Johns Hopkins Engineering

625.464 Computational Statistics

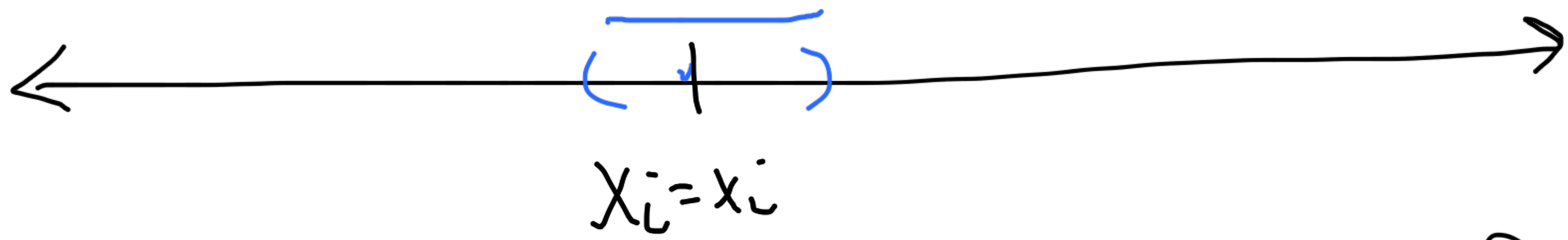
Introduction to Kernel Estimators

Module 11 Lecture 11B

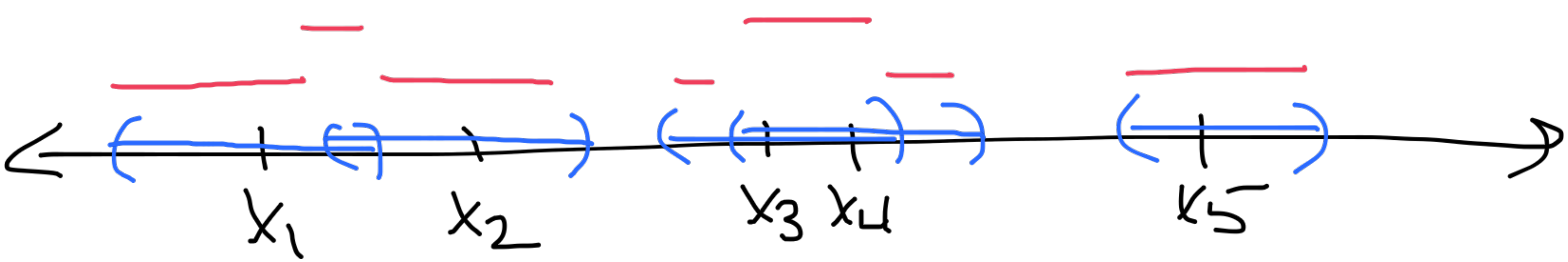
JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Motivation for Kernel Estimators

If we obs. $X_i = x_i$, we assume f assigns density to some interval around x_i.

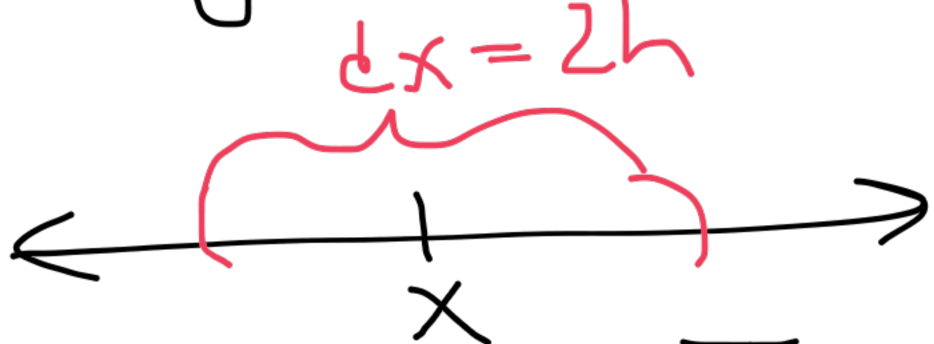


$\therefore$ to estimate f from $X_1, \dots, X_n \sim iid\ f$ it makes sense to accumulate contrib. to the region.



Kernel Estimators

To estimate the density at point x, we consider the region $dx = 2h$ (h fixed) centered at x.



