Multiple Linear Regression Model – Part V

Johns Hopkins Engineering

625.461 Statistical Models and Regression

Module 4 – Lecture 4C

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Test General **Linear** Hypothesis

Testing equality of regression coefficients

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

Test H_0: $\beta_1 = \beta_3$

Let $\mathbf{T} = [0, 1, 0, -1]$

Then H_0: $\mathbf{T\beta} = \beta_1 - \beta_3 = 0$

Test General **Linear** Hypothesis

Under the reduced model with $\mathbf{T\beta} = \beta_1 - \beta_3 = 0$,

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_1 x_3 + \varepsilon \\ &= \beta_0 + \beta_1 (x_1 + x_3) + \beta_2 x_2 + \varepsilon \\ &= \gamma_0 + \gamma_1 z_1 + \gamma_2 z_2 + \varepsilon \end{aligned}$$

Regressing y on z_1, z_2, we have

$SS_{\text{Res}}(RM)$ with df $= n - 4 + 1 = n - 3$

Test General **Linear** Hypothesis

Under the full model, regressing y on x_1, x_2, x_3, we have

$$SS_{\text{Res}}(FM) \quad \text{with df} = n - 4$$

The SS due to H_0 is

$$SS_{\text{H}} = SS_{\text{Res}}(RM) - SS_{\text{Res}}(FM) \quad \text{with df} = 1$$

The F test for H_0 is

$$F_0 = (SS_H / 1) / [SS_{\text{Res}}(FM) / (n - 4)]$$

$$\sim \; F_{1,n\text{-}4} \text{ under } H_0$$

Test General **Linear** Hypothesis

General setting:

To test $H_0\colon \mathbf{T}\boldsymbol{\beta} = \mathbf{c}, \quad H_1\colon \mathbf{T}\boldsymbol{\beta} \neq \mathbf{c}$

where **T** contains r independent rows.

$$F_0 = \frac{\left(\mathbf{T}\hat{\boldsymbol{\beta}} - \mathbf{c}\right)' \left[\mathbf{T}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{T}'\right]^{-1} \left(\mathbf{T}\hat{\boldsymbol{\beta}} - \mathbf{c}\right) / r}{SS_{\mathrm{Res}}(FM)/(n-p)}$$

$\sim F_{r,n\text{-}k\text{-}1}$ under H_0 $\qquad p = k+1$

Confidence Interval (CI) for Regression Coefficient

$$\hat{\beta}_j - t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 C_{jj}} \le \beta_j \le \hat{\beta}_j + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 C_{jj}}$$

Confidence Interval (CI) for Mean Response

CI of the mean response at $\mathbf{x} = \mathbf{x}_0$

$$\hat{y}_0 = \mathbf{x}_0'\hat{\boldsymbol{\beta}}$$

$$\hat{y}_0 - t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2\mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0} \le E(y|x_0)$$

$$\le \hat{y}_0 + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2\mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0}$$

$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ x_{01} \\ x_{02} \\ \vdots \\ x_{0k} \end{bmatrix}$$

The Delivery Time Data (Ex 3.8, 3.9, page 98-99 of Textbook)

$$\hat{\beta}_1 - t_{0.025,22}\sqrt{\hat{\sigma}^2 C_{11}} \le \beta_1 \le \hat{\beta}_1 + t_{0.025,22}\sqrt{\hat{\sigma}^2 C_{11}}$$

$$1.61591 - (2.074)\sqrt{(10.6239)(0.00274378)}$$
$$\le \beta_1 \le 1.61591 + (2.074)\sqrt{(10.6239)(0.00274378)}$$

$$1.61591 - (2.074)(0.17073) \le \beta_1 \le 1.61591 + (2.074)(0.17073)$$

The Delivery Time Data (Ex 3.8, 3.9, page 98-99 of Textbook)

$$\mathbf{x}_0 = \begin{bmatrix} 1 \\ 8 \\ 275 \end{bmatrix}$$

$$\hat{y}_0 = \mathbf{x}_0'\hat{\boldsymbol{\beta}} = [1 \quad 8 \quad 275]\begin{bmatrix} 2.34123 \\ 1.61591 \\ 0.01438 \end{bmatrix} = 19.22 \text{ minutes}$$

$$\begin{aligned}\hat{\sigma}^2\mathbf{x}_0'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x}_0 &= 10.6239[1 \quad 8 \quad 275] \\ &\times \begin{bmatrix} 0.11321518 & -0.00444859 & -0.00008367 \\ -0.00444859 & 0.00274378 & -0.00004786 \\ -0.00008367 & -0.00004786 & 0.00000123 \end{bmatrix}\begin{bmatrix} 1 \\ 8 \\ 275 \end{bmatrix} \\ &= 10.6239(0.05346) = 0.56794\end{aligned}$$

$$19.22 - 2.074\sqrt{0.56794} \le E(y|x_0) \le 19.22 + 2.074\sqrt{0.56794}$$

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