625.664 Computational Statistics

Problem Set 4

Associated Reading: Chapter 1: Section 1.5,
Chapter 6: Introduction - 6.2.3.1

Complete the problems either by hand or using the computer and upload your final document to the Blackboard course site. All final submittals are to be in PDF form. Please document any code used to solve the problems and include it with your submission.

0. Read Example 6.2.

1. Use the Monte Carlo Method of Integration to evaluate the following integral:

$$\int_0^1 4\sqrt{1-x^2}\,dx$$

2. Consider the (triangular) probability density function defined as

$$f(x) = \begin{cases} 4x & 0 \le x \le 1/2 \\ 4-4x & 1/2 \le x \le 1 \end{cases}$$

   (a) Draw plots of the density and of the CDF.
   (b) Use the Inverse CDF method to analytically find a function $G(u)$ that transforms random numbers from $U(0,1)$ distribution into random numbers from $f(x)$.
   (c) Use part (b) to generate 1000 data points from $f(x)$ and plot their histogram.
   (d) Use rejection sampling to generate 1000 data points from $f(x)$ and plot their histogram. What instrumental distribution and envelope did you use?
   (e) Use the Monte Carlo method to estimate $E[X^2]$ when X is drawn from the distribution with density f and compare your estimate to the actual value. You may use the data generated in (c) or (d) for your estimation.

3. Assume that you want to sample from a $N(0,1)$ distribution using a $N(1,2)$ distribution as instrumental distribution.
   (a) Draw a sample of size 10000 using rejection sampling, plot the histogram and calculate the mean and variance of the sample.
   (b) What were your choices for α and the envelope $e(x)$?
   (c) Now consider the same problem, but using squeezed rejection sampling. What advantage is there to this method over standard rejection sampling? Give an example of a squeezing function that could be used to solve this problem. Note: You do not need to generate the random samples for this part.