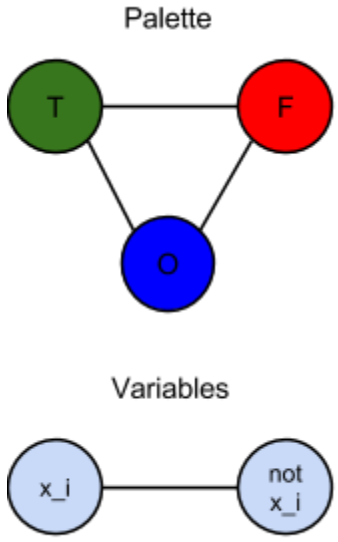
Answer the questions below.

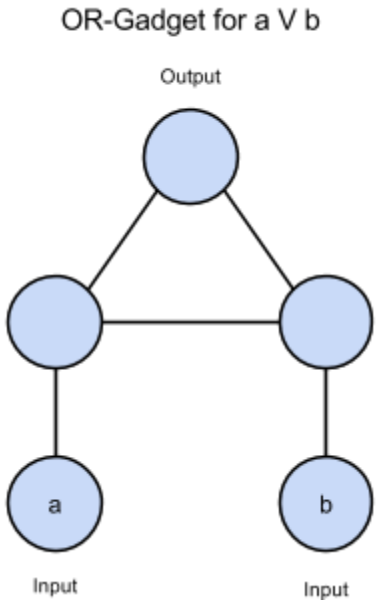
1. The Hamiltonian path problem asks if there is a path in a graph that visits every vertex exactly once. Give a polynomial time reduction of the Hamiltonian path problem (in an undirected graph) to the CNF SAT problem.

https://www.udacity.com/course/viewer#!/c-ud557/l-1209378918/m-2463548775

2. Show that if P = NP then there is a polynomial-time computable function f such that if ϕ is satisfiable boolean formula then $f(\phi)$ is a satisfying assignment. Hint: Use a polynomial-time algorithm for satisfiability repeatedly to find an assignment variable by variable.

3. A k-coloring of a graph is an assignment of one of k colors to each vertex of the graph such that no edge is incident on vertices of the same color. Show that 3-coloring a graph is NP-complete. (Hint: Reduce 3-CNFSAT. You may find the subgraphs below useful).





4. Show that the following problem can be decided in polynomial time.
Input: A graph $G = (V, E)$.

Decision: Is there a 2-coloring of the graph?

5. Given a complete graph G = (V, E) with non-negative integer costs on the edges $c : E \rightarrow Z^+$ and an integer bound k, the Traveling Salesperson problem (TSP) is to decide if there is a Hamiltonian cycle in G whose total cost is at most k. Recall that a Hamiltonian cycle visits every vertex once.

   Metric TSP is the constrained case where the edge cost function c satisfies the triangle inequality. That is, for all vertices $u, v, w \in V$, we have $c(u, w) \leq c(u, v) + c(v, w)$.

   Give a polynomial time reduction from TSP problem to Metric TSP. Hint: You only need to change the capacity function *c.*

6. The Knapsack problem may be stated as follows. Given n items with weights $w_1 \ldots w_n$ and corresponding values $v_1 \ldots v_n$, is there a subset of items $S \subseteq \{1 \ldots n\}$ such that the sum of the weights is at most some capacity W (i.e. $\sum_{i \in S} w_i \leq W$) and the sum of the values is at least V (i.e. $\sum_{i \in S} v_i \geq V$)? The traditional interpretation is that a robber wants to rob an art gallery and make off with most valuable collection of pieces that he can carry in his knapsack. Show that Knapsack is NP-complete (Hint: reduce from Subset-Sum).