Johns Hopkins Engineering

625.464 Computational Statistics

Newton's Method

Module 2 Lecture 2C

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Newton's Method

$g'(x)$ $\quad$ $g''(x) \neq 0$

$g'(x^*)$

$0 = g'(x^*) \approx g'(x^t) + (x^* - x^t) g''(x^{(t)})$

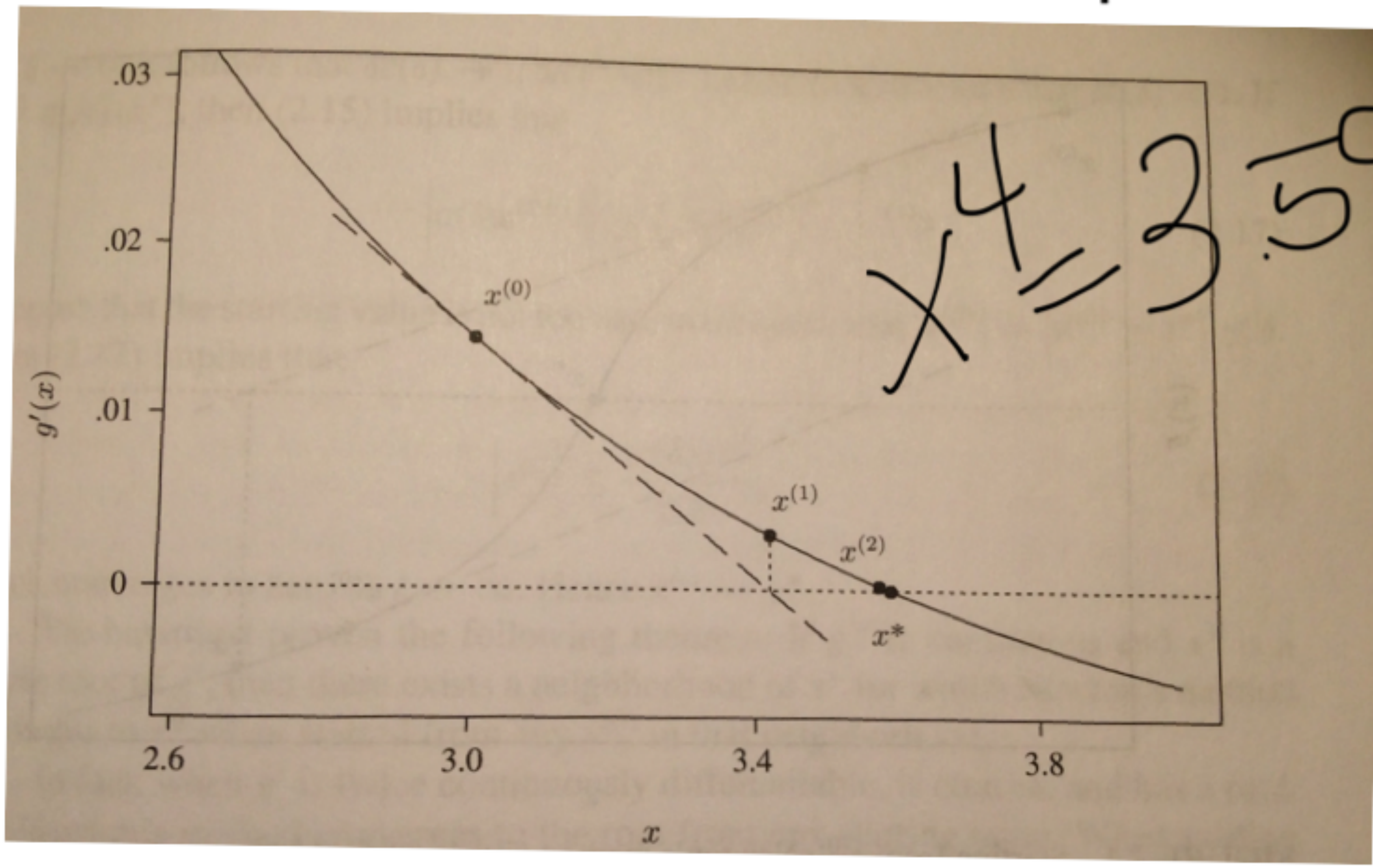
$x^* \approx x^{(t)} - \frac{g'(x^t)}{g''(x^t)}$

