AI1103-Assignment 1

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Download all python codes from

https://github.com/SHASHANK-1-ALL/Assignment1/blob/main/Assignment1.py

and latex-tikz codes from

https://github.com/SHASHANK-1-ALL/Assignment1/blob/main/Assignment1.tex

Question

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1,2,3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1,2,3 or 4 with the die?

Solution

Let $X \in \{0, 1\}$ where X=0 represents that we get 1,2,3 or 4 when a die is rolled and X=1 represents that we get 5 or 6 when a die is rolled.
Let $Y \in \{0, 1, 2, 3\}$ where Y=1 represents that we get exactly one head.Here Y represents the number of heads obtained.

We are required to find probability of getting X=0 when Y=1.
Here we use Bayes' theorem.

$$\Pr(X = 0|Y = 1) = \frac{\Pr(X = 0)\ \Pr(Y = 1|X = 0)}{\sum_{i=0}^{1} \Pr(X = i)\ \Pr(Y = 1|X = i)} \tag{0.0.1}$$

Note that

$$\Pr(X = 0) = \frac{4}{6} = \frac{2}{3} = 0.6666666667 \tag{0.0.2}$$

$$\Pr(X = 1) = \frac{2}{6} = \frac{1}{3} = 0.3333333333 \tag{0.0.3}$$

Also we get

$$\Pr(Y = 1|X = 0) = \frac{1}{2} = 0.5 \tag{0.0.4}$$

$$\Pr(Y = 1|X = 1) = \frac{3}{8} = 0.375 \tag{0.0.5}$$

Substituting values, we get

$$\Pr(X = 0|Y = 1) = \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{8}} \tag{0.0.6}$$

$$\implies \Pr(X = 0|Y = 1) = \frac{8}{11} = 0.7272727273 \tag{0.0.7}$$