Simple Linear Regression Model – Part IV

Johns Hopkins Engineering

625.461 Statistical Models and Regression

Module 2 – Lecture 2C

Interval estimation: 100(1-α)% Confidence Interval (CI)

Following the basics in last lecture, we can obtain

$$\hat{\beta}_1 - t_{\alpha/2;n-2} se(\hat{\beta}_1) \le \beta_1 \le \hat{\beta}_1 + t_{\alpha/2;n-2} se(\hat{\beta}_1)$$

$$\hat{\beta}_0 - t_{\alpha/2;n-2} se(\hat{\beta}_0) \le \beta_0 \le \hat{\beta}_0 + t_{\alpha/2;n-2} se(\hat{\beta}_0)$$

Interval estimation: 100(1-α)% Confidence Interval (CI)

Variance is 2nd moment. If y is normally distributed,

$$P\left\{\chi^2_{1-\alpha/2,n-2} \le \frac{(n-2)MS_{\text{Res}}}{\sigma^2} \le \chi^2_{\alpha/2,n-2}\right\} = 1-\alpha$$

$$\frac{(n-2)MS_{\text{Res}}}{\chi^2_{\alpha/2,n-2}} \le \sigma^2 \le \frac{(n-2)MS_{\text{Res}}}{\chi^2_{1-\alpha/2,n-2}}$$

Rocket Propellant Data (Ex 2.5, page 30 of Textbook)

95% CI

$$\hat{\beta}_1 - t_{0.025,18}\,\text{se}\left(\hat{\beta}_1\right) \le \beta_1 \le \hat{\beta}_1 + t_{0.025,18}\,\text{se}\left(\hat{\beta}_1\right)$$

$$-37.15 - (2.101)(2.89) \le \beta_1 \le -37.15 + (2.101)(2.89)$$

$$\frac{18(9244.59)}{31.5} \le \sigma^2 \le \frac{18(9244.59)}{8.23}$$

Estimation of Mean Response $E(y \mid x_0)$ at $x = x_0$

$$E(y \mid x_0) = \mu_{y|x_0} = \beta_0 + \beta_1 x_0 \quad , \qquad \widehat{E(y|x_0)} = \hat{\mu}_{y|x_0} = \hat{\beta}_0 + \hat{\beta}_1 x_0$$

$$\mathrm{Var}(\hat{\mu}_{y|x_0}) = \mathrm{Var}\left(\hat{\beta}_0 + \hat{\beta}_1 x_0\right) = \mathrm{Var}\left[\bar{y} + \hat{\beta}_1 (x_0 - \bar{x})\right]$$

$$= \frac{\sigma^2}{n} + \frac{\sigma^2 (x_0 - \bar{x})^2}{S_{xx}} = \sigma^2 \left[\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right]$$

$$\hat{\mu}_{y|x_0} - t_{\alpha/2, n-2} \sqrt{MS_{\mathrm{Res}} \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$

$$\leq E(y|x_0) \leq \hat{\mu}_{y|x_0} + t_{\alpha/2, n-2} \sqrt{MS_{\mathrm{Res}} \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$

How to predict new y_0 at $x = x_0$?

At $x = x_0$, the new value $y_0 = \beta_0 + \beta_1 x_0 + \varepsilon_0$. Thus, the predicted value should be $\hat{y}_0 = \hat{\beta}_0 + \hat{\beta}_1 x_0$, unbiased for y_0 . Its variance is:

$$\operatorname{Var}(y_0 - \hat{y}_0) = \sigma^2 \left[1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right]$$

