AI1103-Assignment 3

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Download all python codes from

https://github.com/SHASHANK−1−ALL/AI1103−Assignment−3/blob/main/Assignment3.py

and latex-tikz codes from

https://github.com/SHASHANK−1−ALL/AI1103−Assignment−3/blob/main/Assignment3.tex

Question

The lifetime of two brands of bulbs X and Y are exponentially distributed with the mean life of 100 hours. Bulb X is switched on 15 hours after bulb Y has been switched on. The probability that bulb X fails before bulb Y is

(A) $\frac{15}{100}$

(B) $\frac{1}{2}$

(C) $\frac{85}{100}$

(D) 0

Solution

Let X and Y be exponential random variables which represent the lifetime of bulbs X and Y respectively, both with mean = 100.

Using memorylessness property for exponential distribution, which states that :
An exponentially distributed random variable T obeys the relation

$$\Pr(T > t + s | T > s) = \Pr(T > t) \tag{0.0.1}$$

where $s, t \geq 0$

Proof : Using Complementary cumulative distributive function, we get

$$\Pr(T > t + s | T > s) = \frac{\Pr(T > t + s, T > s)}{\Pr(T > s)} \tag{0.0.2}$$
$$= \frac{\Pr(T > t + s)}{\Pr(T > s)} \tag{0.0.3}$$
$$= \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} \tag{0.0.4}$$
$$= e^{-\lambda t} \tag{0.0.5}$$
$$= \Pr(T > t) \tag{0.0.6}$$

Probability that bulb X fails before bulb Y given that bulb Y was functioning when bulb X was switched on

$$\Pr(Y > X + 15 | Y \geq 15) = \Pr(Y > X) \tag{0.0.7}$$

For both X and Y,

$$\lambda = \frac{1}{100} = 0.01 \tag{0.0.8}$$

Probability distribution function of exponential random variables is given by :
For x,y ≥ 0

$$f_X(x) = \lambda e^{-\lambda x} \tag{0.0.9}$$
$$f_Y(y) = \lambda e^{-\lambda y} \tag{0.0.10}$$

Cumulative distribution function of exponential random variables is given by :
For x ≥ 0

$$F_X(x) = 1 - e^{-\lambda x} \tag{0.0.11}$$
$$F_Y(x) = 1 - e^{-\lambda x} \tag{0.0.12}$$

$$\Pr(Y > X) = \int_{-\infty}^{\infty} F_{\mathrm{Y}}(x) f_{\mathrm{X}}(x) dx \tag{0.0.13}$$

$$= \int_{0}^{\infty} (1 - e^{-\lambda x}) \lambda e^{-\lambda x} \tag{0.0.14}$$

$$= \lambda \left(\frac{1}{2\lambda} e^{-2\lambda x} - \frac{1}{\lambda} e^{-\lambda x} \right) \Bigg|_0^{\infty} \tag{0.0.15}$$

$$= \left(\frac{1}{2} e^{-2\lambda x} - e^{-\lambda x} \right) \Bigg|_0^{\infty} \tag{0.0.16}$$

$$= \left(\frac{1}{2} e^{-0.02x} - e^{-0.01x} \right) \Bigg|_0^{\infty} \tag{0.0.17}$$

$$= \frac{1}{2} = 0.5 \tag{0.0.18}$$

$\therefore$The answer is option (b) $\frac{1}{2}$.