Answer the questions below. Submit answers to 4 and 5 to Udacity, the others to T-square.

1a) Show that the following linear program is infeasible.

$$\begin{array}{ll} \text{min} & 3x_1 - 2x_2 \\ \text{s.t.} & x_1 + x_2 \leq 2 \\ & -x_1 + -2x_2 \leq -6 \\ & x_1, x_2 \geq 0 . \end{array}$$

1b) Show that the following linear program is unbounded.

$$\begin{array}{ll} \text{max} & 2x_1 + 5x_2 \\ \text{s.t.} & -2x_1 + x_2 \leq -1 \\ & -x_1 - 2x_2 \leq 2 \\ & x_1, x_2 \geq 0 . \end{array}$$

2a) Give an example of a linear program for which the feasible region is not bound but the optimal value is finite.

2b) Construct an example of a primal problem that has no feasible solutions and whose dual problem also has no feasible solutions.

3. Consider the following optimization problem

$$\begin{array}{ll} \text{min} & |u| + |v| + |w| \\ \text{s.t.} & u + v \leq 1 \\ & 2u + w = 3 . \end{array}$$

and convert it into a linear program of the form

$$\begin{array}{ll} \text{min} & c^T x \\ \text{s.t.} & Ax = b \\ & x \geq 0 . \end{array}$$

(Hint: you will want to introduce two new non-negative variables for each of u, v, w.)

4. Express the following problem as a linear program. Given an $m \times n$ matrix A and a vector b of length m, find a vector x such that $||Ax - b||_1$ is minimized. In other words, find $x_1 \ldots x_n$ such that $\sum_{i=1}^{m} |b_i - \sum_{j=1}^{n} a_{ij} x_j|$ is minimized. Implement your solution here.m

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5. Let $A = (a_{ij})$ for $1 \le i \le m$ and $1 \le j \le n$ be a matrix with m rows and n columns. Such a matrix defines a two-person game as follows. Two players, Row and Column play a game where Row selects a row i and Column selects a column j. If $a_{ij} > 0$ Row receives a payoff amount of a_{ij} from Column. If $a_{ij} < 0$, Row pays an amount of $-a_{ij}$ to Column. The payoff matrix A is known to both players.

   Suppose Row picks the i-th row with probability p_i and announces this vector p. Knowing this vector, Column will choose column j that minimizes Row's expected payout. Thus, the expected payout is $z = \min_j \sum_{i=1}^{m} p_i a_{ij}$. Naturally, Row then will want to choose the vector $(p_1, \ldots, p_m)$ so as to maximize this quantity. Express Row's problem as a linear program. (Of course $p_1, \ldots p_m \ge 0$ and $\sum_{i=1}^{m} p_i = 1$.)

   Implement your procedure here

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6. Prove that for any $m \times n$ matrix A and vector b of length m, exactly one of the following holds.
   a. There is a vector $x \ge 0$ such that $Ax = b$.
   b. There is a vector y such that $y^T A \ge 0$ and $y^T b < 0$.

   Hint: Use substitution to show both statements cannot be true for the same matrix A. To show that at least one must be true, consider the following linear program.

   min $\quad b^T y$

   s.t. $\quad A^T y \ge 0$

   and find its dual. Use the Duality Theorem to complete the result.