Johns Hopkins Engineering

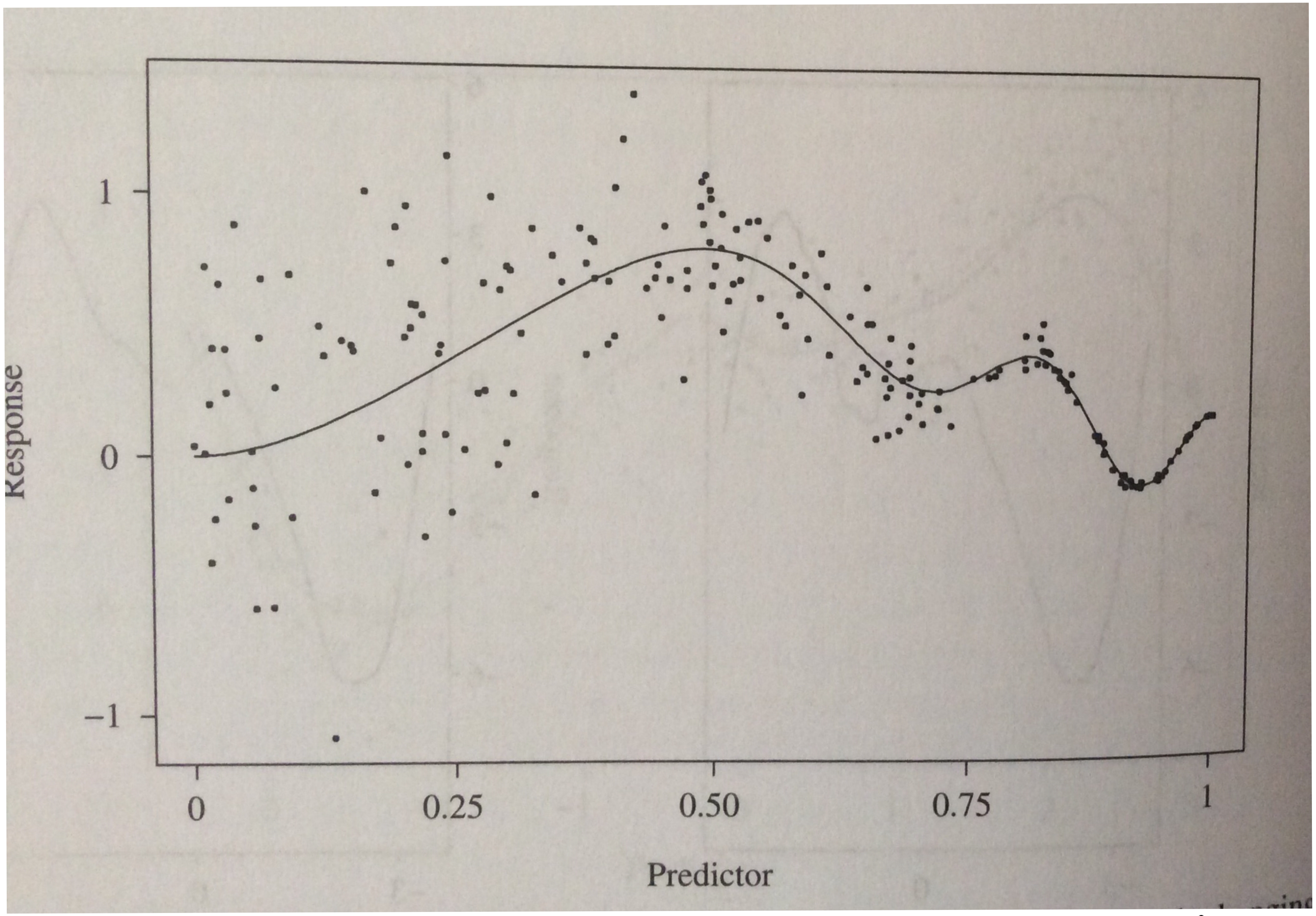
625.464 Computational Statistics

Nonlinear Smoothers

Module 12 Lecture 12D

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

The Supersmoother



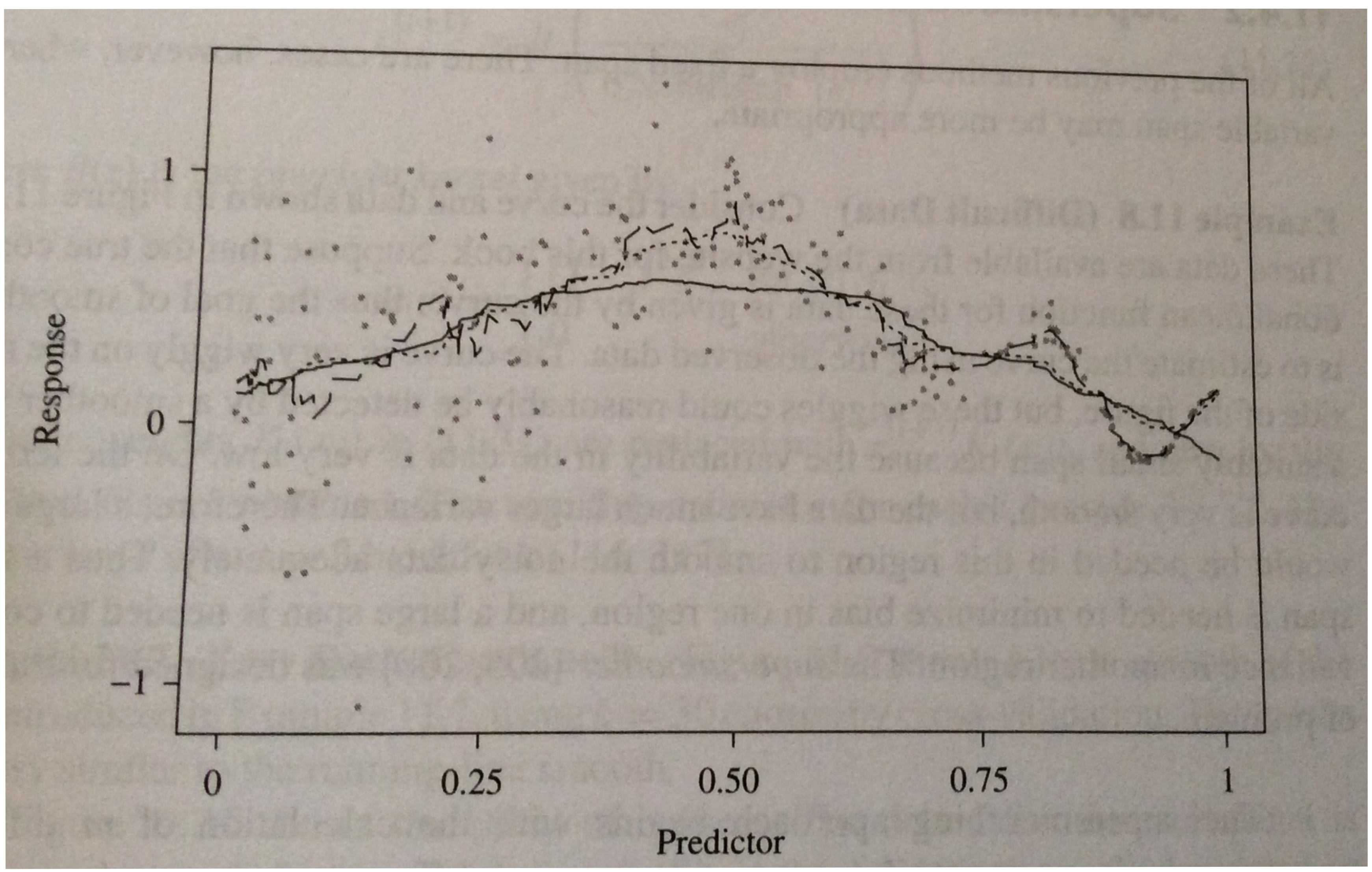
left side
⇒ large span

right side
⇒ small span

need a
variable
span

Supersmoothing Approach

Step 1: Calculate m different smooths $\hat{S}_1(x), \ldots, \hat{S}_m(x)$ each with a different fixed span $h_1, \ldots, h_m$



$m = 3$

$h_1 = .05n$

$h_2 = .2n$

$h_3 = .5n$

Supersmoothing Approach

Step 2: Define $p(h_j, x)$ to be a measure of performance of the j^{th} smooth at point x

