Johns Hopkins Engineering

625.464 Computational Statistics

Implementation Concerns
Part 2

Module 6 Lecture 6E

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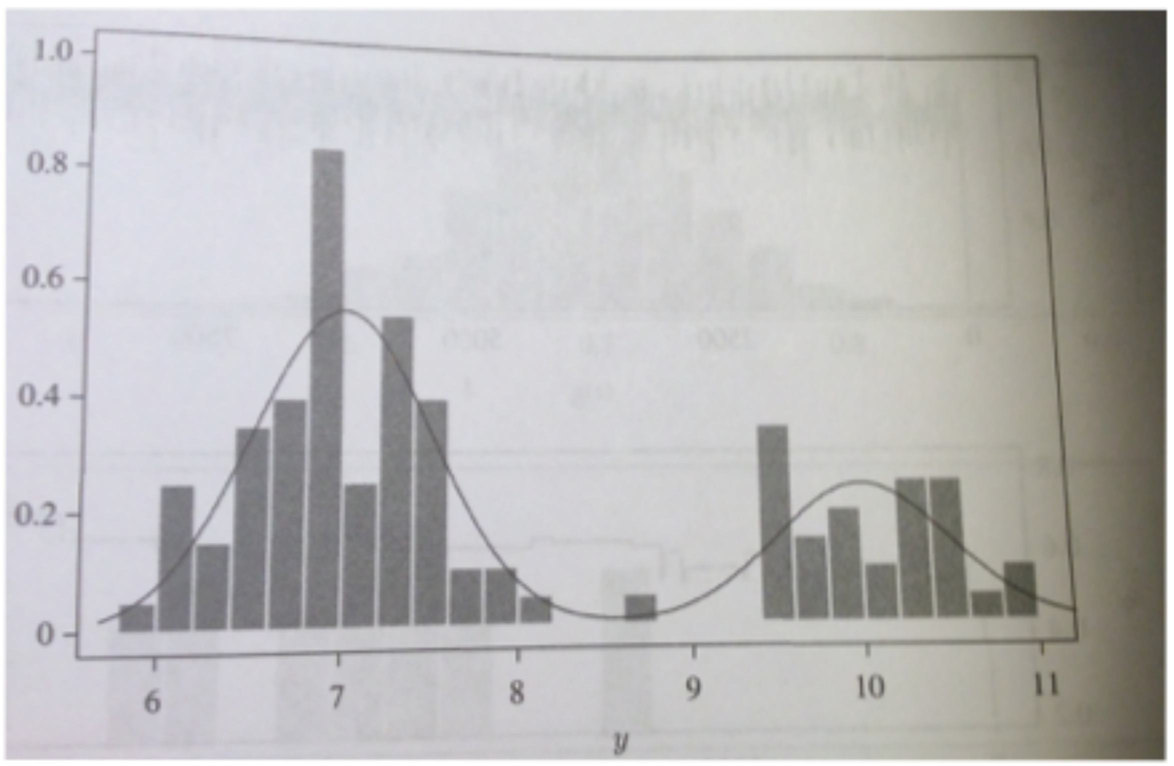
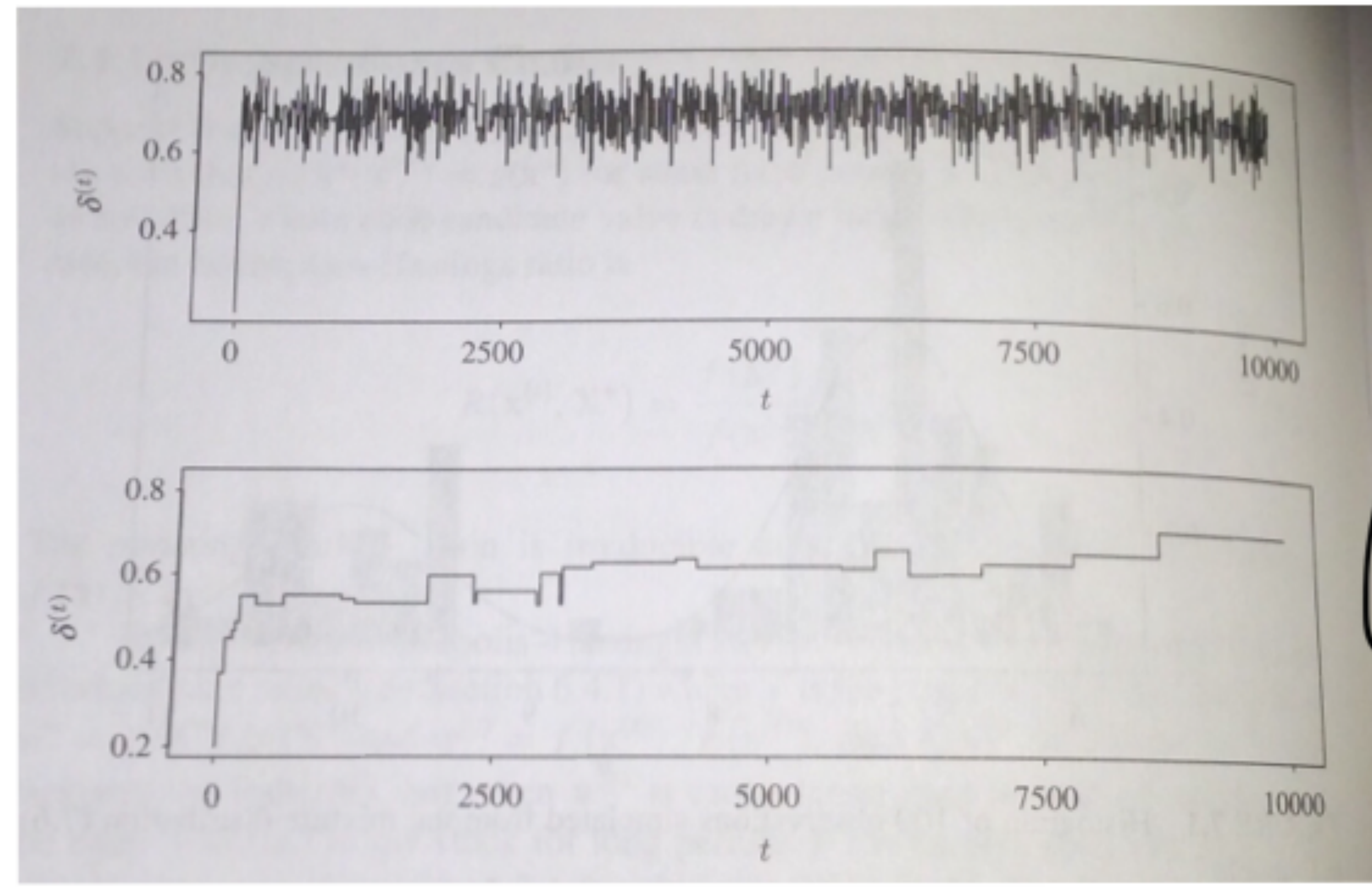
Sample Path

