Johns Hopkins Engineering

625.464 Computational Statistics

Jackknife Methods

Module 8 Lecture 8C

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Jackknife Methods

Suppose we have a random sample $Y_1, \dots, Y_n$ which we use to compute a statistic T as an estimator of some parameter Θ of the population from which the Y_i were drawn.

We would like to know characteristics of the distribution of the estimator T of Θ.

- expected value of T
- variance of T
- bias of T

Jackknife Estimators

If we had enough time we could generate S samples $\bar{Y}^{(1)}, \ldots, \bar{Y}^{(S)}$ and compute $T^{(i)}$ for each.

The Jackknife

Sample $y_1, \dots, y_n$ $\quad$ T estimates θ and is based on $y_1, \dots, y_n$

In the Jackknife method we partition the y_i into r groups of size K. $\quad n = Kr$

Consider removing the j^{th} group from the sample & computing the new estimator $T_{(-j)}$ from the remaining $r-1$ groups

$\hookrightarrow n-K \qquad T.$

The Jackknife

Let $\bar{T}_{(\cdot)} = \frac{1}{r} \sum_{j=1}^{r} T_{(-j)}$ the mean of the $T_{(-j)}$

$\bar{T}_{(\cdot)}$ can be used as an estimate for θ.

Consider weighting differences in the estimates from the full sample and the reduced samples

$$T_j^* = rT - (r-1)\,T_{(-j)}$$

pseudovalues

$$\boxed{J(T) = \frac{1}{r} \sum_{j=1}^{r} T_j^* = \bar{T}^*}$$

The Jackknifed T

If T is a linear functional of the ECDF then $\bar{T}_{(\cdot)} = T$

Comments on the Jackknifed T

$$J(T) = \frac{1}{r}\sum_{j=1}^{r} T_j^* = \bar{T}^*$$

① $J(T) = T + (r-1)(T - \bar{T}_{(\cdot)})$

② $J(T) = rT - (r-1)\bar{T}_{(\cdot)}$

③ In most applications $k=1$

$\&$ $r = n$.

Jackknife Variance Estimate

Basic idea: Although pseudovalues T_j^* are not independent, we treat them as if they were and use $Var(J(T))$ to estimate $Var(T)$.

We estimate $V(T)$ with the sample variance of the mean of the T_j^*

$$\widehat{V(T)}_J = \frac{\sum_{j=1}^{r} \left(T_j^* - J(T)\right)^2}{r(r-1)}$$

Comments on Jackknife Variance Estimate

0. $\hat{V}(T)_J$ estimates $V(T)$
1. If T = the mean of $\underline{Y}$ & $K=1$, then $\hat{V}(T)_J$ is the standard variance est.
2. There are other ways to obtain est variance & MC studies have shown that $\hat{V}(T)_J$ is often conservative.
3. Another variant of $\hat{V}(T)_J$ is sometimes used

$$\frac{\sum_{j=1}^{r} (T_j^* - T)^2}{r(r-1)} \geq \hat{V}(T)_J$$

The Delete-k Jackknife

Consider the Jackknife estimator of the variance of the sample median. If we leave out only one observation at a time the median of the reduced sample will always be one of two values.

Wont produce a good estimate of $V(T)$

Delete K observations at a time.

$$\sqrt{n}/K \to 0 \quad \text{and} \quad n-K \to \infty$$

$$\binom{n}{K}$$