A Stochastic Model for Credit Spreads under a Risk-Neutral Framework through the use of an Extended Version of the Jarrow, Lando and Turnbull Model

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ABSTRACT

The model derives risky corporate bond prices (or equivalently credit spreads) subject to credit default and migration risk, based on an extended version of the Jarrow, Lando and Turnbull model, under a risk-neutral framework, as a result of the simulation of a continuous time, time-homogeneous Markov chain. The inclusion of credit default and migration risk is made possible due to an allowance for a credit risk premium that varies stochastically[2]. While the standard Jarrow-Lando-Turnbull model assumes that the credit risk premium is a deterministic function of time which, along with the assumption of a constant "real-world" transition matrix and constant recovery rate, leads to deterministic credit spreads, the extension proposed through this article captures a stochastic risk premium in order to better fit with historical observation[3]. The model is of particular importance in the European Embedded Value (i.e. EEV) context where risk-neutral scenarios[4] are required for calculating the Time Value of Options and Guarantees (i.e. TVOG[5]) covering all material options and guarantees embedded following the requirements of EEV principles[6]. Moreover, the model can also be used in a real-world framework for pricing government and risky corporate debts with the exclusion of the Markov chain[7]. This allows to capture the marginal impact of credit default and migration risk at the TVOG level due to the corresponding changes that arise on the economic scenarios. The methodology is applied to corporate debts, but the extension proposed is flexible enough to be applicable to other securities as well.

Keywords: bond pricing, stochastic credit spreads, enhanced Jarrow, Lando and Turnbull model, risk-neutral valuation, Markov chain, arbitrage-free condition, European Embedded Value, Time Value of Options and Guarantees

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[2] Such an allowance is performed by relaxing the assumptions of the standard Jarrow, Lando and Turnbull model.

[3] In practice, credit spreads vary randomly.

[4] A set of risk neutral scenarios is partly composed of risky corporate bond returns.

[5] In order to determine TVOG, a set of economic scenarios that captures the potential distribution of various economic variables such as asset returns, yields, interest rates and inflation rates over a long-term horizon is required. Moreover, a market-consistent valuation analysis is adopted using a 'risk-neutral' set-up (i.e. by using deflators to value future cash flows). The risk-neutral approach is concerned with valuing risky cash flows in a way which is consistent with the prices of traded assets (typically bonds, equities and derivatives).

[6] A general overview covering those principles is proposed in CRO Forum, [1].

[7] The exclusion of risk-neutral transition probabilities leads to relax the arbitrage-free condition. This leads to the generation of corporate bond returns that exclude credit default and migration risk but include spread risk.

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I. INTRODUCTION

This article presents a model for the risk-neutral valuation of debt securities involving credit default and migration risk, and considering as a prerequisite a stochastic process for the evolution of the default-free term structure as well as the term structure of risky debts for credit spreads. Said differently, the model takes as given a stochastic term structure of default-free interest rates and a stochastic maturity specific credit spread[8]. Given these two term structures and the insertion of a time-homogeneous Markov chain based on a risk-neutral framework, risky corporate bonds are then priced in an arbitrage-free manner using the Martingale measure technology. Denote that while the pragmatic approach to the risk-neutral pricing of securities involving credit risk is to ignore credit risk, this solution is inconsistent with the absence of arbitrage and the existence of spreads. The model considers therefore an arbitrage-free dynamics for the term structures, and applies a risk-neutral valuation procedure using a continuous time Markov chain.

This model is an extension and a refinement of the Jarrow, Lando and Turnbull model that explicitly incorporates credit rating information into the risk-neutral valuation of corporate bonds. The model proposed allows to consider different seniority debts for obligors to be incorporated via different recovery rates in the event of default. In addition, it can be combined with any desired term structure model for default-free debt. The model uses historical transition probabilities for rating classes in order to determine the real-world probabilities used in the valuation. Moreover, the Cox, Ingersoll, Ross model is used to link real-world and risk-neutral probabilities though the proposed model extension that aims to generate stochastic spreads as it has the attractive advantage of avoiding the generation of negative risk-neutral probabilities.

As introduced in A. Jarrow, David Lando, Stuart M. Turnbull, [2], it is also assumed the independence between the obligor's default and the stochastic process for the default-free short rate[9] in order to derive a risk-neutral closed-form bond formula.