A plot of the iteration number t versus the realization $x^{(t)}$.





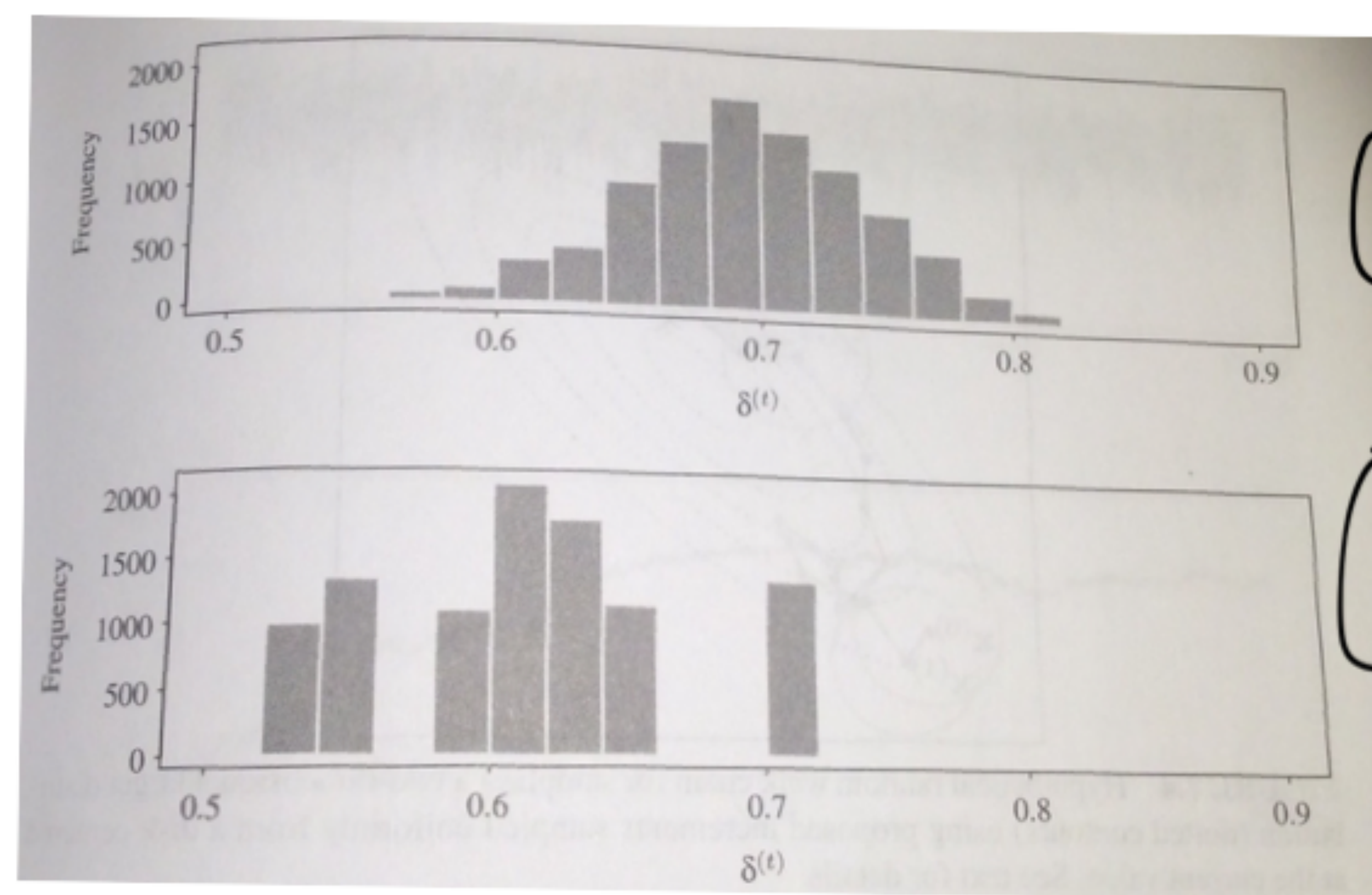
1

2

100 observations
$\delta N(7, 0.5^2) + (1-\delta) N(10, 0.5^2)$

1 Beta (1,1)

2 Beta (2,10)



1

2

Cusum (Cumulative Sum) Plot

- used to assess convergence of an est. of a one dimensional parameter $\Theta = E[h(x)]$