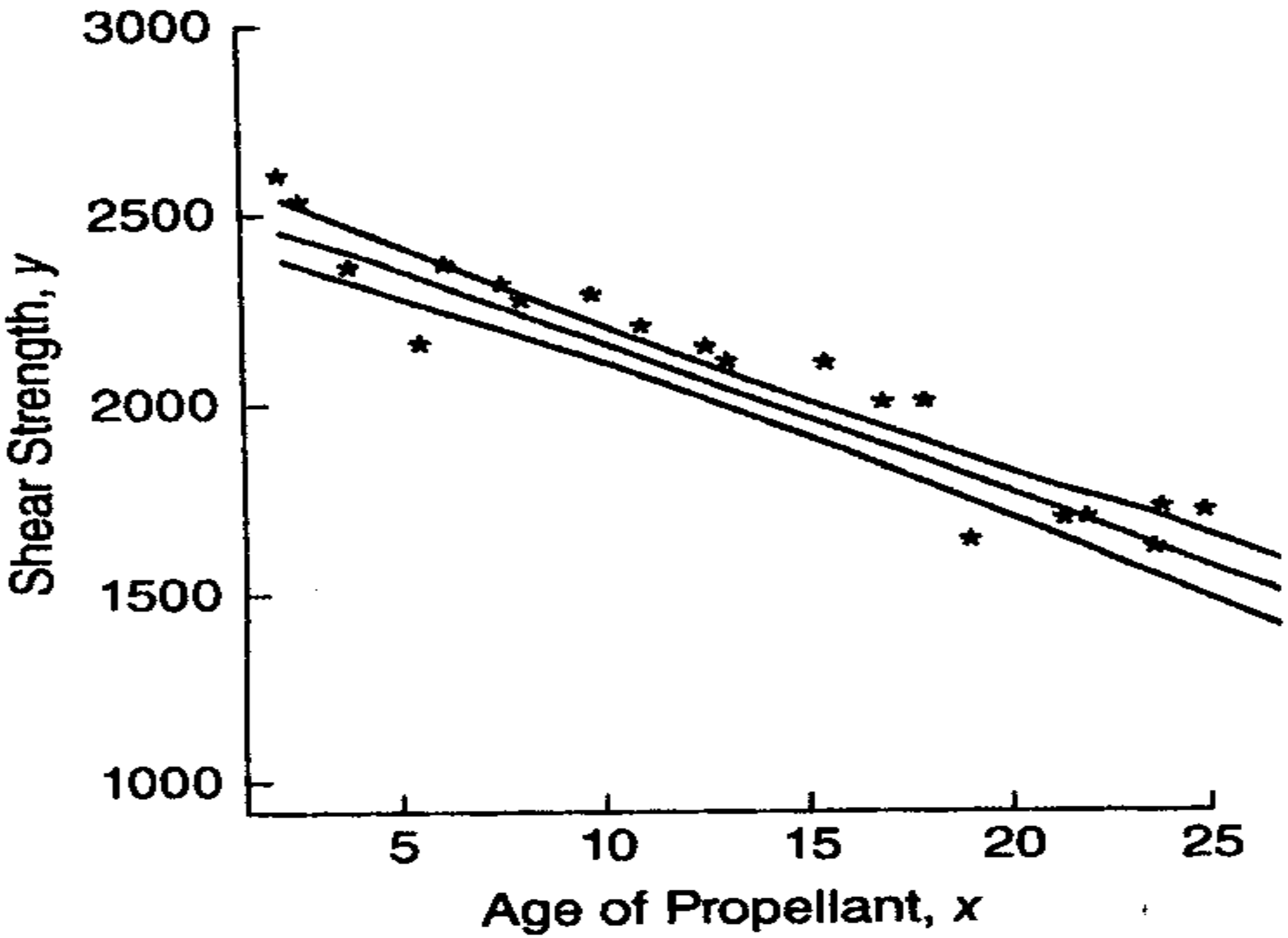
$$\hat{y}_0 - t_{\alpha/2, n-2} \sqrt{MS_{\text{Res}} \left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right)}$$
$$\leq y_0 \leq \hat{y}_0 + t_{\alpha/2, n-2} \sqrt{MS_{\text{Res}} \left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}} \right)}$$

Rocket Propellant Data (Ex 2.6, page 31 of Textbook)