$\gamma = [x-h, x+h]$ The proportion of obs in γ is an indic. of the density at x.

$$\hat{f}(x) = \frac{1}{2hn} \sum_{i=1}^{n} I\{|x - x_i| < h\}$$

Kernel est uniform Kernel

$nh \to \infty$ and $h \to 0$ as $n \to \infty$

need for $\hat{f}$ to be reasonable and pt. wise consistent

Kernel Estimators

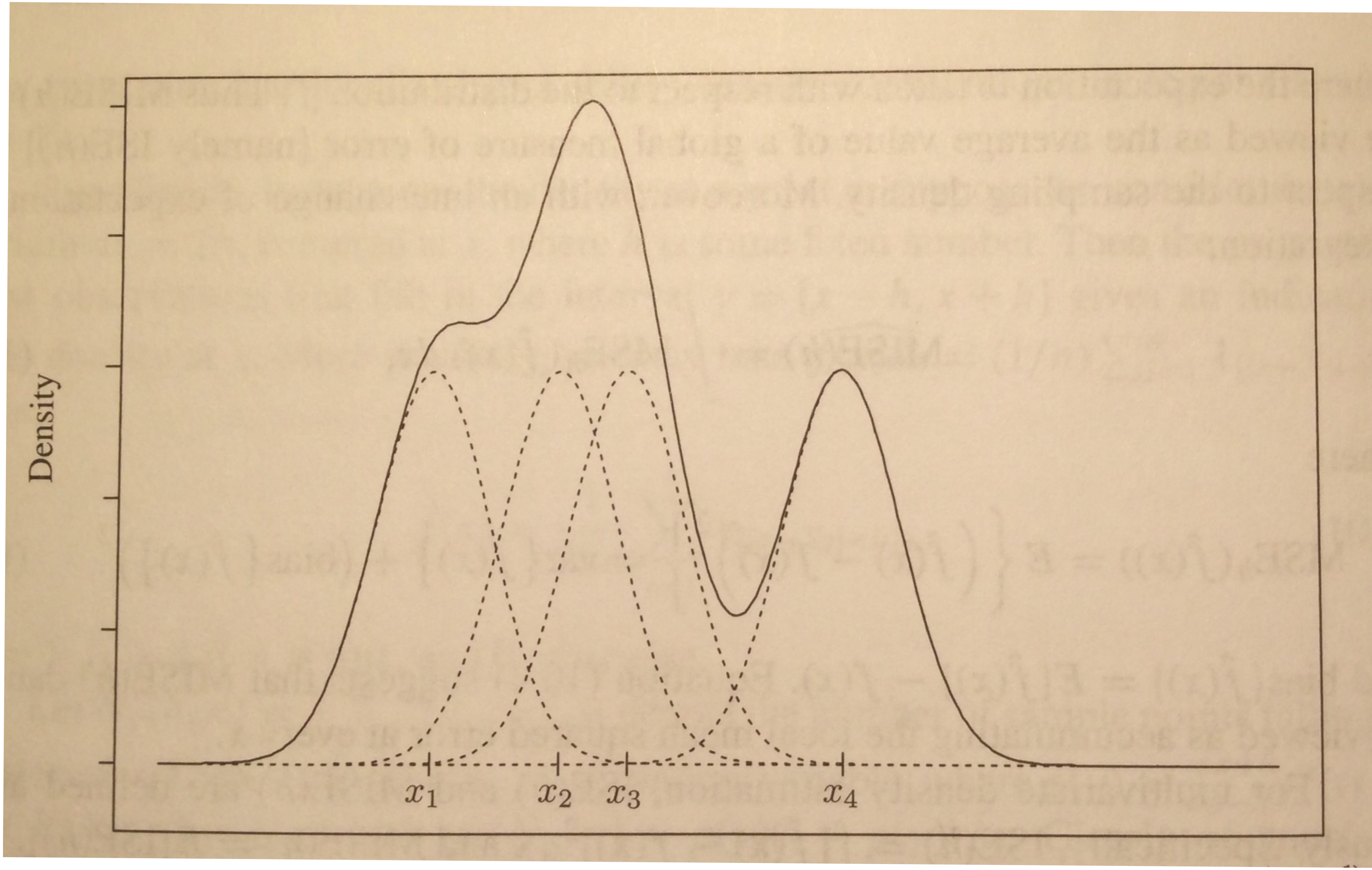
$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

where K is a kernel function and h is the bandwidth.

Comments:

1. The kernel function weights the contributions given by each x_i to the density est $\hat{f}(x)$ based on the proximity of x_i to x.
2. K is usually pos. everywhere and sym. about 0.
3. K is often a density, but doesn't have to be.
4. The kernel function on the previous slide was the uniform.
5. bandwidth adjusts range of influence.

Kernel Estimator Example



When constructing a Kernel Density Estimator you need to consider two things:

1. the Kernel K
2. the bandwith h

#2 is of much greater importance.