$\tilde{q}_0 = q_0 = 1$

$\tilde{q}_1 = x - \frac{\langle 1, x \rangle}{\langle 1, 1 \rangle} 1$

$= x$

$\tilde{q}_2 = x^2 - \frac{\langle 1, x^2 \rangle}{\langle 1, 1 \rangle} 1 - \frac{\langle x, x^2 \rangle}{\langle x, x \rangle} x$

$= x^2 - \frac{\pi}{8} \frac{2}{\pi} - 0$

$= x^2 - \frac{1}{4}$

$\tilde{q}_3 = x^3 - \cancel{\frac{\langle 1, x^3 \rangle}{\langle 1, 1 \rangle} 1}$ 0

$- \frac{\langle x, x^3 \rangle}{\langle x, x \rangle} x$

$- \cancel{\frac{\langle (x^2 - \frac{1}{4}), x^3 \rangle}{\langle (x^2 - \frac{1}{4}), (x^2 - \frac{1}{4}) \rangle}} (x^2 - \frac{1}{4})$ 0

$= x^3 - \left(\frac{\pi}{16} \cdot \frac{8}{\pi}\right) x$

$= x^3 - \frac{x}{2}$

$\langle q_i, q_j \rangle = \int_{-1}^{1} q_i(x)\, q_j(x)\, w(x)\, dx$

$\langle 1, x \rangle = \int_{-1}^{1} (1)\, x\, (1 - x^2)^{1/2}\, dx$

$= \int_{-1}^{1} x \sqrt{1 - x^2}\, dx \quad u = 1 - x^2$

$du = -2x\, dx$

$= \int x \sqrt{u} \frac{du}{(-2x)} = -\frac{1}{2} \int_0^0 \sqrt{u}\, du = 0$

$\langle 1, 1 \rangle = \int_{-1}^{1} \sqrt{1 - x^2}\, dx \; ; \; x = \sin\theta$

$dx = \cos\theta\, d\theta$

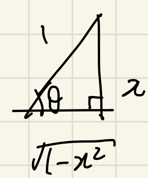
$\Rightarrow \int \sqrt{1 - \sin^2\theta} \cdot \cos\theta\, d\theta = \int \sqrt{\cos^2\theta} \cos\theta\, d\theta$

$= \int \cos^2\theta\, d\theta \quad \left(\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))\right)$

$= \frac{1}{2} \int 1 + \cos(2\theta)\, d\theta$

$= \frac{1}{2} \left(\theta + \frac{1}{2} \overbrace{\sin(2\theta)}^{2\sin\theta\cos\theta}\right)$

$= \frac{1}{2}\theta + \frac{1}{2} \sin\theta \cos\theta$



$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{x}{1}\right)$

$\theta = \sin^{-1}(x)$

$= \int_{-1}^{1} \frac{1}{2} \sin^{-1}(x) + \frac{1}{2} x \sqrt{1 - x^2}\, dx$