Homework 3

Problem

Non-expansions

Note: Submission challenges will come soon.

The convergence of generalized Markov decision processes depends heavily on establishing that value operators are non-expansions. In this problem, you will work with non-expansions to help improve your intuition about this class of operators.

Given a vector v in R^n , where $n > 0$ is odd, define $median(v)$ to equal the middle-largest of its components. For example, if $v = (6, -42, -19, 25, 37)$, then $median(v) = 6$.

Define the L_∞ norm of v, denoted $\|v\|_\infty$, to be the maximum of the absolute values of its components. For example, if $v = (6, -42, -19, 25, 37)$, then $\|v\|_\infty - 42$.

Like max , min , and $mean$, the $median$ operator is a non-expansion. That is, for all vectors x and y in R^n (with n odd), the following inequality holds:

$$|median(x) - median(y)| \leq \|x - y\|_\infty.$$

Of course, we know that the L_∞ distance of x and y is at least as big as the difference of their medians. But what is the smallest it can actually be? For this homework, given the value of $|median(x) - median(y)|$, you will determine the minimum possible value of $\|x - y\|_\infty$.

The catch is that you will not be given all the components of x and y: one coordinate of each will be missing. You will need to determine the minimum possible value of $\|x - y\|_\infty$ given that those components could have any integer value in addition to the constraint on $|median(x) - median(y)|$.

$x = (50, 200, 154, -55, -244, 103, -249, -105, ???, -146, -80, -242, -76, 53, 32, -251, 199)$

$y = (164, ???, 69, -51, -135, 136, -199, 51, 202, -81, 10, 17, -196, 24, 242, 7, 207)$

Find the missing components of x and y such that $|median(x) - median(y)| = 100$ and which minimize $\|x - y\|_\infty$. What is the minimum value of $\|x - y\|_\infty$ in this case?