ THETACEIL(ε underflow), $\varepsilon \leftarrow$ THETACEIL $= 4$
2. MINIMIZE$_\theta$ $h(\theta) = \frac{(\theta-3)\theta!}{\theta-2} > 1/\delta$ ($h(\theta)$ is incremental)
   - If $\theta >$ THETACEIL (θ overflow), $\theta \leftarrow$ THETACEIL, $\delta \leftarrow 1/h(\theta)$
3. $\gamma \leftarrow$ MIN(THETACEIL$/\theta$, $\frac{\sqrt{n}}{\theta+1}$)

TrieHH Insights and Improvements

	$\varepsilon = 1$	$\varepsilon = 2$	$\varepsilon = 3$
$n = 1e3$	infeasible	infeasible	infeasible
$n = 5e3$	infeasible	infeasible	$\theta = 11$
$n = 1e4$	infeasible	$\theta = 12$	$\theta = 12$
$n = 2e4$	$\theta = 12$	$\theta = 12$	$\theta = 12$
$n = 3e4$	$\theta = 13$	$\theta = 13$	$\theta = 13$
$n = 1e5$	$\theta = 14$	$\theta = 14$	$\theta = 14$

Table: Choices of γ, θ to achieve $\varepsilon = 1, 2, 3$ and $\delta <= 1/n^2$

Remark

Infeasible means θ overflow or ε underflow $\Rightarrow$ reduce θ & increase δ or increase ε

TrieHH Insights and Improvements

```
Total number of clients: 10000
Theta overflow, infeasible r:: Reduce theta, and Increase delta
((1, 3.215020576131687e-06))-DP; Gamma used: 1.06
Theta: 9
Batch size used by TrieHH: 105
Discovered 1 heavy hitters in run #1
['the']
Total number of clients: 10000
Theta overflow, infeasible r:: Reduce theta, and Increase delta
((1, 3.215020576131687e-06))-DP; Gamma used: 1.06
Theta: 9
Batch size used by TrieHH: 105
Discovered 1 heavy hitters in run #2
['the']
```

```
Total number of clients: 10000
(1, 1e-08)-DP
Theta used by TrieHH: 12
Batch size used by TrieHH: 79
Discovered 1 heavy hitters in run #1
['the']
Discovered 0 heavy hitters in run #2
[]
Discovered 0 heavy hitters in run #3
[]
Discovered 0 heavy hitters in run #4
[]
Discovered 0 heavy hitters in run #5
[]
```

Figure: Finding heavy hitters under given $n = 10^4, \varepsilon = 1, \delta = 1/n^2$. (Left: decrease δ to solve δ overflow)

- *Remark 1*: Server goal is to maximum utility, i.e. finding heavy hitters. However, when client size is small, γ, θ are increased. (For example, $n = 1000 \Rightarrow \varepsilon = 31.4$, and $n = 2000 \Rightarrow \delta = 0.083$)
- *Remark 2*: $\varepsilon < 1$ is hard to achieve, since it positively relative to L (the max word length, default to 10)

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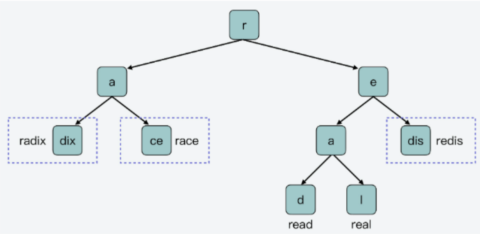
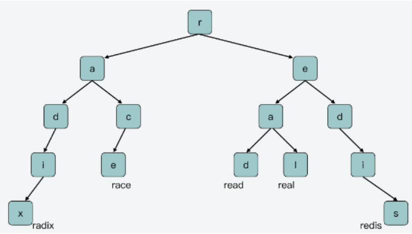
Prefix Tree Structures



Radix Trie



Standard Trie

- Radix Tree is the compact vision of the standard trie.
- Radix Tree could reduce the tree depth to achieve searching efficiency.

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Sample and Threshold Differential Privacy[2]

Research Question: How differential privacy can be obtained via a simple sample-and-threshold mechanism?

Model: Bernoulli sampling with threshold.

Contributions:

- Bernoulli sampling is sufficient to provide differential privacy.
- The resulting mechanism can also answer heavy hitter, quantile and range queries.
- The associated counts provide accurate frequency estimates for items from the input.

[2] Akash Bharadwaj and Graham Cormode. "Sample and Threshold Differential Privacy: Histograms and applications". In: *CoRR* abs/2112.05693 (2021). arXiv: 2112.05693. URL: https://arxiv.org/abs/2112.05693.

Sample and Threshold Differential Privacy

Insights

Lemma 1

If we set the sampling rate $p_s = \alpha\left(1 - e^{-\varepsilon}\right)$ for some $0 < \alpha \leq 1$ and $\varepsilon \leq 1$, then sample-and-threshold achieves (ε, δ) differential privacy for $\delta = \exp\left(-C_\alpha \tau\right)$, where $C_\alpha = \ln(1/\alpha) - 1/(1+\alpha)$.

For example, for $\varepsilon = 1$ and $\alpha = 1/6$, the sampling rate is $p_s = 0.105 \approx 0.1$ and, choosing $\tau = 20, \delta < 10^{-8}$ using $C_\alpha = 0.935$

Sample and Threshold Differential Privacy

Lemma 2

The TrieHH++ protocol using L sample-and-threshold histograms with (ε, δ)-DP achieves an overall guarantee of $(L\varepsilon, L\delta)$-DP.

Remark 1: Instead of proceeding in rounds, simply applying the basic histogram protocol to the full inputs (without build the Trie), and reporting the items which survive the threshold achieves (ε, δ)-DP (L can be dropped from these bounds.)

Remark 2: Hyperparamters are $\varepsilon, \theta, \alpha$

$\Rightarrow$ Sampling rate $p_s = \alpha\,(1 - e^{-\varepsilon})$

$\Rightarrow \delta(\theta, \alpha)$

Question:

- Sample-and-threshold is not related to client size.
- Hyperparameters is not intuitive.

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Dynamic Prefix Tree as a predict model[3]

Ideas

- Radix Trie can reduce deadly depth caused by long bit strings.
- Dynamic extend a Radix Trie within several rounds, then, clients only send a few prefix to infer heavy hitters in the rest rounds.

Remaining Questions:

- Fault Tolerance: client may lose connection.
- DP decision: ε, δ is decided by the sever, whose goal is maximizing the utility.

[3] Xiang Lisa Li and Percy Liang. "Prefix-Tuning: Optimizing Continuous Prompts for Generation". In: *CoRR* abs/2101.00190 (2021). arXiv: 2101.00190. URL: https://arxiv.org/abs/2101.00190.