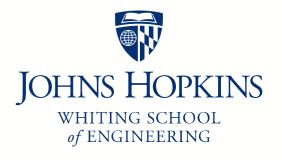
Johns Hopkins Engineering

625.464 Computational Statistics

Introduction to Function Estimation Orthogonal Functions

Module 10 Lecture 10C



A Few Definitions

Def: If each function in a linear space It can be expressed as a linear combination of functions in a set 6, then 6 is a basis (generating set, spanning set) of 1+. Def: A set of functions sqicxi3 is said to be onthogonal if $\frac{\text{ganal}}{\text{gis}} = \frac{1}{\text{gissing}(x)} = \frac{1}{\text{gissing}(x)}$ Additionally, if <a>Zqi,qi, = [qi&x)wx)dx= | \tilde{V}\tilde\tilde{V}\tilde{V}\tilde{V}\tilde{V}\tilde{V}\tilde{V}\tilde{V}\t

A Few More Definitions

- If a set 590003 is orthogonal with some Nutl then

79000 3 is orthonormal.

- If Zgills is a set of orthogonal functions, then it is a linearly independent set.

5 1, cosx, sinx, coszx, sin2x,)

0 < x < 2 th is an orthogenal family

known as the Fourier Fig Family.

Gram-Schmidt Otrhonomalization From any linearly ind. set, 290(x)3, we can always construction orthogonal set. 290 (x)7 bram - Schmidt Drilho. $\frac{\sqrt{9}, 9379}{\sqrt{8}, 917}$ $-\frac{\sqrt{92}, 937}{\sqrt{90}, 927}$ 9 K = 9 K - S (81192) 9i

Why Otrhonormal Functions? Often useful to rep. function of interest functions gold, gills. —. "Simpler" + (x) = \(\int \C_K 9 \kx \) - gooditea to use Egicus 3 that is an orthonormal basis. Given Egycos how do I find a.

Finding the Linear Representation

f(x) = \(\Sigma \) Ck gk (x)

R: Given \(\g_{\text{X}} \omega_{\text{X}} \omega_{\text{Y}} \)

A: If \(f \) is cont and integrable over

Do then

$$C_{K} = \langle f, g_{K} \rangle$$

\(\Sigma \)

\(\Sigma \)

\(\Gamma \)

\(\Sigma \)

\(\Gamma \

Finding the Linear Representation

In practice we approx & with f= = CKQK(X) has error $f = \frac{1}{5} ckgk(x) = f - f$ WSE 11 t- f112= 101 11 t- 5 cx 9x112 The Fourier wob 2 We will 3 what basis?

SCX3 minimize the opprox

mSE when Egks of then est - Fourier trig

one orthonormal the cool - ortho poly

- spling