Questions 1-4 should be completed on the Udacity site. The graders will download and evaluate your answers. The others should be typed and uploaded to T-Square (on-campus students may upload all answers to T-square if they prefer).

For problems 5-7 we are not looking for a formal transition function and inductive proof, but rather a detailed argument that convinces the reader of its correctness.

1. Complete the exercise of tracing through the configuration sequence for the given Turing machine .

<https://www.udacity.com/course/viewer#!/c-ud557/l-1740638623/e-1740278552/m-1740278553>

1. Program a Turing machine the shifts its input one square to the right and places a $ sign at the beginning of the tape. See

<https://www.udacity.com/course/viewer#!/c-ud557/l-1740638623/e-1732309017/m-1740278556>

1. Program a Turing machine that tests if the input string has an equal number of zeros and ones. See

<https://www.udacity.com/course/viewer#!/c-ud557/l-1740638623/e-1740278557/m-1740278558>

4. Program a two-tape Turing machine to perform substring search. See <https://www.udacity.com/course/viewer#!/c-ud557/l-1728138752/e-1718598811/m-1751158600>

5. Another alternative Turing machine model has a single, one-way infinite tape, but two read-write heads. The transition function has the form , the same as a multi-tape machine. Describe how you would program such a machine to decide the language .

Hint: You can use the S (stay put) movement to achieve the effect of having one head move faster than the other.

Note: One-way infinite tape means the tape extends to the right with infinitely many blanks. In a two-way infinite tape, the tape extends both ways with infinitely many blanks. In both cases the heads may move right or left or (in this problem) stay-put. We say the machine rejects if it falls of the left end of a one-way tape.

6. Suppose we have a one-tape Turing machine whose head instead of having to move just left or right in each computation step, can move left or right or stay put. We called these “stay-put machine” in the lesson. Argue that it is possible to create a new standard Turing machine (no “stay put”, just moves left and right) that recognizes the same language as by changing only the transition function , keeping the same and .

7. Suppose that we constrained the standard 1-tape Turing machine to only be able to change the symbol on each square two times (clarification: overwriting a symbol with itself doesn’t count; overwriting a blank symbol with a non-blank symbol does). Prove that this model can decide every language that a standard Turing machine can.

Hint: Use lots of tape and do a lot of copying.

Bonus (hard): Describe how you would modify your machine so that it only changes the symbol once.

8. Consider a Turing machine with a two-dimensional tape where the head can move up and down as well as right and left. Assume that the paper is infinite in the right and downward directions. Give a formal definition of this machine.