1. Give the contrapositive of the following statement. “If every bird flies, then there is a hungry cat.”

If there is not a hungry cat, then every bird does not fly.

Or equivalently, “if a cat is not hungry, then every bird does not fly”

1. A proposition is a statement that can be true or false but not both. Let A, B, and C be propositions. Let denote logical AND, let denote logical OR, and let denote logical NOT. Argue that if is true, then must be true as well.

Here is the truth table for this scenario:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | C | (A OR B) AND (NOT B OR C) | A OR C |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

We want to show that if is true, then is also true. From the definition of a conditional statement, this means that if is true, then must also be true. Otherwise, can be either true or false. From the truth table above, we see that this condition is satisfied. When is true, is also true. When is not true, then does not have any condition on it.

A different way of thinking about this is to look at the statement itself. In order for to be true, must be true and must also be true. Observe that if A is true and C is true, then is also true. Also observe that the only case where A is not true and is true is when B and C are both true. In all these cases, will also be true because at least A or C will be true in each of these cases.

1. We use the notation to indicate that A implies B. This new proposition is true except when A is true and B is false. We write when either both A and B are true or both are false. Argue that if and only if and .

Using the truth table below, we see that the two statements are equivalent. From the definition of a biconditional statement, this proves that if and only if and

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | A implies B | B implies A | (A implies B) AND (B implies A) |  |
| F | F | T | T | T | T |
| F | T | T | F | F | F |
| T | F | F | T | F | F |
| T | T | T | T | T | T |

1. We will use the notation to indicate the number of elements in the set or its *cardinality*, e.g. is the number of elements in the set A. Consider four sets A,B,C,D such that the intersection of any three is empty. Use the inclusion-exclusion principle to give an expression for without using any union ( symbols.

From the definition, of the inclusion-exclusion principle, we know that the cardinality of a union of sets is the sum of the cardinality of each individual set minus the cardinality of where those sets intersect. Since the intersection of any three sets is empty, we have the following simplified expression:

1. State the formal definition of , and use the definition to show that the function is .

Big O notation is used to describe the performance of an algorithm in the worst case scenario as the input size grows. Formally, assuming real valued functions f and g and real variable x, “if, for sufficiently large values of *x*, the values of are less than those of a multiple of , then f is of order at most g, or f(x) is O(g(x))” (*Discrete Mathematics with Applications*, Epp). So in this case, as n tends to infinity, we want to show that

As n goes to infinity, f(n) will be dominated by the largest terms. So

So, in this case, M = 1 and we have shown that f(n) is O(n).

1. For every positive integer n, let denote the set . Use an inductive argument to show that

An alternative way to phrase the question is as follows. Let A be a set. We use the notation to indicate the power set of A, which consist of all subsets of A. For example, if , then . Considerand use an inductive argument to show that the sum .

(For example, the expansion for is )

**Step 1:** Proof for n=1

Q(n) = {1}

So

So we have proven the sum is correct for n=1

**Step 2:** Show that if the sum is true for n=k is true, then the sum is also true for n=k+1

So we assume that

is true. Now we need to show that

is also true.

We have

Writing in terms of the summation from step 1, we have

Substituting in our assumptions, we have

So we have shown that

1. Prove that the set of all languages over that have a bounded maximum string length is countable.

I adapted the proof from the quiz “A False Proof”

1. Let Si be the set of languages containing strings of length at most i where i ≤ M, the maximum string length
2. Each Si is finite.
3. The set of all languages is
4. A countable union of finite sets is countable.

The reason the proof is acceptable now is because the maximum string length is bounded, where before it was not.