Johns Hopkins Engineering

625.464 Computational Statistics

Introduction to Function Estimation
Inner Products and Norms

Module 10 Lecture 10A

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WHITING SCHOOL
of ENGINEERING

Estimation of Functions

Basic Problem:

1. We have observations of the function at specific points.
2. We wish to estimate the function with two goals.
   (i) providing a good fit to the obs data
   (ii) predicting values at other points

Some Notation

We wish to estimate

$$f : \mathbb{R}^d \longrightarrow \mathbb{R} \text{ by estimator } \hat{f}$$

$\hat{\Theta}$ est. Θ

- $\hat{f}$ is a random variable w/ an underlying prob. dist.

Inner Products and Norms - A Crash Course

Def: Let f & g be real valued functions over the domain D, then the <u>inner product</u> of f & g denoted $\langle f, g \rangle$

$$\langle f, g \rangle = \int_D f(x)\, g(x)\, dx$$

if the integral exists.

① $\langle f, g \rangle_D$

② Lebesgue integral

③ def holds w/ slight var for complex functions

④ we will assume

$$\langle f, f \rangle = \int_D |f(x)|^2 dx$$

Inner Product Comments Continued

$$\langle f, g\rangle = \int_D f(x)\, g(x)\, dx$$

(5) Cauchy-Schwarz

$$\langle f, g\rangle \leq \langle f, f\rangle^{1/2} \langle g, g\rangle^{1/2}$$

(6) Sometimes we define IPS in terms of a weight function. $w(x)$ w.r.t a measure μ so that $d\mu = w(x)\,dx$

$$\langle f, g\rangle_{(\mu, D)} = \int_D f(x)\, g(x)\, w(x)\, dx$$

(7) Linear: $\langle af + g, h\rangle = a\langle f, h\rangle + \langle g, h\rangle$

Definition of a Norm

Def: The <u>norm</u> of a function f, denoted by $\|f\|$, is a mapping into the nonneg reals such that

(i) If $f \neq 0$, then $\|f\| > 0$

(ii) $\|0\| = 0$

(iii) $\|af\| = |a| \, \|f\| \quad \forall a \in \mathbb{R}$

(iv) $\|f+g\| \leq \|f\| + \|g\|$

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① often norms are defined using IP of f w/ itself however NOT ALL NORMS ARE DEFINED THIS WAY!

② L_p norm

$$\|f\|_p = \left(\int_D |f(x)|^p w(x) \, dx \right)^{1/p}$$

More Comments on Norms

3. The L_2 norm

$$\|f\|_2 = \langle f, f \rangle^{1/2}$$

4. The L_∞ norm

$$\|f\|_\infty = \sup |f(x)w(x)|$$

5. $\|f-g\|$

6. A normal function is one whose $\|f\| = 1$.

7. Assume necessary convergence.