ps5\_rmd

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library(latex2exp)  
### Problem 2  
importance\_weights <- function(x) {  
 dnorm(x = x, mean = 0, sd = 1) /  
 dnorm(x = x, mean = 1, sd = sqrt(2))  
}  
  
sample\_size <- 1e3  
set.seed(664)  
target\_draw <- rnorm(sample\_size, # draw from target distribution  
 mean = 0, sd = 1)  
instrumental\_draw <- rnorm(sample\_size, # draw from instrumental distribution  
 mean = 1, sd = sqrt(2))  
weighted\_sample <- instrumental\_draw \* # create weighted sample  
 importance\_weights(instrumental\_draw)  
  
mean(weighted\_sample); var(weighted\_sample) # 0.02561563, 1.226633  
  
par(mfrow = c(1,2)) # plot  
hist(weighted\_sample, main = 'Histogram of Weighted Sample',  
 xlab = 'Weighted Sample Values')  
hist(target\_draw, main = 'Histogram of Target Sample',  
 xlab = 'Target Sample Values')  
dev.off()  
  
### Problem 4  
target\_distribution <- function(x, delta = 0.7) { # target distribution  
 delta \* dnorm(x = x, mean = 7, sd = 0.5) +  
 (1 - delta) \* dnorm(x = x, mean = 10, sd = 0.5)  
}  
  
proposal\_distribution <- function(input, given) { # proposal distribution  
 dnorm(x = input, mean = given, sd = 1e-2)  
}  
  
proposal\_sample <- function(x\_t) { # sample from proposal  
 rnorm(n = 1, mean = x\_t, sd = 1e-2)  
}  
  
mh\_ratio <- function(x\_t, x\_star) { # M-H ratio  
 (target\_distribution(x = x\_star) \* # numerator  
 proposal\_distribution(input = x\_t, given = x\_star)) /  
 (target\_distribution(x = x\_t) \* # denominator  
 proposal\_distribution(input = x\_star, given = x\_t))  
}  
  
x\_0s <- c(0, 7, 15, 20) # possible x^(0)'s  
  
mh <- function(x\_init = x\_0s[1], num\_iterations = 1e4) {  
 x\_t <- x\_init # Initialize variables  
 sampling\_matrix <- matrix(NA, nrow = num\_iterations)  
   
 for (index in 1:num\_iterations) {  
 x\_star <- proposal\_sample(x\_t = x\_t)  
 ratio\_r <- mh\_ratio(x\_t = x\_t, x\_star = x\_star)  
   
 if (ratio\_r >= 1) { # first check  
 sampling\_matrix[index] <- x\_star  
 x\_t <- x\_star  
 } else {  
 u <- runif(n = 1)  
 if (u < ratio\_r) { # second check  
 sampling\_matrix[index] <- x\_star  
 x\_t <- x\_star  
 } else {  
 sampling\_matrix[index] <- x\_t  
 }  
 }  
 }  
 return(sampling\_matrix)  
}  
  
sample\_x0\_1 <- mh(x\_init = x\_0s[1]) # generate samples  
sample\_x0\_7 <- mh(x\_init = x\_0s[2])  
sample\_x0\_15 <- mh(x\_init = x\_0s[3])  
  
par(mfrow = c(2,2)) # plot sample paths and histograms  
plot(1:1e4, sample\_x0\_1, type = 'l', main = TeX('Sample Path for $x^{(0)}=0$'),  
 ylab = TeX('$x^{(t)}$'), xlab = TeX('$t$'))  
plot(1:1e4, sample\_x0\_7, type = 'l', main = TeX('Sample Path for $x^{(0)}=7$'),  
 ylab = TeX('$x^{(t)}$'), xlab = TeX('$t$'))  
plot(1:1e4, sample\_x0\_15, type = 'l', main = TeX('Sample Path for $x^{(0)}=15$'),  
 ylab = TeX('$x^{(t)}$'), xlab = TeX('$t$'))  
dev.off()  
  
par(mfrow = c(2,2))  
hist(sample\_x0\_1, main = TeX('Histogram of Samples from $x^{(0)}=1$'),  
 xlab = TeX('Values of x^{(t)}'))  
hist(sample\_x0\_7, main = TeX('Histogram of Samples from $x^{(0)}=7$'),  
 xlab = TeX('Values of x^{(t)}'))  
hist(sample\_x0\_15, main = TeX('Histogram of Samples from $x^{(0)}=15$'),  
 xlab = TeX('Values of x^{(t)}'))  
dev.off()  
  
  
# part (b)  
proposal\_distribution2 <- function(input) {  
 dunif(x = input, min = 0, max = 20)  
}  
  
proposal\_sample2 <- function() {  
 runif(n = 1, min = 0, max = 20)  
}  
  
mh\_ratio2 <- function(x\_t, x\_star) {  
 (target\_distribution(x = x\_star) \* # numerator  
 proposal\_distribution2(input = x\_t)) /  
 (target\_distribution(x = x\_t) \* # denominator  
 proposal\_distribution2(input = x\_star))  
}  
  
mh2 <- function(x\_init = x\_0s[4], num\_iterations = 1e4) {  
 sampling\_matrix <- matrix(NA, nrow = num\_iterations)  
 x\_t <- x\_init  
   
 for (index in 1:num\_iterations) {  
 x\_star <- proposal\_sample2()  
 ratio\_r <- mh\_ratio2(x\_t = x\_t, x\_star = x\_star)  
   
 if (ratio\_r >= 1) {  
 sampling\_matrix[index] <- x\_star  
 x\_t <- x\_star  
 } else {  
 u <- runif(n = 1)  
 if (u < ratio\_r) {  
 sampling\_matrix[index] <- x\_star  
 x\_t <- x\_star  
 } else {  
 sampling\_matrix[index] <- x\_t  
 }  
 }  
 }  
 return(sampling\_matrix)  
}  
  
set.seed(664)  
sample\_x0\_20 <- mh2(x\_init = x\_0s[4])  
par(mfrow = c(1,2))  
hist(sample\_x0\_20, main = TeX('Histogram of Samples from $x^{(0)}=20$'),  
 xlab = TeX('Values of x^{(t)}'))  
plot(1:1e4, sample\_x0\_20, type = 'l', main = TeX('Sample Path for $x^{(0)}=20$'),  
 ylab = TeX('$x^{(t)}$'), xlab = TeX('$t$'))  
dev.off()