Nonlinear Regression – Part IV

Johns Hopkins Engineering

625.461 Statistical Models and Regression

Module 14 – Lecture 14B

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Puromycin Data (Ex 12.5, page 402)

$$y = f(x, \theta_1, \theta_2) + \varepsilon = \frac{\theta_1 x}{\theta_2 + x} + \varepsilon$$

$$\frac{\partial f(x, \theta_1, \theta_2)}{\partial \theta_1} = \frac{x}{\theta_2 + x} \quad \text{and} \quad \frac{\partial f(x, \theta_1, \theta_2)}{\partial \theta_2} = \frac{-\theta_1 x}{(\theta_2 + x)^2}$$

Set initial value: $\theta_{10} = 205,\ \theta_{20} = 0.08.$

Puromycin Data (Ex 12.5, page 402)

$$\hat{\beta}_0 = \begin{bmatrix} 8.03 \\ -0.017 \end{bmatrix}$$

$$S(\theta_0) = 3155$$

TABLE 12.2 **Data, Fitted Values, Residuals, and Derivatives for the Puromycin Data at $\hat{\theta}_0' = [205, 0.08]'$**

i	x_i	y_i	f_i^0	$y_i - f_i^0$	Z_{i1}^0	Z_{i2}^0
1	0.02	76	41.00	35.00	0.2000	–410.00
2	0.02	47	41.00	6.00	0.2000	–410.00
3	0.06	97	87.86	9.14	0.4286	–627.55
4	0.06	107	87.86	19.14	0.4286	–627.55
5	0.11	123	118.68	4.32	0.5789	–624.65
6	0.11	139	118.68	20.32	0.5789	–624.65
7	0.22	159	150.33	8.67	0.7333	–501.11
8	0.22	152	150.33	1.67	0.7333	–501.11
9	0.56	191	179.38	11.62	0.8750	–280.27
10	0.56	201	179.38	21.62	0.8750	–280.27
11	1.10	207	191.10	15.90	0.9322	–161.95
12	1.10	200	191.10	8.90	0.9322	–161.95

Puromycin Data (Ex 12.5, page 402)

$$\hat{\theta}_1 = \hat{\beta}_0 + \theta_0$$

$$= \begin{bmatrix} 8.03 \\ -0.017 \end{bmatrix} + \begin{bmatrix} 205.00 \\ 0.08 \end{bmatrix} = \begin{bmatrix} 213.03 \\ 0.063 \end{bmatrix}$$

$$S(\hat{\theta}_1) = 1206$$

The Gauss-Newton algorithm converges at

$$\hat{\theta} = [212.7, 0.0641]' \qquad S(\hat{\theta}) = 1195$$

Puromycin Data (Ex 12.5, page 402)