Newton's Method

Given $x^{(t)}$

$$x^{(t+1)} = x^{(t)} - \underbrace{\frac{g'(x^{(t)})}{g''(x^{t})}}_{h(t)}$$

Newton's Method Example



$x^4 = 3.59112$

$$g(x) = \frac{\log x}{1+x}$$

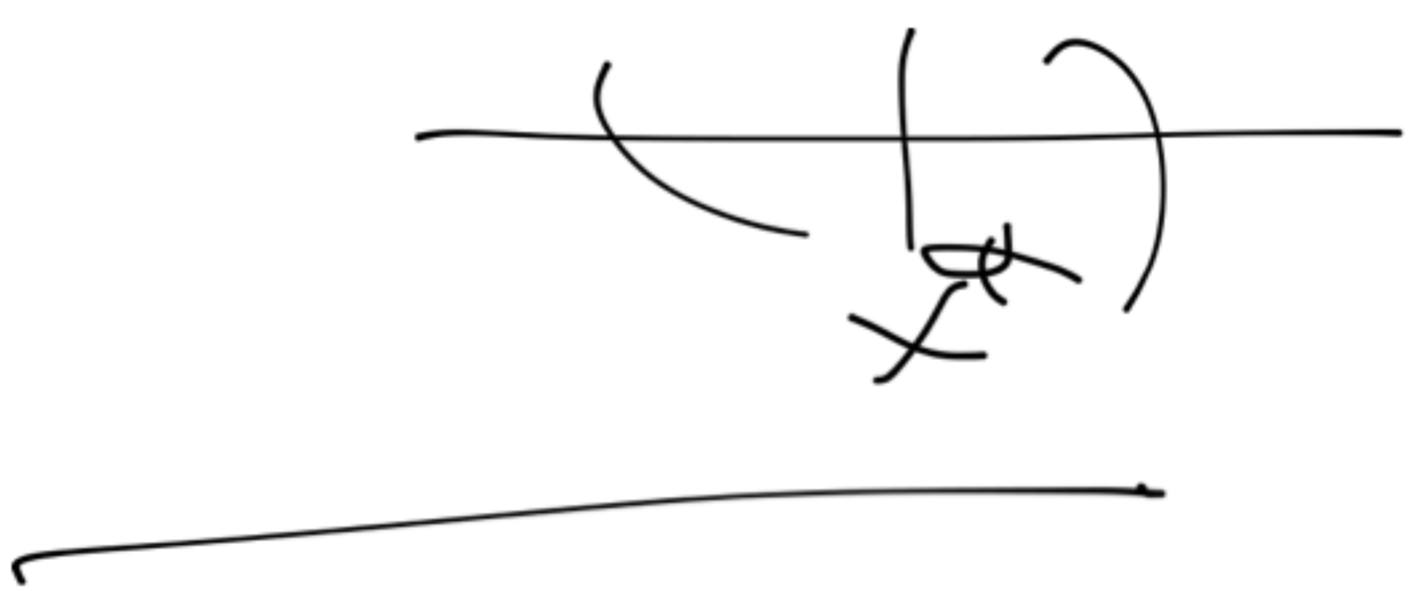
$$g'(x) = \frac{1 + 1/x - \log x}{(1+x)^2}$$

$$h(t) = \frac{(x^{(t)}+1)\left(1 + 1/x^{(t)} - \log x^{(t)}\right)}{3 + 4/x^{(t)} + 1/(x^{(t)})^2 - 2\log x^{(t)}}$$

Newton's Method

+ fast

− not gaureteed to converge.



Convergence Order

convergence order β

$$x^{(t)} - x^{*} = \varepsilon^{(t)}$$

① $$\lim_{t \to \infty} \varepsilon^{(t)} = 0$$

② $$\lim_{t \to \infty} \frac{|\varepsilon^{(t+1)}|}{|\varepsilon^{(t)}|^{\beta}} = C$$

$C \neq 0$ and $\beta > 0$.

Convergence Order

$$0 = g'(x^*) = g'(x^{(t)}) + (x^* - x^{(t)})\, g''(x^{(t)}) + (x^* - x^{(t)})^2\, g'''(\xi)/2$$

for some ξ between $x^{(t)}$ & x^*.

$$\underbrace{x^{(t+1)} - x^*}_{\varepsilon^{(t+1)}} = x^{(t)} + h(t) - x^* = \underbrace{(x^* - x^{(t)})}_{\varepsilon^{(t)}}{}^2\, g'''(\xi)/2g''(x^{(t)})$$

$$\varepsilon^{(t+1)} = \varepsilon^{(t)^2}\, \frac{g'''(\xi)}{2g''(x^{(t)})}$$

Convergence Order

$$\frac{\varepsilon^{(t+1)}}{\left(\varepsilon^{(t)}\right)^2} = \frac{g'''(\xi)}{2g''(x^{(t)})} \qquad \xi \quad x^{(t)} \quad x^*$$

$$g'''(\xi) \longrightarrow g'''(x^*) \qquad c = \left|\frac{g'''(x^*)}{2g''(x^*)}\right|$$

$$g''(x^{(t)}) \longrightarrow g''(x^*)$$

$$\lim_{t\to\infty} \frac{|\varepsilon^{(t+1)}|}{|\varepsilon^{(t)}|^2} = c \qquad \beta = 2$$

Convergence Order

(1) $\lim\limits_{t \to \infty} |\varepsilon^{(t)}$