Johns Hopkins Engineering

625.464 Computational Statistics

A Few More Bootstrap Applications

Module 9 Lecture 9D

Bootstrapping for Hypothesis Testing

Consider a null hypothesis based on a para whose estimator can be bootstrapped.

- obtain a conf. int. by percentile method
- if the $(1-\alpha)100\%$ b.c.i does not cover the null value, we reject w/ p-value no greater than α.

Want the Bootstrap Sample to Reflect the Null Hypothesis

Consider a null hyp, H_0, about a univariate para θ w/ null value θ_0.

$$R(X, F) = \hat{\theta} - \theta_0$$

reject if $|\hat{\theta} - \theta_0|$ is large v/s ref dist

$$R(X^*, \hat{F}) = \hat{\theta}^* - \theta_0$$

θ_0 far from θ then $|\hat{\theta} - \theta_0|$ $|\hat{\theta}^* - \theta_0|$

What's a better choice?

instead of resampling $R(\mathcal{X}^*, \hat{F}) = \hat{\theta}^* - \theta_0$

we use

$$R(\mathcal{X}^*, \hat{F}) = \hat{\theta}^* - \hat{\theta}$$

Thon if θ_0 is far from θ the values

$|\hat{\theta}^* - \hat{\theta}|$ will be small compared to

$|\hat{\theta}^* - \theta_0|$

Jackknife After Bootstrap

Technique to reduce Monte Carlo variance

1. Est variance of B.S. est.
   How? Use the jackknife.

   - draw n separate B.S. on the original sample w/ a diff observation removed each time. then apply jackknife.

- Better way
- draw m b.s. samples & Store indices in m×n matrix.

- for each sample not containing x_i, Treat it like it came from the original sample w/ x_i omitted

$$\begin{array}{cccc} 1 & 2 & \cdots & m \end{array}$$
$$\begin{pmatrix} x_1 & x_4 \\ x_7 & x_{13} \\ x_{13} & x_2 \\ \vdots & \vdots \end{pmatrix}$$

Balanced Bootstrap

Consider a bootstrapped bias correction for the sample mean. Should be 0 since $\overline{X}$ is unbiased.

$$R(\mathcal{X}, F) = \overline{X} - \mu$$

bootstrapped

$$R(\mathcal{X}_j^*, \hat{F}) = \overline{X}_j^* - \overline{X} \qquad j = 1, \dots, B$$

ie, $\frac{1}{B}\sum_{j=1}^{B} R(\mathcal{X}_j^*, \hat{F})$ should be zero

Idea: Force each data value to occur in $\{X_1^*, X_2^*, \dots, X_B^*\}$ w/ same freq as in $\mathcal{X}$