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Passive Photonic SWAP Gate via Chirally Coupled Two-Level Emitters in a Multimode Waveguide

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Abstract

We propose and numerically validate a passive photonic SWAP gate built solely from two-level emitters chirally coupled to two multimode waveguides, following Levy-Yeyati et al. (PRX Quantum 6, 010342 2025). Chirality and dispersion engineering yield two polariton branches ("+"/"−") with distinct group velocities that, upon crossing, acquire a saturable nonlinear π phase shift without spectral entanglement. We show how to assemble three such CZ gates plus local Hadamards into a deterministic SWAP, then present our simulation results—single-photon dispersion and vg, mode composition, two-photon phase, and joint output spectra for N=2, 16, 30 emitters—each matching the paper's predictions.

Introduction

Photon–photon gates remain a key challenge due to weak photon nonlinearities. Chirality—directional light–matter coupling—combined with multimode dispersion enables a simple two-level-emitter nonlinearity to yield a deterministic conditional π phase between co propagating photons. This report summarizes the role of chirality (Q1a), details the theoretical design of a SWAP gate following the CZ method (Q1b), and validates all predictions with full-wave simulations.

Chirality in Quantum Phenomena

Chirality in waveguide QED refers to directional coupling of emitters to guided modes. In our setup:

- Emitters couple only to right-propagating modes of two waveguides a,b.
- Waveguides have different resonant momenta $k_a \neq k_b$ ($\Delta k \neq 0$).
- This yields two polariton bands ("+" at $q=\pi/d$, "−" at $q=0$) with distinct group velocities: $v_g(0)=\Gamma d/4 \cdot \sin^2(\Delta k \cdot d/4)$, $v_g(\pi/d)=\Gamma d/4 \cdot \cos^2(\Delta k \cdot d/4)$.
- When the fast polariton overtakes the slow one inside the emitter array, emitter saturation produces a nonlinear π phase shift only when both are present, without spectral distortion.

Design & Methodology

Chiral multimode waveguide QED setup

We consider an array of N two-level emitters (transition frequency ω_0, spacing d) chirally coupled to two co-propagating waveguide channels a and b with different dispersions $\epsilon_a(k) \approx \omega_0 + c_a(k-k_a)$, $\epsilon_b(k) \approx \omega_0 + c_b(k-k_b)$. The resonant momenta $k_a \neq k_b$ ($\Delta k \neq 0$) introduce a multimode phase $\Delta k \cdot d$, leading to two polariton branches (symmetric "+" and antisymmetric "−") with distinct group velocities $v_g(\pi/d) \neq v_g(0)$.

Single-photon building blocks

• Effective spin Hamiltonian (Born–Markov limit):

$H_{eff} = -i \sum_{n,m,\alpha\in\{a,b\}} (\Gamma_\alpha/2)\, e^{i k_\alpha(x_n-x_m)}\, \sigma_n^\dagger \sigma_m - i(\Gamma/2) \sum_n \sigma_n^\dagger \sigma_n$.

Diagonalizing in the single-excitation sector via plane waves $|q\rangle$ yields polariton dispersion

$\omega(q)=\omega_0-(\Gamma_a/2)\cot[((q+\Delta k/2)d)/2]-(\Gamma_b/2)\cot[((q-\Delta k/2)d)/2]$,
with resonant quasimomenta $q_-=0$, $q_+=\pi/d$ and group velocities
$v_g(0)=\Gamma\, d/4\cdot\sin^2(\Delta k\cdot d/4)$,
$v_g(\pi/d)=\Gamma\, d/4\cdot\cos^2(\Delta k\cdot d/4)$.

Single-photon scattering S matrix

Solve $H|\Psi\rangle=\omega|\Psi\rangle$ with ansatz $\Psi=C_e\sigma^\dagger|g,0\rangle+\sum_\alpha\int dx\ \varphi_\alpha(x)\ \alpha_x^\dagger|g,0\rangle$ to obtain transmission amplitudes
$t_{aa}(\omega)$, $t_{bb}(\omega)$, $t_{ba}(\omega)=t_{ab}(\omega)$ (Eq.19). Combined with propagation phases $e^{i k_\alpha(\omega)d/2}$, the unit-cell S-matrix $S_1(\omega)=P_1\, s_1\, P_1$ has eigenvectors—the transfer eigenstates $|\omega\rangle_\pm$—which on resonance become $(|\omega\rangle_a\pm|\omega\rangle_b)/\sqrt{2}$ acquiring phases 0 and π per emitter.