The model proves useful for the risk-neutral pricing of corporate bonds required for the determination of options and guarantees embedded within life insurance policies. Such a valuation is based on a risk-neutral framework in accordance with the requirements of CRO Forum, [1]. As the model can be used to generate returns of corporate bonds over the term of insurance contracts, for a particular rating class and under a risk-neutral framework, this model can participates to the determination of TVOG, which considers risky corporate bonds as a specific asset class that is part of the (re)investment strategy of life insurance companies.

[8] Such structures, being exogenously specified, require the use of standard stochastic processes broadly described in the financial literature with the application of an appropriate calibration. A similar approach is used in Robert A. Jarrow, Stuart M. Turnbull, [3].

[9] It is assumed that the two processes are statistically independent under the risk-neutral probabilities. Alternatively stated, the Markov process for credit ratings is independent of the spot level of interest rates under the risk-neutral probabilities. Justifications regarding the reasonableness of the approach are provided in Jarrow, David Lando, Stuart M. Turnbull, [2], p 483.

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II. MODEL FORMALISM

A filtered probability space (Ω, Q, F_t) with $(F_t)_{0\leq t\leq T}$ is considered[10]. It is assumed that there exists a unique equivalent martingale measure Q such as the discounted value of credit default and migration-free bond values are martingales[11] when using stochastic discount factors. Following the definition of martingales, it leads that the market for credit default and migration-free bonds is complete[12] and arbitrage-free[13]. Moreover, the credit default and migration-free short rate, noted $r(t)$ is assumed determined. One can make use of the Cox, Ingersoll, Ross stochastic process[14] but others particular stochastic processes such as O. Vasicek, [5] or F. Black, P. Karasinski, [6] might be considered. Through a continuous time framework, the cash return accumulates returns at the spot rate such as:

$$C(t) = e^{\int_{t=0}^{T} r(t)dt} \tag{1}$$

A. Risky Corporate Bond Price under a Risk-Neutral Probability Measure

In an arbitrage-free market, the risk-neutral pricing values an asset by discounting its cash flows at the risk-free rate and taking the average under the "risk-neutral measure". Consider a risky bond with maturity T following a bond-at-par strategy[15] (i.e. the bond pays a price of 100 if it survives until maturity, or some fraction δ, the recovery rate[16], if it has defaulted), the risk-neutral valuation of the bond can be written as follows:

$$v(0,T) = E_Q\left[\frac{\left(I(\tau>T)+\delta I(\tau\leq T)\right)}{C(t)}\right] \tag{2}$$

where r is the credit default and migration-free short rate, τ denotes the time at which default occurs ($I(\tau > T)$ and $I(\tau \leq T)$ are indicator functions denoting whether or not the bond has defaulted) and E_Q is the risk-neutral expectation under the probability measure Q. Under the additional simplifying assumption[17] that default i.e. τ is independent of the stochastic process for the default-free short rate i.e. $\{r(t)_{0\leq t\leq\tau}\}$, it leads:

$$v(0,T) = v^{gov}(0,T)\left[1 - P_Q(\tau \leq T) + \delta P_Q(\tau \leq T)\right] \tag{3}$$

where $v^{gov}(0,T)$ is the price of government bond (i.e. credit default and migration-free bond) and $P_Q(\tau \leq T)$ the risk-neutral probability that the bond defaults before maturity. As the price of a risky corporate bond is the credit default and migration-free bond multiplied by the expected payoff at time T, it leads that the evolution of the term-structure of risky corporate bonds is uniquely determined by the probability that the bond defaults under the risk-neutral probability measure. Consequently, the remaining part of this chapter focuses on the government bond price and the risk-neutral probability in order to complete the model.

[10] The filtration $(F_t)_{0\leq t\leq\tau}$ is an increasing family of σ-algebra included in Q.

[11] A process $(X_t)_{0\leq t\leq T}$ is a Martingale if and only if: $E^Q[X_t|F_s] = X_s$.

[12] The interest of complete markets is that it allows to derive a theory of pricing and hedging such as it exists a unique probability measure Q equivalent to P under which discounted credit default and migration-free bonds are martingales.

[13] Market prices do not allow for profitable arbitrage. An arbitrage-free market is a prerequisite for a general economic equilibrium.

[14] See Cox, Ingersoll, Ross, [4].