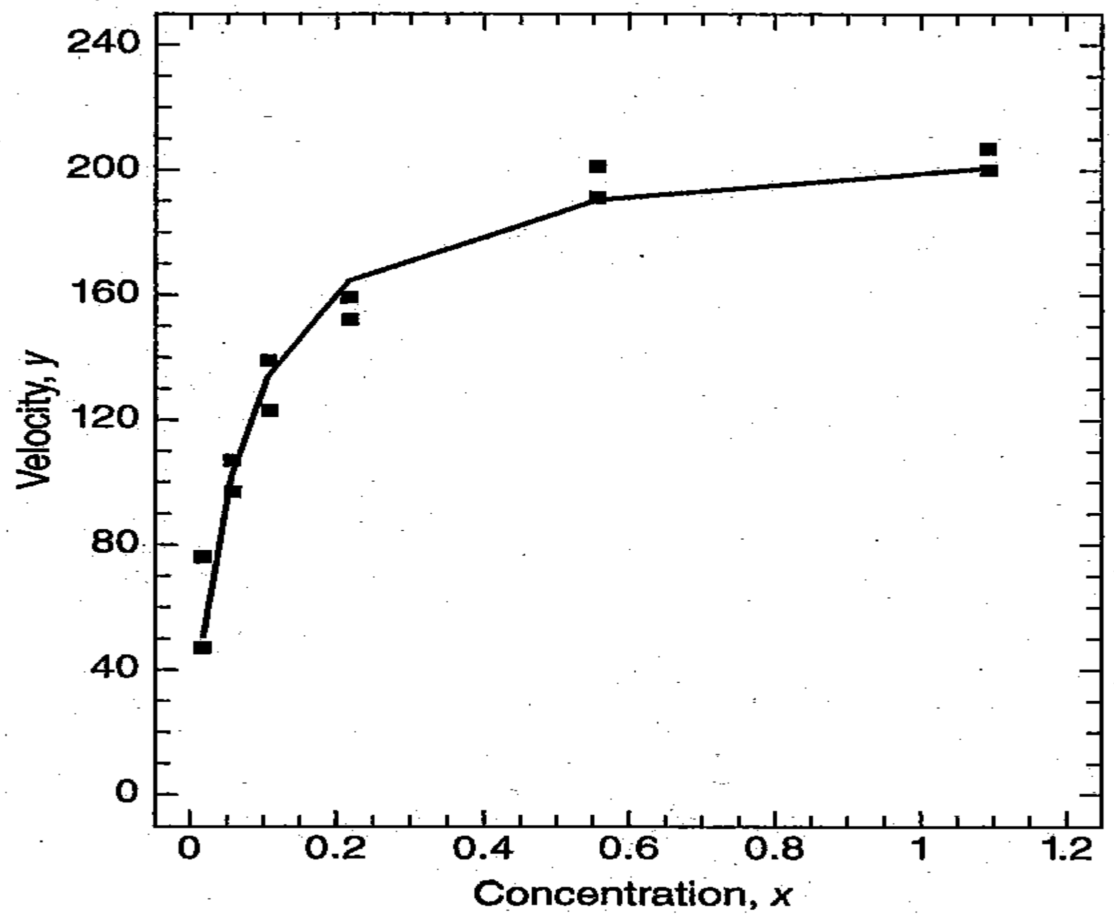

Figure 12.4 Plot of fitted nonlinear regression model, Example 12.5.

Puromycin Data (Ex 12.5, page 402)

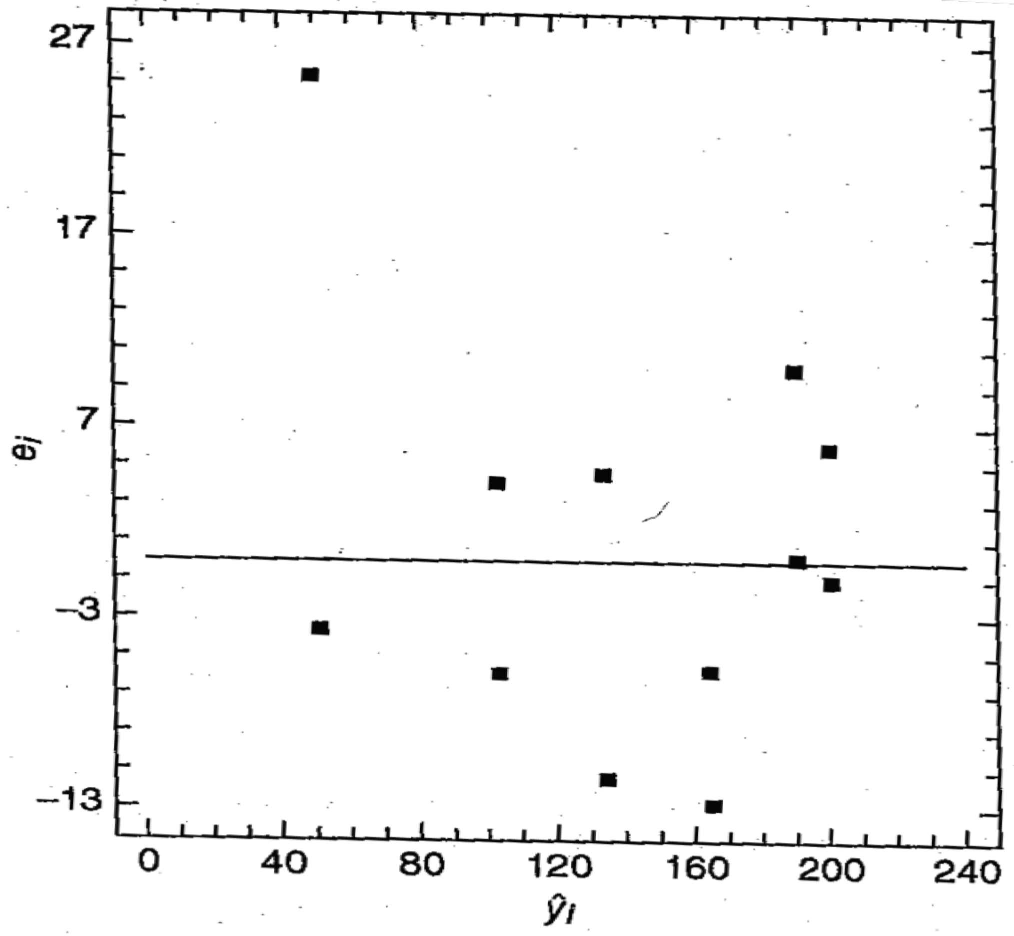

Figure 12.5 Plot of residuals versus predicted values, Example 12.5.

