

# Johns Hopkins Engineering

## 625.464 Computational Statistics

### Randomization Methods

#### Module 8 Lecture 8A



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# Randomization Methods

Basic idea: Compare an observed configuration of the outcomes with all possible configurations.

Process 1:  $X_1, X_2, \dots, X_{n_1}$

Process 2:  $Y_1, Y_2, \dots, Y_{n_2}$

null Hypothesis:  $H_0 = \text{the means are equal}$

$$t_0 = \bar{X} - \bar{Y}$$

# Randomization Test

$$t_0 = \bar{x} - \bar{y}$$

$y_1, x_2, x_3, \dots, x_{n_1}$

$x_1, y_2, y_3, \dots, y_{n_2}$

$\bar{x}_{new}$  is mean

$\bar{y}_{new}$  is mean

$$t_1 = \bar{x}_{new} - \bar{y}_{new}$$

$\binom{n_1+n_2}{n_2}$  different configurations  $\Rightarrow \binom{n_1+n_2}{n_2}$  test statistics

if  $\binom{n_1+n_2}{n_2} = m$  compare to other  $m$ . It will be  $k^{th}$  largest. And we will reject  $H_0$  with sig level  $\frac{k}{m}$

## Randomization Example

For each rat  $i$  the outcome  $X_i$  is measured.

- null hypothesis: outcome does not depend on if the rat was treatment or control
- alternate hypothesis: outcomes for rats labeled treatment will be larger

$$T = \text{mean}(\text{treatment}) - \text{mean}(\text{control})$$

and has value  $T = t_0$  for the observed data

calculate  $t_2, t_3, \dots, t_m$  for all  $m$  possible permutations and compare to  $t_0$