Johns Hopkins Engineering

625.464 Computational Statistics

Random Number Generation
The Inverse CDF Method

Module 4 Lecture 4B

Johns Hopkins
Whiting School of Engineering

Generation of Random Numbers / Simulation

1. generate pseudo random numbers from $U(0,1)$
2. use these to obtain variates from $f(x)$

1. computer. $r_{n+1} = (k r_n + a) \bmod m$

r_0 $\quad \frac{r_{n+1}}{m} \to [0,1]$ $\quad k.a < m$

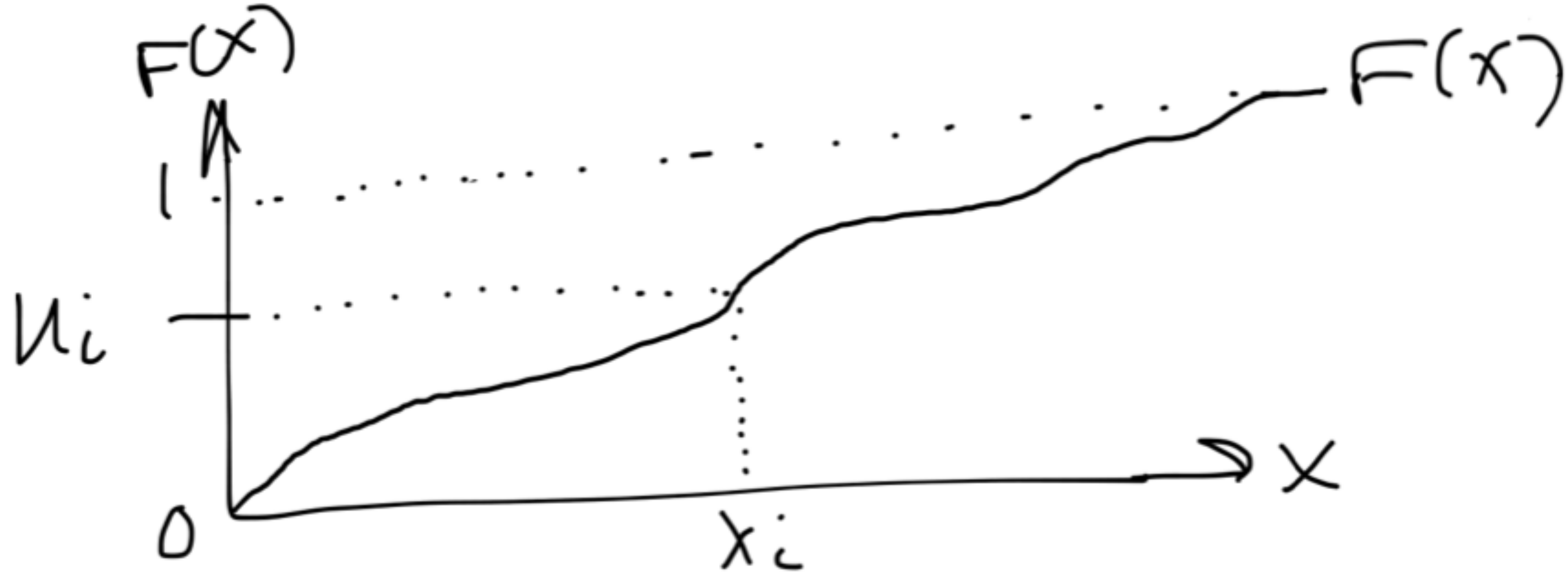
2. CDF $F(x)$

Standard Parametric Families

- software

Inverse CDF Method or Probability Integral Transformation Approach

For any continuous dist function F of a R.V. X, we can define the inverse F^{-1} on $[0,1]$ mapping into R_X



Inverse CDF Method

Proposition

Suppose F con't CDF. Let $U \sim U[0,1]$
then the R.V. $X = F^{-1}(U)$ has
CDF F.

~~Pf~~ $F_X(x) = P(X \leq x) = P(F^{-1}(U) \leq x)$

$= P(U \leq F(x)) = F(x)$ $\square$

Inverse CDF Method

If we can find F^{-1} ....
we can generate r.v.
w/ CDF F by simply:

① generate $u_1, \ldots, u_n \sim \mathcal{U}(0,1)$

② $x_i = F^{-1}(u_i)$ simple

An Inverse CDF Example

Generate Exponential r.v θ

$$F(x) = 1 - e^{-\theta x}$$

$$r = 1 - e^{-\theta x}$$

$$e^{-\theta x} = 1 - r$$

$$-\theta x = \ln(1-r)$$

$$x = \frac{\ln(1-r)}{-\theta}$$

$$\boxed{x = \frac{\ln r}{-\theta}}$$