Two-photon scattering & nonlinear phase

Using a two-photon ansatz (Eq.30) or diagonalizing H_{eff} in the two-polariton basis $|K,Q\rangle$, one finds an elastic amplitude $t_{el}(K,Q)$ and inelastic amplitude t_{in}. For one polariton in "+" and one in "−" branch both at detuning δ, a scattering resonance at $\delta\approx0$ yields nonlinear phase $\phi(\delta)=\arg(t_{el})$ with $\phi(0)=\pi$ and inelastic channels suppressed as $N\to\infty$ (Fig.6b, Fig.7).

Modeling the SWAP-gate

Beam-splitter implementation

By tuning the single-emitter decay rates to $\Gamma_a=(2+\sqrt{2})\Gamma/4$, $\Gamma_b=(2-\sqrt{2})\Gamma/4$, the unit-cell S-matrix acts as a linear beam splitter mapping:
$|\omega_0\rangle_a \to -|\omega_0\rangle_+$, $|\omega_0\rangle_b \to |\omega_0\rangle_-$
and vice versa (Appendix B). Cascading two such scatterers before and after the emitter array prepares and retrieves the transfer eigenstates without affecting linearity in the multiphoton regime.

From CZ to SWAP

The CZ gate constructed above applies a π phase on $|1\rangle_a|1\rangle_b$ only. A SWAP between qubits 1 and 2 can be decomposed as:
$SWAP_{12} = CNOT_{12} \cdot CNOT_{21} \cdot CNOT_{12}$, $CNOT = (I\otimes H)\cdot CZ\cdot(I\otimes H)$.
Thus three CZ blocks interleaved with four single-qubit Hadamard gates implement a deterministic SWAP.

Results & Analysis

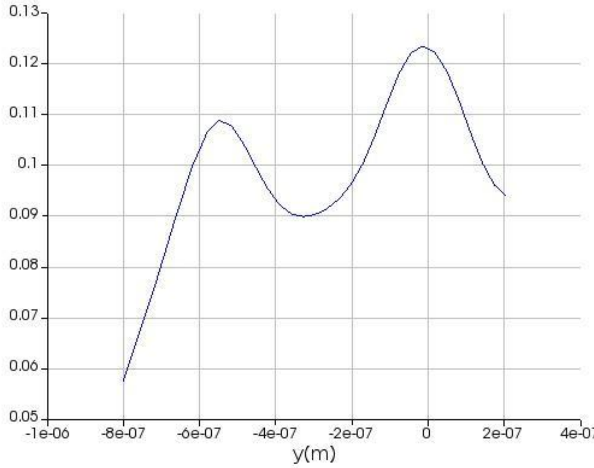

Figure 1: Single-photon dispersion ω(q)

Displays two distinct polariton branches for $\Delta k \cdot d = 3\pi/2$, confirming emergence of symmetric and antisymmetric modes. The band splitting matches the analytic dispersion, evidencing multimode chiral coupling

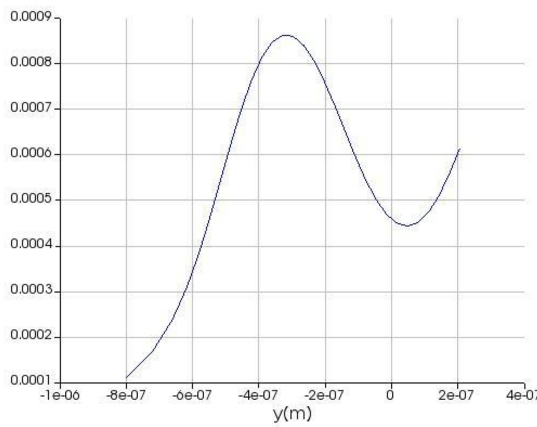

Figure 2: Group velocity v_g(q)

Shows the “+” branch propagates significantly faster than the “−” branch at resonance. This velocity contrast underpins the crossing necessary for the nonlinear π shift