Puromycin Data (Ex 12.5, page 402)

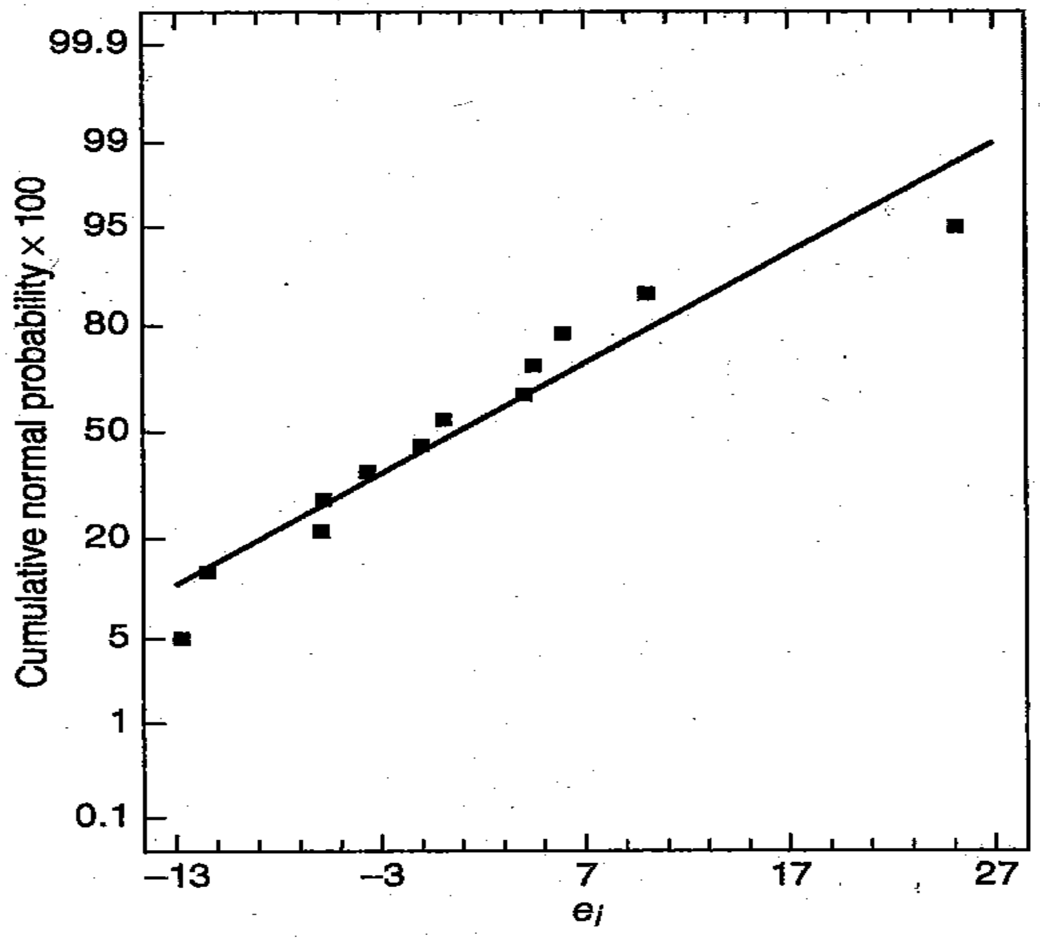

Figure 12.6 Normal probability plot of residuals, Example 12.5.

Puromycin Data (Ex 12.5, page 402)

$$\hat{\sigma}^2 = MS_{\text{Res}} = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-p} = \frac{\sum_{i=1}^{n}\left[y_i - f\left(\mathbf{x}_i, \hat{\boldsymbol{\theta}}\right)\right]^2}{n-p} = \frac{S\left(\hat{\boldsymbol{\theta}}\right)}{n-p}$$

$$\hat{\sigma}^2 = \frac{S\left(\hat{\boldsymbol{\theta}}\right)}{n-p} = \frac{1195}{12-2} = 119.5$$

$$\text{Var}\left(\hat{\boldsymbol{\theta}}\right) = \hat{\sigma}^2(\mathbf{Z}'\mathbf{Z})^{-1} = 119.5\begin{bmatrix} 0.4037 & 36.82\times10^{-5} \\ 36.82\times10^{-5} & 57.36\times10^{-8} \end{bmatrix}$$

Puromycin Data (Ex 12.5, page 402)

The correlation between $\hat{\theta}_1$ and $\hat{\theta}_2$

$$\frac{36.82\times10^{-5}}{\sqrt{0.4037(57.36\times10^{-8})}}=0.77$$

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