AI1103-Assignment 6

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Download all python codes from

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https://github.com/SHASHANK−1−ALL/AI1103−ASSIGNMENT−6/blob/main/Assignment6.py
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and latex-tikz codes from

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https://github.com/SHASHANK−1−ALL/AI1103−ASSIGNMENT−6/blob/main/Assignment6.tex
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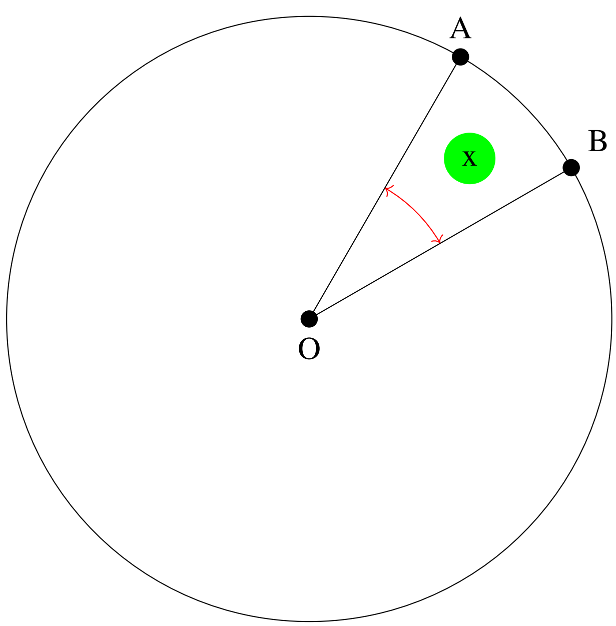
Question

A point is chosen at random from a circular disc shown below. What is the probability that the point lies in the sector OAB?



(where $\angle AOB$ = x radians)

1) $\frac{2x}{\pi}$
2) $\frac{x}{\pi}$
3) $\frac{x}{2\pi}$
4) $\frac{x}{4\pi}$

Solution

Let $X \in \{0, 1\}$ be a random variable such that X=0 means we choose a point lying in sector OAB and X=1 means that we choose a point lying outside sector OAB and inside the circle.

Area of a sector subtending an angle θ at the centre of circle with radius a is given by :

$$A = \frac{1}{2}a^2\theta \tag{0.0.1}$$

where θ is in radians.

Let the radius of circle shown in figure be r. It is given that sector OAB subtends an angle of x radians at the centre of the circle.

Probability that the chosen point lies in sector OAB is:

$$\Pr(X = 0) = \frac{\text{Area of sector OAB}}{\text{Area of circle}} \tag{0.0.2}$$

$$= \frac{\frac{1}{2}r^2 x}{\pi r^2} \tag{0.0.3}$$

$$= \frac{x}{2\pi} \tag{0.0.4}$$

$\therefore$The correct answer is **option (3)** $\frac{x}{2\pi}$.

Alternate Solution

The joint pdf is given by:

$$f_{r\theta}(r, \theta) = \begin{cases} \frac{r}{\pi R^2} & \text{if } 0 < \text{r} < \text{R} \text{ , } 0 < \theta < 2\pi \\ 0 & \text{otherwise} \end{cases} \tag{0.0.5}$$

Let A $\equiv$ (R, θ_2) and B $\equiv$ (R, θ_1).
Hence,

$$(\theta_2 - \theta_1) = x \tag{0.0.6}$$

We want $\theta \in (\theta_1 , \theta_2)$ and r $\in$ (0,R) for point to lie in the sector. Let the point to be chosen be (r, θ).

So, Required probability is:

$$\Pr(\theta_1 < \theta < \theta_2, 0 < r < R)$$

$$= \int_{\theta_1}^{\theta_2} \int_0^R \frac{r}{\pi R^2}\, dr\, d\theta \tag{0.0.7}$$

$$= \int_{\theta_1}^{\theta_2} \frac{1}{\pi R^2} \frac{r^2}{2} \bigg|_0^R \tag{0.0.8}$$

$$= \int_{\theta_1}^{\theta_2} \frac{R^2}{2\pi R^2} \, d\theta \tag{0.0.9}$$

$$= \int_{\theta_1}^{\theta_2} \frac{1}{2\pi} \, d\theta \tag{0.0.10}$$

$$= \frac{\theta}{2\pi} \bigg|_{\theta_1}^{\theta_2} \tag{0.0.11}$$

$$= \frac{\theta_2 - \theta_1}{2\pi} \tag{0.0.12}$$

$$= \frac{x}{2\pi} \tag{0.0.13}$$

$\therefore$The correct answer is **option (3)** $\frac{x}{2\pi}$**.**