# Johns Hopkins Engineering

## 625.464 Computational Statistics

### A Gibbs Sampling Example

Module 6 Lecture 6C

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# Gibbs Sampling
## A Stream Ecology Example

$Y = (Y_1, \ldots, Y_c)$ – denotes the counts of insects of different classes

$P = (P_1, \ldots, P_c)$ – denotes the prob of each class & depends on $\alpha_1, \ldots, \alpha_c$

$N$ – denotes the random ~~total~~ # of insects collected & depends on parameter $\lambda$.

Want to compare two test statistics $T_1(Y)$ and $T_2(Y)$

$c = 3$

# The Stream Ecology Model

$$(Y_1, Y_2, Y_3) \mid (N = n, P_1 = p_1, P_2 = p_2, P_3 = p_3)$$

$$\sim \text{Multinomial}\,(n, p_1, p_2, p_3)$$

$$(P_1, P_2, P_3) \sim \text{Dirichlet}\,(\alpha_1, \alpha_2, \alpha_3)$$

$$N \sim \text{Poisson}\,(\lambda)$$

How to sample $(Y_1, Y_2, Y_3)$?

$$Y_1 + Y_2 + Y_3 = N$$

$$P_1 + P_2 + P_3 = 1$$

$$\underline{X} = (Y_1, Y_2, P_1, P_2, N)$$

$$\lambda, \alpha_1, \alpha_2, \alpha_3$$

# Conditionals?

$$\{N, Y_1, Y_2, Y_3, P_1, P_2, P_3\}$$

$$(Y_1, Y_2, Y_3) \mid (N=n, P_1=p_1, P_2=p_2, P_3=p_3) \sim \text{Multinomial}(n, p_1, p_2, p_3)$$

$$(P_1, P_2, P_3) \mid (N=n, Y_1=y_1, Y_2=y_2, Y_3=n-y_1-y_2)$$

$$\sim \text{Dirichlet}(y_1+\alpha_1, y_2+\alpha_2, n-y_1-y_2+\alpha_3)$$

$$N-y_1-y_2 \mid (Y_1=y_1, Y_2=y_2, P_1=p_1, P_2=p_2, P_3=p_3)$$

$$\sim \text{Poisson}(\lambda(1-p_1-p_2))$$

# The Full Univariate Conditionals

$$\underline{X} = (Y_1, Y_2, p_1, p_2, N).$$

The Gibbs sampler

$$Y_1^{(t+1)} \sim \mathrm{Bin}\left(n^{(t)} - Y_2^{(t)}, \frac{p_1^{(t)}}{1-p_2^{(t)}}\right)$$

$$Y_2^{(t+1)} \sim \mathrm{Bin}\left(n^{(t)} - Y_1^{(t+1)}, \frac{p_1^{(t)}}{1-p_2^{(t)}}\right)$$

$$\frac{p_1^{(t+1)}}{1-p_2^{(t)}} \sim \mathrm{Beta}\left(y_1^{(t+1)} + \alpha_1, n^{(t)} - y_1^{(t+1)} - y_2^{(t+1)} + \alpha_3\right)$$

$$\frac{p_2^{(t+1)}}{1-p_1^{(t+1)}} \sim \mathrm{Beta}\left(y_2^{(t+1)} + \alpha_2, n^{(t)} - y_1^{(t+1)} - y_2^{(t+1)} + \alpha_3\right)$$

$$N^{(t+1)} - y_1^{(t+1)} - y_2^{(t+1)} \sim \mathrm{Poisson}\left(\lambda\left(1 - p_1^{(t+1)} - p_2^{(t+1)}\right)\right)$$