625.661 Statistical Models and Regression

Module 1-2 Assignment

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Please complete all the problems.

Do not use any math/stat software to produce statistical results, but you can use any math/stat software to generate the percentile of normal, t, F, chi-square distribution or do basic mathematical calculations. State assumptions in your analyses or analytic derivations.

1. In a simple linear regression analysis, n *independent* paired data (y_1, x_1), …., (y_n, x_n) are fitted to the model

$$y_i = \beta_1(x_i - \mu) + \varepsilon_i \ , \ \ i = 1, \ldots, n \ ,$$

where x is the only non-random independent variable (or so-called regressor), μ is a known real number, and ε is the random error that has mean zero and unknown constant variance σ^2. Before the data for (y , x) are available, we need to construct estimators for the parameters.

   a) Construct the ordinary least squares (OLS) estimator of β_1 .
   b) Construct the variance of the OLS estimator of β_1 in a) and construct an unbiased estimator of this variance.

2. In Problem 1, add the intercept term β_0 to the model. Then do a) and b).

3. In Problem 1, the μ is a real number but the value is unknown. Please do a) and b).

4. Consider a regression model $y = \beta_0 + \beta_1 x + \epsilon$, where x is a non-random regressor. Discuss whether the ordinary least-squares estimator of the slope β_1 is always unbiased and whether it always has the smallest variance than **any** estimator of β_1, irrespectively of what the value of β_0 is. State assumptions in your discussion. Be careful about the word "**any**".

5. Use any math/stat software (e.g., www.**numbergenerator**.org/**randomnumbergenerator**) of your choice to find a random number generator to randomly select 15 rows of Table for Problem 2.18 (page 63-64) of Textbook and then do (a), (b), (c), (d). **State assumptions for all steps in your analyses.**