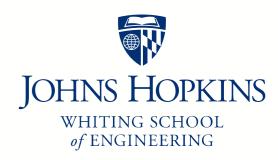
Johns Hopkins Engineering 625.464 Computational Statistics

Orthogonal Polynomials

Module 10 Lecture 10D



Orthogonal Polynomials

Can be developed from 1, X,X,X,X,X,

by applying 5-S w/aprop- weight normalize 1 2 gis g v 7 Ed/Logendre Poly Rang (-1)1 J wext=1 $g_0 = g_0 = 1$ $< g_1, g_1 > = \int g_2(x)g_1(x) dx$

Orthogonal Polynomial Recurrences For the Kth paynomial in an orthograph Janak (not dep on X) st. gk(x)-axxgk-(x) is a poly of degree k) · · · qx(x) - ax x qx= = = (i,=) (i,3) for some ci $g_{K}(x) = (a_{K}x + c_{K-1})g_{K-1}(x) + c_{K-2}g_{K-1}(x) + \cdots$ $(o = c_{1} = \cdots c_{K-3} = 0)$ $f_{K}(x) = (a_{K}x + c_{K-1})g_{K-1}(x) - c_{K}g_{K-1}(x)$ $f_{K}(x) = (a_{K}x + c_{K-1})g_{K-1}(x) - c_{K}g_{K-1}(x)$

Polynomial Recurrence Example

Legendre Polynomials $q_{K}(x) = \frac{2k-1}{K} \times q_{K-1}(x) + \frac{k-1}{K} q_{K-2}(x)$

Using Orthognal Polynomial for Statistics

Q: How to apply O.P. to a data set X= \(\times \ A: Let's suppose for = g(x)pcx) where pox) is a probo. donsity function. Then for any orthonormal set [giop] we can approx f $f(x) = \sum_{K=0}^{\infty} (Q_K Q_K(X)) \sum_{K=0}^{\infty} (Q_K Q_K(X))$

$$C_{K} = \sqrt{f_{1}g_{K}} = \int_{D} g_{K}(x)g(x)p(x)dx$$

$$= E \left[g_{K}(x)g(x)p(x) \right]$$

whole X ~ P :. X₁,... X_n then we can unbiasedly est Gc by $G_c = \frac{1}{n} \sum_{i=1}^{n} g_i(x_i)g(x_i)$

Our Function Estimator... Finally

$$f(x) = \frac{1}{h} \underbrace{\sum_{k=0}^{\infty} \frac{1}{k} x_{k}} \underbrace{\sum_{k=0}^{\infty} \frac{1}{k} x_{k}}$$