Notes for Module 11 Lecture 11D

Slide #7 to #8

Without the two regressors, the model is the constant-probability-of-success model

$$\pi_i = \frac{e^{\beta_0}}{1+e^{\beta_0}} \quad ,i = 1, \dots , n \ .$$

Since π_i is constant for all i, the maximum likelihood estimator of π_i is y/n , where $y = \sum_{i=1}^{n} y_i$.

Thus, using Eq (13.9), p.425 of the Textbook,

$$L(RM) = \sum_{i=1}^{n} y_i \ln\left(\frac{y}{n}\right) + \sum_{i=1}^{n} (n_i - y_i) \ln\left(1 - \frac{y}{n}\right)$$

$$= y\ln(y) + (n-y)\ln(n-y) - n\ln(n) \ .$$

Slide 11

The equation

$$D = 2\ln\frac{L(\text{saturated model})}{L(FM)} = 2\sum_{i=1}^{n}\left[y_i \ln\left(\frac{y_i}{n_i \pi_i}\right) + (n_i - y_i)\ln\left(\frac{n_i - y_i}{n_i(1-\hat{\pi}_i)}\right)\right]$$

is revised to

$$D = 2\ln\left[\frac{L(\text{saturated model})}{L(FM)}\right]$$

$$= 2\sum_{i=1}^{n}\left[y_i \ln\left(\frac{y_i}{n_i \hat{\pi}_i}\right) + (n_i - y_i)\ln\left(\frac{n_i - y_i}{n_i(1-\hat{\pi}_i)}\right)\right]$$

where n_i is the sample size of group i, $n = \sum n_i$.