[15] Refer to section C. 'Risk-Free (Government) Bond price' for further details about the strategy of bonds.

[16] The recovery rate δ is a given constant depending on the bond seniority. Nevertheless, random recovery rates could also be fitted into the pricing model.

[17] This assumption is imposed for deriving a closed-form formula for bonds.

B. Risk-Free (Government) Bond Price

The derivation of risk-free bond prices depends on the bond strategy implemented. The approach adopted is referred to as 'Bond-at-Par' strategy. Bonds are assigned a coupon (given their maturity) so that, at the beginning of each time increment, the bond trades at par (i.e. a price of 100). The bond coupon and maturity are re-set once each year. The total return for government bonds is derived based on the determination of the bond value before and after rebalancing. The bond value before rebalancing delivers a coupon at maturity that is based on the bond-at-par strategy ensuring a bond present value of 100. Given the rebalancing feature, the risk-free bond portfolio value after rebalancing is equal to a price of 100. The government bond total return is analytically expressed below in a general case gathering the production of a set of economic scenarios[18] and over a projection term T :

$$r_{\lambda}^{gov}(t,n) = \frac{v_{\lambda}^{B}(t,n)}{v_{\lambda}^{A}(t,n)} - 1 \tag{4}$$

Where $r_{\lambda}^{corp}(t,n)$ is the risk-free (governemnt) total relative return[19] of a n-year default and migration-free bond for scenario λ with $\lambda \epsilon [1;L]$ generated at time t of the projection with $t \epsilon [0;T]$; $v_{\lambda}^{B}(t,n)$ and $v_{\lambda}^{A}(t,n)$ are the n-year government bond value (for scenario λ at time t of the projection) before and after rebalancing respectively. Both bond values are based on the replication of a bond-at-par strategy with a fixed annual coupon payment[20]. More precisely, using the same conventions introduced above, for a risk-free government bond, the bond value before rebalancing is derived as follows:

$$v_{\lambda}^{B}(t,n) = y_{\lambda}(t-1,n) \sum_{i=1}^{n-2} \left(\frac{1}{1+r_{\lambda}(t,i)}\right)^{i} + \frac{1+y_{\lambda}(t-1,n)}{\left(1+r_{\lambda}(t,i)\right)^{n-1}} + c_{\lambda}(t,n) \tag{5}$$

Where $y_{\lambda}(t,n)$ refers to the n-year yields and $c_{\lambda}(t,n)$ the n-year income yields (also referred to as the coupon at maturity) for scenario λ at time t of the projection. In a similar way, the government bond value after rebalancing adapted for default and migration-free bonds is given by:

$$v_{\lambda}^{A}(t,n) = y_{\lambda}(t,n) \sum_{i=1}^{n-1} \left(\frac{1}{1+r_{\lambda}(t,i)}\right)^{i} + \frac{1+y_{\lambda}(t,n)}{\left(1+r_{\lambda}(t,i)\right)^{n}} \tag{6}$$

C. Risk-Neutral Probability

The approach to estimating default probabilities is to use a credit migration model[21] described by a time-homogeneous Markov chain[22] with a generator matrix[23]. The Markov property can be mathematically formalized as follows: A process $(X_t)_t$ is a Markov process with respect to the filtration $(F_t)_t$ when X_t is adapted to the filtration F_t, and, for any $s > t$, X_s is independent of F_t given X_t. If the process $(X_t)_t$ is also conditionally stationary, then it is a time-homogenous Markov process.

[18] The formalism introduced is compliant with the derivation of TVOG requiring the production of a full set of economic scenario over a long-term projection period.

[19] The main reason for working with relative returns rather than absolute returns is that relative returns are independent from the absolute level of bond prices and are therefore comparable among each others (i.e. the latter does not measure changes in term of the given bond price level).

[20] The presented formula is only correct for an annual frequency for coupon payments. However, one could easily extend it in order to encompass for a semi-annual coupon payment for instance.

[21] A credit migration model characterizes the change in credit quality of obligors.

[22] A Markov chain is a discrete-time random process endowed with the Markov property (i.e. the conditional probability distribution of the process at the next step given its current state depends only on the current state of the system, and not additionally on the state of the system at previous steps). While the process changes randomly and does not generally allow to predict the exact state in the future, the statistical properties of the process's future can be predicted.