- After discarding initial iterations — calculate the estimator

$$\hat{\theta}_n = \frac{1}{n} \sum_{j=1}^{n} h(X^{(j)})$$ and then plot

$$\sum_{i=1}^{t} [h(x^{(i)}) - \hat{\theta}_n] \text{ v/s } t$$

- good mixing $\Rightarrow$ wiggly plot w/ small excursions from zero

Autocorrelation Plot

plot i v/s correlation between iterations that are i apart

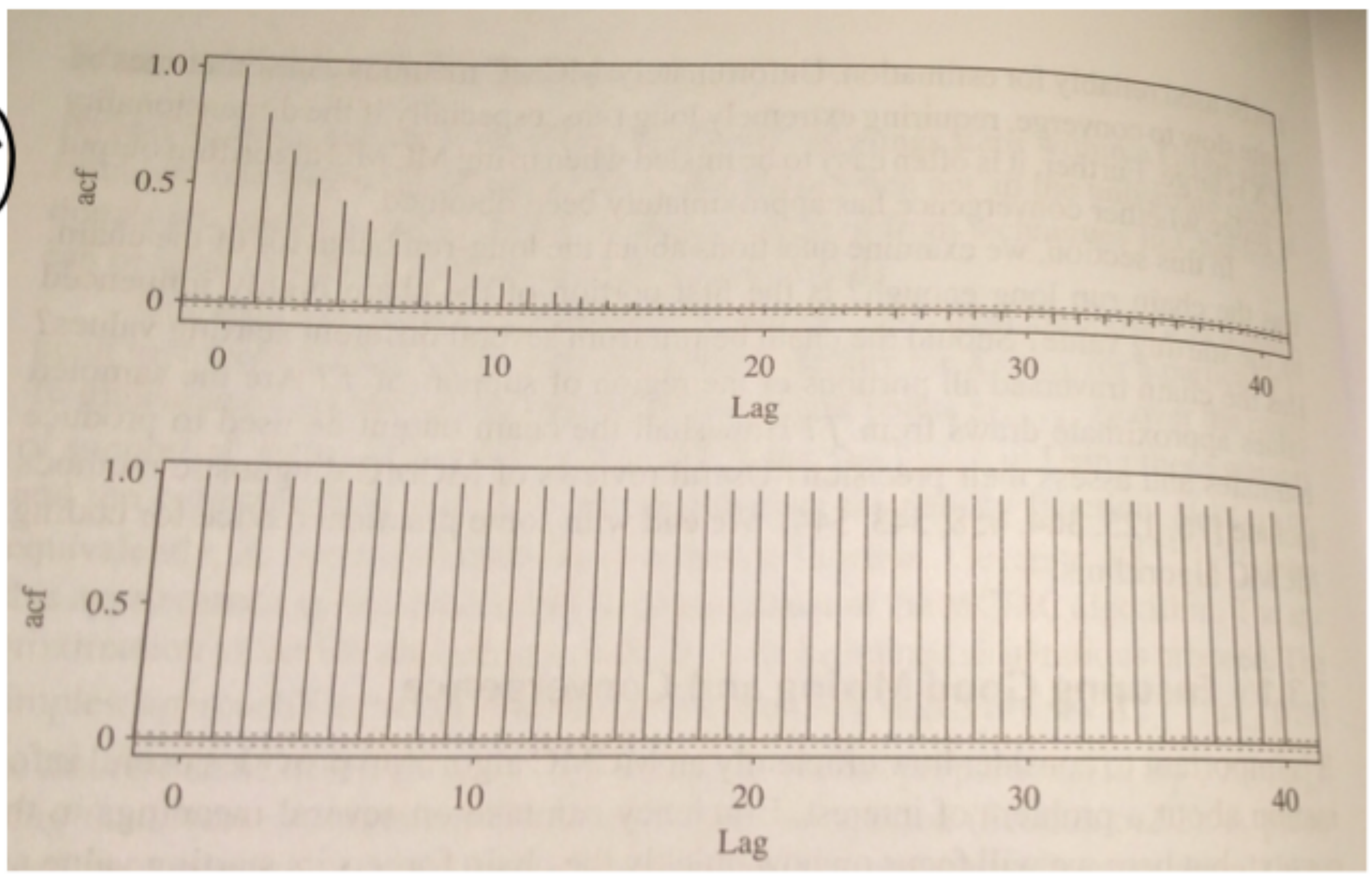
Plot i v/s $R(i) = \frac{C_i}{C_0}$

C_i = autocovariance function

C_0 = variance function

1

2



Burn-in and Run Length

$X^{(t)} \sim f$ only in the limit

- usually throw away D iterations
- need L iterations
- need to run J chains

Burn-in and Run Length

Chain 1 $X_1^{(0)}, \ldots, X_1^{(D)}, X_1^{(D+1)}, \ldots, X_1^{(D+L-1)}$

$\vdots$

Chain J $X_j^{(0)}, \ldots, X_j^{(D)}, X_j^{(D+1)}, \ldots, X_j^{(D+L-1)}$

Let $\overline{X_j} = \frac{1}{L}\sum_{t=D}^{D+L-1} X_j^{(t)}$; $\overline{X} = \frac{1}{J} \sum_{j=1}^{J} \bar{X}_j$

Define Between-chain variance and within chain variance

$$B = \frac{L}{J-1}\sum_{j=1}^{J}(\overline{X}_j - \overline{X})^2 \qquad S_j^2 = \frac{1}{L-1}\sum_{t=D}^{D+L-1}(X_j^{(t)} - \bar{X}_j)^2$$

Burn-in and Run Length

$$B = \frac{L}{J-1} \sum_{j=1}^{J} (\bar{x}_j - \bar{x})^2 \qquad s_j^2 = \frac{1}{L-1} \sum_{t=D}^{D+L-1} (x_j^{(t)} - \bar{x}_j)^2$$

$$W = \frac{1}{J} \sum_{j=1}^{J} s_j^2 \leftarrow \text{overall ave}$$

Define

$$R = \frac{\frac{L-1}{L} W + \frac{1}{L} B}{W}$$

$$\boxed{\sqrt{R} < 1.2}$$

$$\sqrt{R} \to 1 \qquad L \to \infty$$