Johns Hopkins Engineering

625.464 Computational Statistics

Independence Chains and Random Walk Chains

Module 6 Lecture 6A

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Review Basic Metropolis-Hastings Algorithm

Given $x^{(t)}$ compute $x^{(t+1)}$

① generating $x^* \sim g(x \mid x^{(t)})$

② $r = R(x^{(t)}, x^*) = \dfrac{f(x^*)\, g(x^{(t)} \mid x^*)}{f(x^{(t)})\, g(x^* \mid x^{(t)})}$

③ Accept $x^{(t+1)} = x^*$ w prob. $\min\{r, 1\}$

o.w. $x^{(t+1)} = x^{(t)}$

$x^{(t+1)} \sim f$

Independence Chains

$x^* \sim g$

$$g(x^* \mid x^{(t)}) = g(x^*)$$

$$r = \frac{f(x^*)\, g(x^{(t)})}{f(x^{(t)})\, g(x^*)} = \frac{w(x^*)}{w(x)}$$

Ergodic $\longrightarrow$ if $\forall x$ with $f(x) > 0$, $g(x) > 0$

$$w(x) = \frac{f(x)}{g(x)}$$

1. choice of g like choice of envelope e
2. be careful, if $w(x^{(t)})$ is much larger than the typical $w^{(t)}$ the chain gets stuck

Random Walk Chains

x^* $\quad \varepsilon \sim h(\varepsilon)$

$x^* = x^{(t)} + \varepsilon$ $\quad g(x^* | x^{(t)}) = h(x^* - x^{(t)})$

common h: uniform ball centered at origin
- scaled normal
- scaled student's t

Support of f is connected & h is pos in a neighborhood of 0 $\Rightarrow$ ergodic

Random Walk Chain Example