[23] This part is based on findings of Robert A. Jarrow, David Lando, Stuart M. Turnbull, [2].

Following Robert A. Jarrow, David Lando, Stuart M. Turnbull, [2], a continuous time, time-homogenous Markov chain on a finite state space $S = \{1, \dots, k\}$ with generator matrix Δ is used to describe credit rating transitions. For instance, the probability of the bond making a transition from credit rating i to credit rating j over a small time-interval δt is given by the $(i, j)^{\text{th}}$ elements of the matrix:

$$I + \delta t \Delta \tag{7}$$

Add that the generator matrix is specified by a $k * k$ transition matrix where λ_{ij} represents the actual probability of going from state i to state j in one time step with $\lambda_{ij} \geq 0$ for all $i, j, i \neq j$ and $\lambda_{ii} = 1 - \sum_{j=1}^{k} \lambda_{ij}$ for all i[24].

$$\Delta = \begin{bmatrix} \lambda_{1,1} & & \cdots & \lambda_{1,k} \\ \lambda_{2,1} & & \cdots & \lambda_{1,k} \\ \vdots & \vdots & \dots & \vdots \\ \lambda_{k-1,1} & \lambda_{k-1,2} & \dots & \lambda_{k-1,k} \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{8}$$

The final row of the matrix corresponds to the bond being in default. It is assumed to be an absorbing state (i.e. once the bond has defaulted, it cannot be upgraded). Hence, $\lambda_{kj} = 0$ for $j = 1, \dots, k-1$ and $\lambda_{kk} = 1$. Powering up the matrix allow to calculate transition matrices over time-periods. It is known that the n-step $k * k$ transition matrix entry, $\Delta_{0,n}$ whose (i, j)th entry is $\lambda_{ij}(0, n)$ satisfies $\Delta_{0,n} = \Delta^n$, the n-fold matrix product of Δ. As a result, if a bond has credit rating i at time 0, then the probability that it will be in state j at time T is given by the (i, j)th element of the following matrix[25]:

$$P(0, T) = exp[\Delta T] \tag{9}$$

In particular, if there are k credit states, then the real-world probability of a bond initially in state i defaulting by time T is the (i, j)th element such as:

$$P_R(\tau \leq T) = \lambda_{ik}(0, T) \tag{10}$$

So far, the generator matrix corresponds to real-world transition probabilities estimated using historical transition data. However, given that the objective is to value bonds under a risk-neutral probability measure, it is desirable to set up a transformation process to determine risk-neutral probabilities based on the estimated real-world probabilities. Following Robert A. Jarrow, David Lando, Stuart M. Turnbull, [2], equation (6), the authors derive a deterministic function of time π_i to characterize the risk premium required by investors for bearing credit default and migration risk. Denoting $\tilde{\lambda}_{ij}$ the (i, j)th element of the risk-neutral generator matrix, the authors assume the following relationship:

$$\tilde{\lambda}_{ij} = \pi_i . \lambda_{ij} \tag{11}$$

In order to derive positive risk-neutral probabilities, the π_i are positive. However, the problem with assuming deterministic π_i is that the resulting model produces deterministic credit spread. To allow for stochastic spreads, the next section proposes an appropriate extension[26] to the Jarrow, Lando and Turnbull model.

[24] The probabilities of each row sum to one. State 1 is the highest one and state k represents bankruptcy.

[25] The exponential of a matrix being defined in terms of its series expansion is: $exp[\Delta t] = 1 + \Delta t + \frac{1}{2}(\Delta t)^2 + \cdots$

[26] Add that Robert A. Jarrow, David Lando, Stuart M. Turnbull, [2] allows the factors to change over time i.e. $\pi_i = \pi_i(t)$ in order to closely fit observed bond prices. There are however assumed to be deterministic function of time.

III. MODEL EXTENSION

A. Overall Framework

As introduced before, the standard Jarrow, Lando and Turnbull model relies on the use of deterministic spreads by assuming deterministic scaling factors π_i. This section provides an extension to the model based on Arvanitis, Angelo, Jonathan Gregory, Jean-Paul Laurent, [7] in order to allow for stochastic spreads.