Ideally we would use $E[g(Y - s_j^{(i)}(x_i)) \mid X = x_i]$

$\hat{p}(h_j, x_i) = \hat{S}^*(g(Y_i - \hat{S}_j^{(i)}(x_i)))$ where

S^* is a fixed span smoother

Step 2 Example

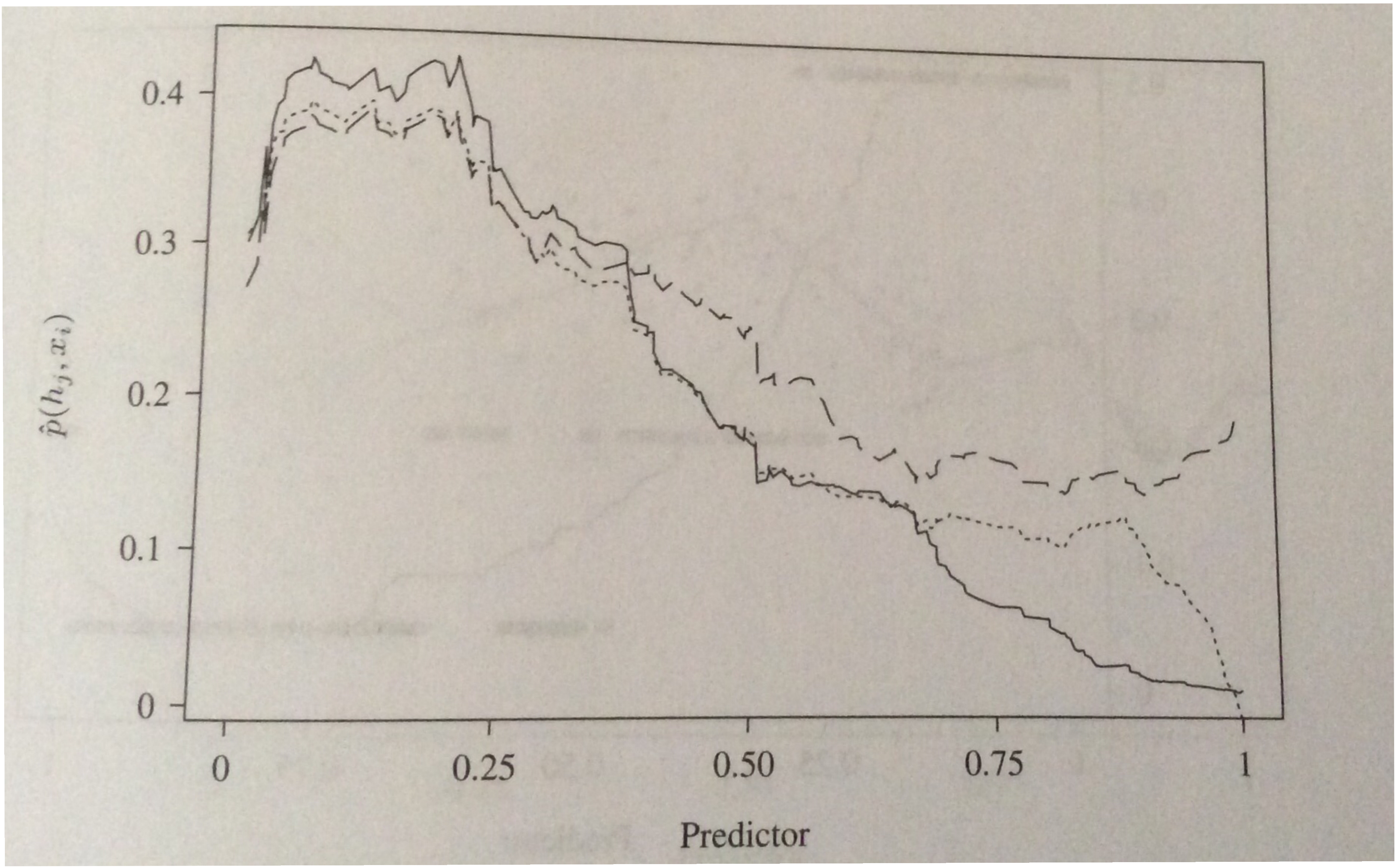
Ex $\hat{S}^* = \hat{S}_2$

$g(x) = |x|$

- - - - $h_3 = .5n$

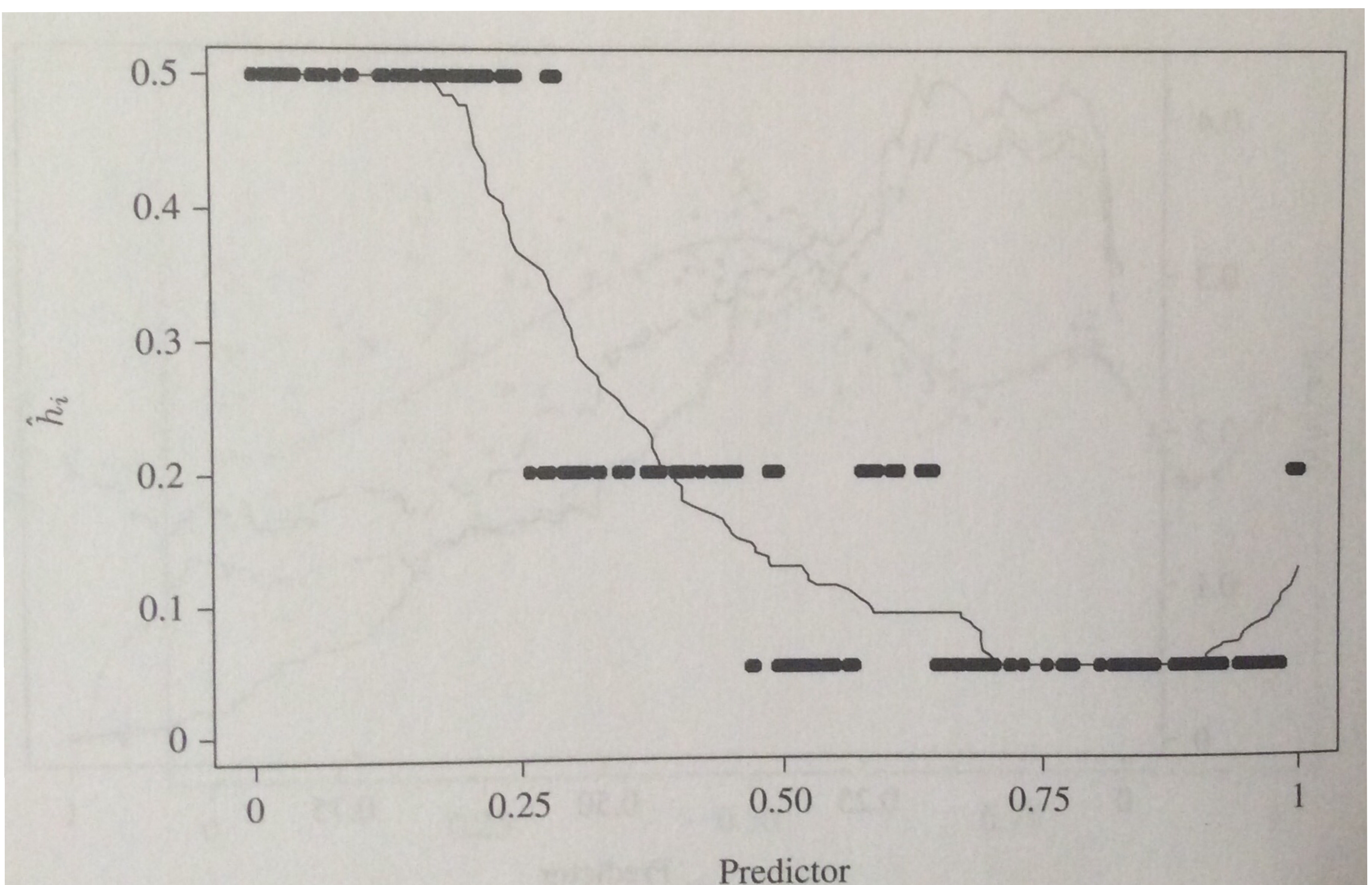
. . . . . . $h_2 = .2n$

—— $h_1 = .05n$



Supersmoothing Approach

Step 3: At each x_i denote the best of the spans as $\hat{h}_i$ — ie the span w/ the lowest $\hat{p}(h_j, x_i)$



- pass $(x_i, \hat{h}_i)$ through some smoother $\hat{s}^*$ to obtain optimal span $\hat{h}(x)$ as a function of x

Final Smooth

The final step is to use a linear interpolation between $\hat{S}_{h^-(x_i)}(x_i)$ and $\hat{S}_{h^+(x_i)}(x_i)$ where among the m fixed spans

$h^-(x_i)$ is the largest span $< \hat{h}(x_i)$

$h^+(x_i)$ is the smallest span $> \hat{h}(x_i)$

Thus

$$\hat{s}(x_i) = \frac{\hat{h}(x_i) - h^-(x_i)}{h^+(x_i) - h^-(x_i)} \hat{s}_{h^+(x_i)}(x_i) + \frac{h^+(x_i) - \hat{h}(x_i)}{h^+(x_i) - h^-(x_i)} \hat{s}_{h^-(x_i)}(x_i)$$

Supersmoothing Final Example