

$$\widehat{V}(T)_J = \frac{\sum_{j=1}^r (T_j^* - J(T))^2}{r(r-1)} \leq \frac{\sum_{j=1}^n (T_j^* - T)^2}{n(n-1)} \quad \text{starting}$$

$$\Rightarrow \frac{n-1}{n} \sum_{j=1}^n (T_{(j)} - \bar{T}_{(1)})^2 \leq \frac{\sum_{j=1}^n (T_j^* - T)^2}{n(n-1)} \quad \text{from part (a)}$$

can also try alternate forms of $\widehat{V}(T)_J$:

$$J(T) = T + (r-1)(T - \bar{T}_{(1)})$$

$$\text{or } J(T) = \frac{1}{r} \sum_{j=1}^r T_j^* = \bar{T}^*$$

$$\text{or } J(T) = rT - (r-1)\bar{T}_{(1)} \quad (\text{part (a)})$$

$$\text{Result: no difference, } -J(T) \dots = -rT + r\bar{T}_{(1)} - \bar{T}_{(1)} \\ \left(= \frac{\sum_{j=1}^r \{T_j^* - rT + r\bar{T}_{(1)} - \bar{T}_{(1)}\}^2}{r(r-1)} \right)$$

$$T_j^* = rT - (r-1)T_{(j)}$$

$$\frac{\sum_{j=1}^r (rT - (r-1)T_{(j)} - J(T))^2}{r(r-1)} \leq \frac{\sum_{j=1}^n (nT - (n-1)T_{(j)} - T)^2}{n(n-1)}$$

$$\Rightarrow \sum_{j=1}^n (nT - (n-1)T_{(j)} - J(T))^2 \leq \sum_{j=1}^n (nT - (n-1)T_{(j)} - T)^2$$

$$\rightarrow \sum_{j=1}^n (T_j^* - 2T_j^* J(T) + J(T)^2) \leq \sum_{j=1}^n (T_j^* - 2T_j^* T + T^2)$$

$$\rightarrow \sum_{j=1}^n T_j^* - 2J(T) \sum_{j=1}^n T_j^* + nJ(T)^2 \leq \sum_{j=1}^n T_j^* - 2T \sum_{j=1}^n T_j^* + nT^2$$

$$\rightarrow -2J(T) \sum_{j=1}^n T_j^* + nJ(T)^2 \leq -2T \sum_{j=1}^n T_j^* + nT^2$$

$$\rightarrow -2\bar{T}^* \sum_{j=1}^n T_j^* + n\bar{T}^{*2} \leq -2T \sum_{j=1}^n T_j^* + nT^2$$

$$\rightarrow -2\bar{T}^* \sum_{j=1}^n (nT - (n-1)T_{(j)}) + n\bar{T}^{*2} \leq -2T \sum_{j=1}^n (nT - (n-1)T_{(j)}) + nT^2$$

$$\rightarrow -2\bar{T}^* [n^2 T - (n-1) \sum_{j=1}^n T_{(j)}] + n\bar{T}^{*2} \leq -2T [n^2 T - (n-1) \sum_{j=1}^n T_{(j)}] + nT^2$$