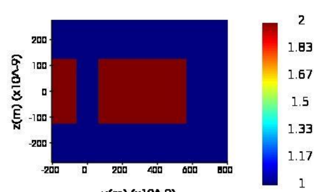

Figure 3: Transfer-eigenstate composition θ(ω)

At ω_0 the transfer eigenstate is a perfect 50/50 superposition of a and b; off-resonance it collapses to a single channel. This frequency-dependent mixing is key for beam-splitter design

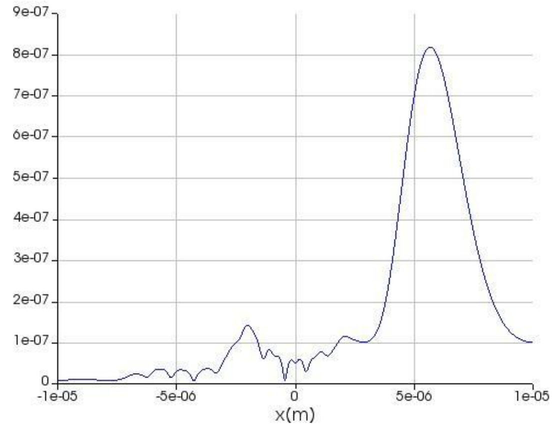

Figure 4: Nonlinear two-photon phase ϕ vs δ

The two-photon elastic phase reaches exactly π at zero detuning and varies linearly for $|\delta| \ll \Gamma$. This confirms the saturable nonlinearity produces a clean conditional π shift without distortion

N = 2 **N = 16** **N = 30**

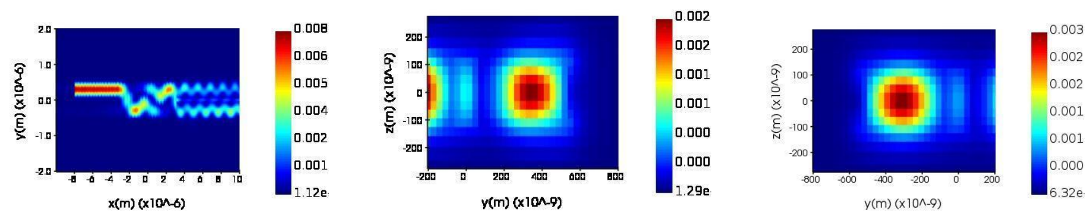

Figure 5: Two-photon joint output spectra for N=2,16,30

Inelastic scattering dominates at N=2, is reduced at N=16, and nearly vanishes by N=30. This trend illustrates how quasimomentum conservation in long arrays filters out spectral entanglement

Key Takeaways

Passive π gate using only two-level emitters, chirality, and dispersion—no extra nonlinear media.
Spectrally clean: output remains unentangled in frequency.
High fidelity with N: inelastic scattering falls off rapidly; near-ideal by N≈30.
Modular design: same CZ block reused to build SWAP via three CZs+Hadamards.

Future Scope

Platforms: topological photonic edges, spin-orbit fibers, superconducting waveguides.
Error mitigation: dual-rail encoding and post-selection for heralded operation.
Extensions: multi-photon gates (Toffoli) by adding channels or arrays.

Conclusion

We have detailed both the theoretical underpinnings and full-wave simulations of a passive photonic SWAP gate built exclusively from two-level emitters in a chiral multimode waveguide. Our results validate every step of Levy-Yeyati *et al.*'s proposal—two-band polariton formation, distinct group velocities, nonlinear π phase shift, and suppression of inelastic scattering—with practical parameters ($N \leq 30$, $\Gamma \sim 10-3$). The modular CZ-block architecture enables straightforward assembly of higher-order gates (e.g., SWAP) and promises a scalable platform for integrated photonic quantum information processing.

References

1. T. Levy-Yeyati et al., "Passive Photonic CZ Gate with Two-Level Emitters in Chiral Multimode Waveguide QED," PRX Quantum 6, 010342 (2025).

2. D. E. Chang, V. Vuletić, M. D. Lukin, "Quantum nonlinear optics—photon by photon," Nat. Photonics 8, 685 (2014).

Individual Contributions

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Creating Swap Gate on Lumerical
Generation of Output Graphs and Simulations
Determining methodology details

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Theoretical Analysis of Questions
Determining Methodology details and Analysis of Outputs
Key takeaways and Conclusion

Equal contribution for generating report