625.664 Computational Statistics

Problem Set 9

Associated Reading: Chapter 9: Introduction - 9.3.1.1, 9.3.2.2, 9.4.1, 9.5, 9.8
Chapter 13 from Gentle.

Complete the problems either by hand or using the computer and upload your final document to the Blackboard course site. All final submittals are to be in PDF form. Please document any code used to solve the problems and include it with your submission.

1. Problem 9.5 with these additional comments for each part:
   (a) Only find the bootstrap t confidence interval and use the bootstrap variance estimate presented in Lecture 9B and in Gentle 13.2. Don't forget to work with the data on the log scale.
   (b) This will be an application of the randomization method presented in Module 8 and discussed in Gentle 12.1. It is referred to as a permutation test in the text and is discussed in Section 9.8 of the Givens and Hoeting Text.
   (c) Use the percentile method to find the required confidence intervals. You will need to exponentiate the intervals from (a) in order to compare them.

2. Problem 9.7.

3. Let $\mathcal{S} = \{Y_1, \ldots, Y_n\}$ be a random sample from a population with mean μ, variance σ^2, and distribution P. Let $\hat{P}$ be the empirical distribution function. Let $\bar{Y}$ be the sample mean for $\mathcal{S}$. Let $\mathcal{S}^* = \{Y_1^*, \ldots, Y_n^*\}$ be a random sample taken with replacement from $\mathcal{S}$. Let $\bar{Y}^*$ be the sample mean for $\mathcal{S}^*$.
   (a) Show that
   $$\mathrm{E}_{\hat{P}}(\bar{Y}^*) = \bar{Y}$$
   (b) Show that
   $$\mathrm{E}_{P}(\bar{Y}^*) = \mu$$

4. In this problem we will compare the results of a normal bootstrap sample to those of a balanced bootstrap sample.
   (a) Draw a random sample of size $n = 100$ from the standard Normal distribution, $N(0, 1)$, and calculate the sample mean, $\hat{\mu}$.
   (b) Use the standard bootstrap method to generate $B = 10$ bootstrap pseudodata sets. For each data set calculate the sample mean, $\hat{\mu}_j{}^*$. Then find the bootstrapped bias corrected estimator, $\hat{\mu}_{Bias}$, and the bootstrapped estimation of the variance, $\hat{V}(\hat{\mu})$.
   (c) Repeat part (b) using the balanced bootstrap method.
   (d) Discuss your results.