Johns Hopkins Engineering

625.464 Computational Statistics

Fisher Scoring, the Secant Method, and Fixed Point-Iteration

Module 2 Lecture 2E

JOHNS HOPKINS
WHITING SCHOOL
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Fisher Scoring

$L(\theta)$ $\quad \ell'(\theta) = 0$

$$\theta^{(t+1)} = \theta^{(t)} - \frac{\ell'(\theta^{(t)})}{\ell''(\theta^{(t)})}$$

$-\ell''(\theta)$

$I(\theta)$

Fisher Scoring

$$\theta^{(t+1)} = \theta^{(t)} + \ell'(\theta^{(t+1)}) I(\theta)^{-1}$$

Secant Method

$\beta = 1.62$

$$g''(x^{(t)})$$

$$\frac{g'(x^{(t)}) - g'(x^{(t-1)})}{x^{(t)} - x^{(t-1)}}$$

Secant method

$$x^{(t+1)} = x^{(t)} - g'(x^{(t)}) \frac{x^{(t)} - x^{(t-1)}}{g'(x^{(t)}) - g'(x^{(t-1)})}$$

for $t \geq 1$

Fixed-Point Iteration

fixed point

$x \ni \quad G(x) = x$

$$G(x) = x \iff g'(x) = 0$$

$$G(x) = g'(x) + x$$

Fixed-Point Iteration

$$x^{(t+1)} = x^{(t)} + g'(x^{(t)})$$

G $[a,b]$

G is contractive on $[a,b]$ if

1) $G(x) \in [a,b]$ whenever $x \in [a,b]$

2) $|G(x_1) - G(x_2)| \leq \lambda |x_1 - x_2|$

$\forall x_1, x_2 \in [a,b]$ & some $\lambda \in [0,1)$

Fixed-Point Iteration

$[a,b]$ $G(x)$

x^* on $[a,b]$

$$|x^{(t)} - x^*| \le \frac{\lambda^t}{1-\lambda} |x^{(1)} - x^{(0)}|$$

$g'(x)$ $[a,b]$

$\alpha \neq 0$ $G(x) = \alpha g'(x) + x$