625.664 Computational Statistics

Problem Set 5

Associated Reading: Chapter 1: Section 1.7
Chapter 6: Introduction - 6.4 - 6.4.1
Chapter 7: Introduction - 7.1

Complete the problems either by hand or using the computer and upload your final document to the Blackboard course site. All final submittals are to be in PDF form. Please document any code used to solve the problems and include it with your submission.

1. Assume that the instrumental distribution g in importance sampling (with unstandardized weights) is chosen such that
$$f(x) < M \cdot g(x)$$
for all x and a suitable $M \in \mathbb{R}$, where f is the target distribution.
(a) Show that $\mathrm{Var}_g(w^*(X)) < M - 1$.
(b) Show that $\mathrm{Var}_g(w^*(X) \cdot h(X))$ is finite, if $\mathrm{Var}_f(h(x))$ is finite.

2. Assume that you want to sample from a $N(0,1)$ distribution using a $N(1,2)$ distribution as instrumental distribution. Draw a sample of size 1000 using importance sampling, calculate the weighted mean and weighted variance, and plot a histogram of the weighted sample, i.e. plot a histogram of the draws from N(1,2) times the appropriate weights. How does this histogram compare to a draw directly from $N(0,1)$?

3. Suppose the general population's opinion of the federal government can be classified as "positive", "negative", or "it could be worse". If it is "positive" one day, then it is equally likely to be either "negative" or "it could be worse" the following day. If it is not "positive", then there is one chance in two chance that opinions will hold steady for another day and if it does change, then it is equally likely to become either of the other two opinions.
(a) What is the transition probability matrix for this Markov Chain?
In the long run
(b) How often does the general population hold a non-"negative" opinion of the government? (Hint: Remember that you need to solve the system of equations $\pi P = \pi$ and $\pi e = 1$.)

4. (a) Problem 7.2(a). You do not need to superimpose the true density on your histograms. Also, the **sample path** of a Markov Chain is a plot of the iteration number t versus the realizations of the random variable $X^{(t)}$ for $t = 0, 1, 2, \ldots$.
(b) Now change the proposal distribution in Problem 7.2 (a) to a Uniform distribution on (0, 20) with starting point 7. Does this proposal do any better? Run your code several times and see if you can get a histogram that resembles the expected double hump, then plot the sample path and discuss your results.