First, it is assumed that the real-world probability matrix generator is diagonalizable in order to make the model tractable.

$$\Delta = \sum D \sum^{-1} \tag{12}$$

where D is the diagonal matrix[27] of eigenvalues of Δ and the column of $\sum \in R^{k*k}$ are the right eigenvectors of Δ. In a similar way, it is assumed that the risk-neutral probability matrix generator is given by:

$$\tilde{\Delta} = \sum \widetilde{D}(t) \sum^{-1} \tag{13}$$

where $\sum \in R^{k*k}$ is the matrix introduced in equation (12) and $\widetilde{D}(t)$ is a time-dependent stochastic diagonal matrix in order to allow for stochastic spreads. The risk-neutral probabilities are obtained by taking the expectation over all possible future paths of $\widetilde{D}(t)$ such as[28]:

$$Q(0,T) = \sum E_Q\left[exp\left(\int_0^T \widetilde{D}(s)ds\right)\right]\sum^{-1} \tag{14}$$

In particular, default probabilities (i.e. the (i,k)th element of the above matrix) are then given by[29]:

$$q_{ik}(0,T) = \sum_{j=1}^{k-1} \sigma_{ij}\tilde{\sigma}_{jk}\left\{E_Q\left[exp\left(\int_0^T \tilde{d}_j(s)ds\right)\right] - 1\right\} \tag{15}$$

where σ_{ij} and $\tilde{\sigma}_{jk}$ are the elements of the matrices $\sum$ and $\sum^{-1}$ respectively. According to this expression, bond prices are analytically derived. To fully specify the model, the stochastic process followed by the matrix elements $\tilde{d}_j$ needs to be specified. It is proposed to extend the approach of A. Jarrow, David Lando, Stuart M. Turnbull, [2] by considering the following case:

$$\widetilde{D}(t) = \pi(t)D \tag{16}$$

with $\pi(t)$ is a stochastic process[30]. As a result, the real-world and risk-neutral world probabilities are related as follows:

$$\tilde{\Delta}(t) = \pi(t)\Delta \tag{17}$$

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[27] The diagonal entries of D are called the singular values of the matrix Δ. The singular values are directly the square roots of the eigenvalues given that the eigenvalues are all positive (Δ is a positive-definite matrix). Let A be a $n*n$ matrix. A real number λ is called an eigenvalue of A if there exists a non zero vector x in R^* such that $Ax = \lambda x$. Each non-zero vector x satisfying this equation is called an eigenvector of A associated with the eigenvalue λ. When a matrix is symmetric, all eigenvalues are real numbers. Furthermore, all eigenvectors that belong to distinct eigenvalues are orthogonal. A symmetric positive-definite is a matrix with non negative eigenvalues.

[28] Assuming that $\widetilde{D}(t)$ is a deterministic function, then the risk-neutral probabilities are then given by: $Q(0,T) = \sum\left[exp\left[\int_0^T \widetilde{D}(s)ds\right]\right]\sum^{-1}$.

[29] Assuming again that $\widetilde{D}(t)$ is a deterministic function, then the default probabilities are then given by: $q_{ik}(0,T) = \sum_{j=1}^{k-1} \sigma_{ij}\tilde{\sigma}_{jk}\left\{exp\left(\int_0^T \tilde{d}_j(s)ds\right) - 1\right\}$.

[30] This stochastic process is specified later on.

In some extend, the approach adopted is a more general case than the Jarrow, Lando, Tunrbull model where a stochastic scaling factor is used for each row of the matrix as shown in equation (18). Given this specification for $\widetilde{D}(t)$, equation (15) can be enriched with the scaling factor:

$$q_{ik}(0,T) = \sum_{j=1}^{k-1} \sigma_{ij}\tilde{\sigma}_{jk} \left\{E_Q\left[exp\left(d_j \int_0^T \pi(s)ds\right)\right] - 1\right\} \tag{18}$$

