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Module 4 Discussion Reply

I think compared to how I approached the problem, your answer to this discussion is great due to its simplicity. What I mean is that you make the now obvious connection between SS_T and $SS_R + SS_{Res}$. I did not actually notice that the only difference between these two equations is the $-\hat{\boldsymbol{\beta}}\mathbf{X}'\mathbf{y} + \hat{\boldsymbol{\beta}}\mathbf{X}'\mathbf{y}$ term in the center. I was also not clear on what was meant exactly by using the "hat" matrix in the decomposition. For example, if you merely substituted the **H** matrix into your resulting decomposition, I am not sure if that would suffice for a "correct" response to this discussion. I also had a similar set of assumptions that you had listed also in your response.

In my response, I tried to rewrite the SS_T, SS_R, and SS_{Res} terms so that they can be seen as some form of $(\mathbf{x})^\top(\mathbf{x})$, where the $(\mathbf{x})$ term would vary for each formula. I then did a similar process of adding and subtracting $\hat{\mathbf{y}}$ within each of the parentheses. This leads to a formula that has both SS_R and SS_{Res}, but it also includes some other terms that are different. My goal then is to show that these other terms zero out through some math. This is when I tried to include the "hat" matrix. However, it's worth noting that in order for me to write SS_R as some $(\mathbf{x})^\top(\mathbf{x})$ term, it also required that I used this same process involving the "hat" matrix.

It's a bit complicated if your linear algebra is rusty, but basically it comes down to the $\bar{\mathbf{y}}$ term. I understood it to be a $n \times 1$ vector where each item is equal to $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$. To rexpress this $\bar{\mathbf{y}}$, I used $\frac{1}{n}\mathbf{J}\mathbf{y}$, where $\mathbf{J}$ is a $n \times n$ matrix of 1's only. The trick is that **H** is a projection matrix based on the design matrix **X**, where the design matrix itself contains a column of 1's for the intercept term. The result then is that $\mathbf{HJ} = \mathbf{J}$, due to the properties of the "hat" matrix. Like I said, I wasn't entirely sure on what the professor had expected for the response. So, I also rewrote SS_R and SS_{Res} in terms of "hat" matrix terms.