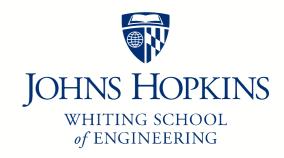
Johns Hopkins Engineering 625.464 Computational Statistics

Gibbs Sampling

Module 6 Lecture 6B



Why Gibbs Sampling?

multidimensional dist f

Basic idea: Build up the random vector element by element, by sequentially sampling from univariate conditional distributions

Basic Gibbs Sampler

(i) Select starting value $X^{(0)} = (X_1, X_2, \dots, X_p)^T$ and set t=0

benerate inturn (t+1) drawn from f(X1/X2) X2, ..., Xp)

X(t+1) drawn from f(X2/X1/X1/X2) X2, ..., Xp)

3) Increment t and go to Step 2

Comments on Gibbs Sampling

- 1. At each draw in Step 2, we are conditioning on the most recent update to all other elements.
- 2. The densities f_1 , f_2 , ..., f_p are called the full conditionals. Gibbs sampling only requires the full conditionals.
- 3. Even for high dimensional problems, all of our simulations are univariate. Advantageous, but can be slow.
- 4. Once convergence of the MC is achieved, the resultant vector $X^{(t)}=(X_1^{(t)}, ..., X_p^{(t)})$ is from f with each component converging individually.
- 5. Each draw X_i is from a marginalization of f.

Simple Gibbs Sampling Example

Want to sample
$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim \mathcal{N}(J, \Sigma)$$
 $\mathcal{N} = \begin{bmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{bmatrix} \stackrel{!}{\leftarrow} \sum = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim \mathcal{N}(J, \Sigma)$
 $\begin{bmatrix} X_1 | X_2 \end{bmatrix} = \mathcal{N}(\mathcal{N}_1 + \rho \frac{J_1}{J_2}(X_2 - \mathcal{N}_2), J_1^2(J - \rho^2))$
 $\begin{bmatrix} X_2 | X_1 \end{bmatrix} = \mathcal{N}(\mathcal{N}_1 + \rho \frac{J_2}{J_1}(X_1 - \mathcal{N}_1), J_2^2(J - \rho^2))$
 $\begin{bmatrix} X_1 | X_1 \end{bmatrix} = \mathcal{N}(\mathcal{N}_1 + \rho \frac{J_2}{J_1}(X_2 - \mathcal{N}_2)) \stackrel{!}{\leftarrow} \sum_{j=1}^{J_2} (X_j - \mathcal{N}_2) \stackrel{!}{\rightarrow} \sum$

Variations and Generalizations of Gibbs Sampling 1. The order of the updates can change. 2. Blocking - updating a group of elements (x_1) (x_1) (x_1) (x_2) (x_3) (x_4) (x_1) (x_2) (x_3) (x_4) $X_{1}^{(t+1)}(t+1) \sim \int (X_{21}X_{3} | X_{1}^{(t+1)} X_{4}^{(t+1)})$ x_{t+1} $(x_{t})_{x_{t}}$ $(x_{t+1})_{x_{t}}$ $(t_{t+1})_{x_{t}}$ $(t_{t+1})_{x_{t}}$

3 Hybrid Lilbr sampling
-add metropolis hastings steps where
Convenient