B. Stochastic Process for $\pi(t)$

The stochastic process suggested for $\pi(t)$ to ensure the generation of stochastic spreads is a mean-reverting model as it allows the consideration of different economic cycles. For instance, when economic conditions start to decline, companies' creditworthiness becomes worse. As a result, spreads increase. Yet, such a situation cannot be persistent over time. Economic conditions will eventually recover in the future, leading to an increase in the creditworthiness of the companies within the rating specific scale and therefore to a decrease of spreads. Hence, a simple analysis at this point is likely to reveal a mean reversion behaviour for spreads. Moreover, it is relevant to select a model that ensures the generation of positive risk-neutral probabilities only. Hence, the Cox-Ingersoll-Ross model, a one-factor equilibrium model[31] is used. The capacity of the model to replicate market movements has made the Cox-Ingersoll-Ross model a popular choice amongst practitioners. The stochastic differential equation for this model is:

$$d\pi(t) = \alpha\big(\mu - \pi(t)\big)dt + \sigma\sqrt{\pi(t)}dW_t \tag{19}$$

The model exhibits mean reversion properties because if spreads are higher than the mean reversion level $(\pi(t) > \mu)$, then mean reversion models tend to have a negative drift so that spreads are pulled down in the direction of the mean reversion level μ. Similarly, if the spread level is lower than the mean reversion level $(\pi(t) < \mu)$, then mean reversion models tend to have a positive drift so that spreads are pulled up in the direction of the mean reversion level μ. In addition, the stronger the mean reversion speed α, the faster the model pulls back to the mean reversion level μ. Finally, the model only produces positive spreads given the inclusion of a square root of time.

Furthermore, the expectation under the risk-neutral probability measure introduced in equation (14) and (15) can be derived analytically in a similar way as performed for the calculation of zero coupon bond prices within the Cox-Ingersoll-Ross model[32]. It leads:

$$E_Q\left[exp\left(d_j \int_0^T \pi(s)ds\right)\right] = exp\left(A_j(0,T) - \pi(0)B_j(0,T)\right) \tag{20}$$

where:

$$A_j(0,T) = \frac{2\alpha\mu}{\sigma^2} ln\left(\frac{2ve^{\frac{1}{2}(\alpha+v_j)T}}{(v_j+\alpha)\left(e^{v_jT}-1\right)+2v_j}\right) \qquad B_j(0,T) = \frac{-2d_j\left(e^{v_jT}-1\right)}{(v_j+\alpha)\left(e^{v_jT}-1\right)+2v_j} \tag{21}$$

$$v_j = \sqrt{\alpha^2 - 2d_j\sigma^2} \tag{22}$$

[31] Equilibrium models usually start with assumptions about economic variables and derive a process. Drift and diffusion terms contain constant parameters. See John C. Hull, [8], chapter 28.2: Equilibrium models", page 650 for further details.
[32] Refer to John C. Hull, [8] for further details.

V. CONCLUSION

The extension of the Jarrow, Lando, Turnbull model proposed through this article derives the risk-neutral valuation of bonds, with an allowance for stochastic spreads, while conserving the analytical traceability of the standard version. Inserting stochastic spreads within the model is of particular importance for life insurance companies, where participating life portfolios require the determination of the value of options and guarantees embedded within the insurance contracts. The determination of such options is based on a long-term projection method, where stochastic simulations are required for any asset class. In this context, given the large exposure of insurance companies to corporate bonds, the determination of economic scenarios for risky corporate bond returns is of utmost importance. Add also that the model proposed is robust and allows to incorporate additional complexity to capture, for instance, a model specification for recovery rates. The model appears also easily understandable and acknowledges the fundamental characteristic of the concept, namely that bond returns must contain a credit risk and a time-dependent spread component when performing the bond valuation.

The reasonableness of the assumption performed for the pricing of bonds, assuming that the stochastic process for risk-free short rates and default is statistically independent under Q, requires further empirical tests and investigations. Moreover, the distribution of $\pi(t)$ might be different under the real-world and risk-neutral probability measure. Within the model specification, the Brownian motion W_t introduced in equation (20) may be different under the real-world probability measure in order to reflect the additional return required by investors to bear the risk. What it is assumed through equation (20) is that the Cox-Ingersoll-Ross stochastic process specified holds in both the real-world and risk-neutral probability measure. It is however a strong assumption. Furthermore, the calibration of such a model is a delicate operation as the stochastic process requires to be calibrated in order to give an appropriate fit for all the different rating classes. This is clearly a difficult operation as finding such a calibration leads to simulations that overestimate AAA spreads and underestimates BB spreads when assuming an “average” fit over all rating classes. Setting up the model is there a trade-off requiring judgmental expertise. Finally, a natural extension of this paper would be to encompass illiquidity premiums within the model specification, as the 2007-2008 financial crisis has highlighted the considerable contribution of illiquidity that arises within the bond valuation when facing severe distressed economic conditions.

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