625.661 Statistical Models and Regression

Module 6 Discussion Question

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Please complete all the following problems.

1. Two different treatments are of interest. The first treatment has two levels (*A* versus *B*). The second treatment has two levels (*a* versus *b*). Denote by y the targeted response variable.

a) Construct a multiple linear regression model to estimate the difference in the expected value of y between *A* and *B* , the difference in the expected value of y between *a* and *b* , and the difference in the expected value of y among the four treatment combinations *Aa*, *Ab*, *Ba*, *Bb*.

Define *X* = 1 for *A* and 0 for *B*. Define *Z* = 1 for *a* and 0 for *b*.

The model for estimating the three differences in expected value of *y* :
$E(y \mid X, Z) = \beta_0 + \beta_x X + \beta_z Z + \gamma XZ$.

From this model, we can derive

For *Aa*, the expected value of *y* is $E(y \mid X = 1, Z = 1) = \beta_0 + \beta_x + \beta_z + \gamma$.

For *Ab*, the expected value of *y* is $E(y \mid X = 1, Z = 0) = \beta_0 + \beta_x$.

For *Ba*, the expected value of *y* is $E(y \mid X = 0, Z = 1) = \beta_0 + \beta_z$.

For *Bb*, the expected value of *y* is $E(y \mid X = 0, Z = 0) = \beta_0$.

When $\gamma = 0$ **,**

for *A*, the expected value of *y* is $E(y \mid X = 1, Z) = \beta_0 + \beta_x + \beta_z Z$.

for *B*, the expected value of *y* is $E(y \mid X = 0, Z) = \beta_0 + \beta_z Z$.

for *a*, the expected value of *y* is $E(y \mid Z = 1, X) = \beta_0 + \beta_z + \beta_x X$.

for b, the expected value of y is $E(y \mid Z = 0, X) = \beta_0 + \beta_x X$.

When $\gamma \neq 0$, the expected value of y depends on the level of the second treatment.

b) Construct an analysis of variance model to estimate the difference in the expected value of y between *A* and *B* , the difference in the expected value of y between *a* and *b* , and the difference in the expected value of y among the four treatment combinations *Aa*, *Ab*, *Ba*, *Bb*.

Let τ_A be the mean effect of treatment *A*.

Let τ_B be the mean effect of treatment *B*.

Let λ_a be the mean effect of treatment *a*.

Let λ_b be the mean effect of treatment *b*.

Let μ be the overall mean effect.

The ANOVA model is

$y_{hki} = \mu + \tau_h + \lambda_k + \gamma_{hk} + \varepsilon_{hki}$, $h = A, B$; $k = a, b$; $i = 1, \dots, n$.

Conditions needed: $\tau_A + \tau_B = 0$; $\lambda_a + \lambda_b = 0$; $\sum_{h=A}^{B} \gamma_{hk} = 0$; $\sum_{k=a}^{b} \gamma_{hk} = 0$