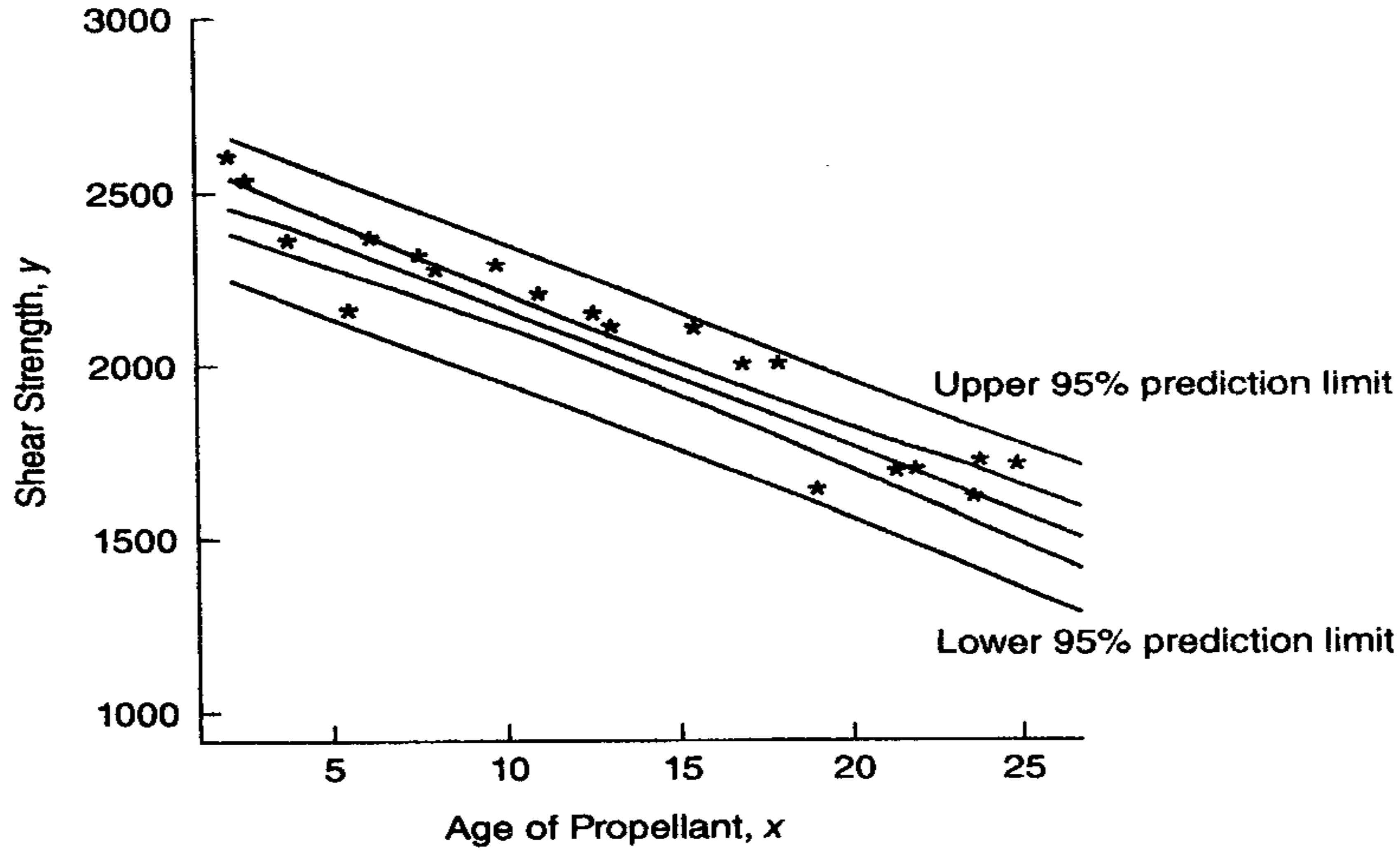
$$\hat{\mu}_{y|x_0} - (2.101)\sqrt{9244.59\left(\frac{1}{20} + \frac{(x_0 - 13.3625)^2}{1106.56}\right)}$$

$$\le E(y|x_0) \le \hat{\mu}_{y|x_0} + (2.101)\sqrt{9244.59\left(\frac{1}{20} + \frac{(x_0 - 13.3625)^2}{1106.56}\right)}$$

Confidence Limits of Estimated Mean Shear Strength (Fig 2.4, page 32 of Textbook)



Prediction Limits of New Shear Strength (Fig 2.5, page 35 of Textbook)



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