Nonlinear Regression – Part V

Johns Hopkins Engineering

625.461 Statistical Models and Regression

Module 14 – Lecture 14C

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Puromycin Data (Ex 12.5, page 402)

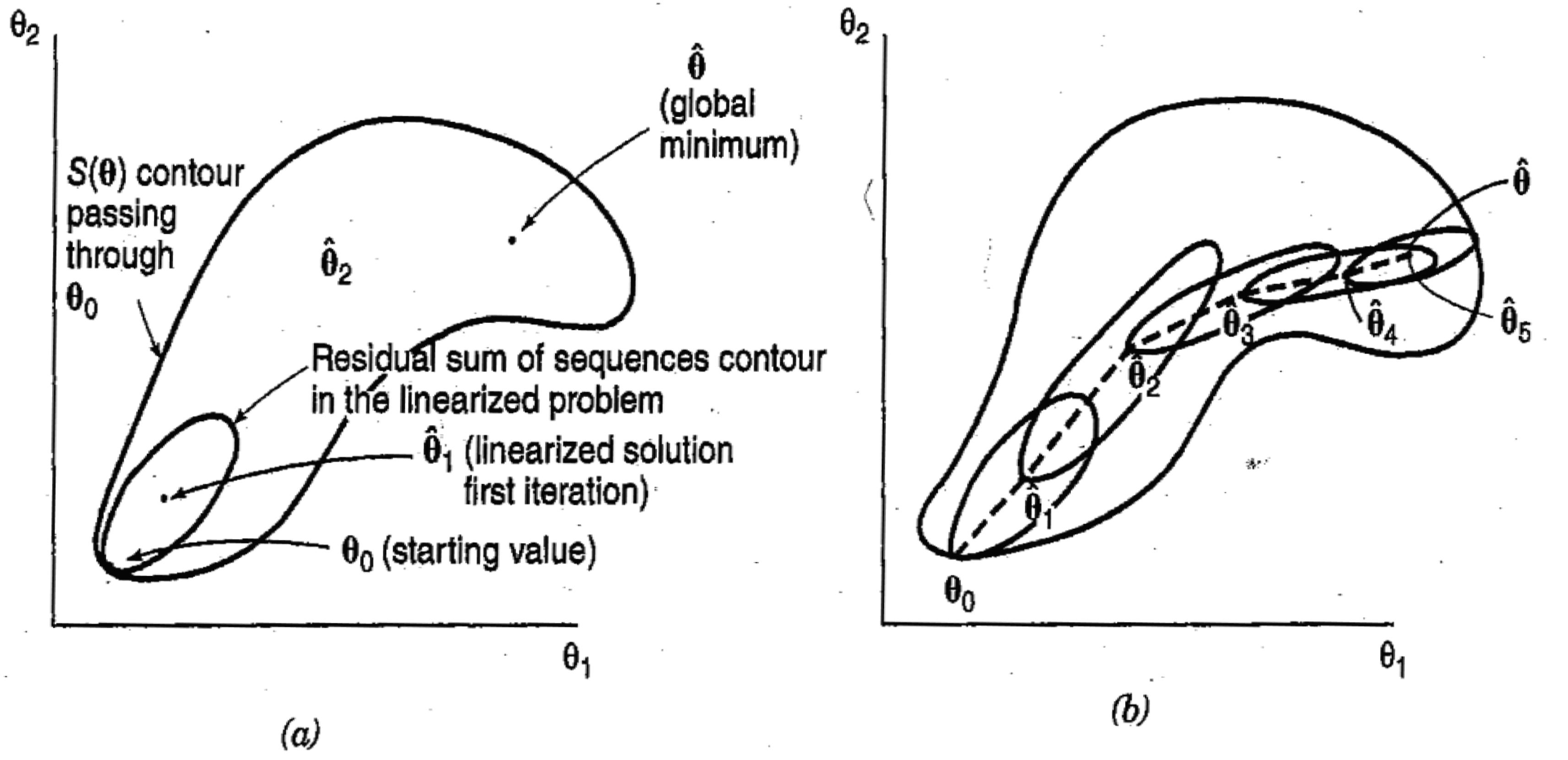

Figure 12.7 A geometric view of linearization: (*a*) the first iteration; (*b*) evolution of successive linearization iterations.

Puromycin Data (Ex 12.5, page 402)

Starting (Initial) value of θ

Good starting values that are close to the true parameter values will minimize convergence difficulties.

In NL regression models the parameters often have physical meaning, and this can be helpful in obtaining starting values.

In the M-M function for the puromycin data, θ_1 is the asymptotic velocity of the reaction, θ_2 is the half concentration.

Statistical Inference in NL regression

Statistical inference depends on large-sample or asymptotic results. The large-sample theory generally applies for both normally and nonnormally distributed errors.

$$\hat{\boldsymbol{\theta}} \approx N_p(\boldsymbol{\theta}, \sigma^2(\mathbf{Z'Z})^{-1})$$

Puromycin Data (Ex 12.6, page 410)

To test H_0: $\theta_1 = \theta_2 = 0$, we could use the model or regression sum squares:

$$
\begin{aligned}
SS_{\text{model}} &= SS_{\text{T}} - SS_{\text{Res}} \\
&= 271{,}410 - 1195.4 \\
&= 270{,}214.6
\end{aligned}
$$

$$F_0 = \frac{SS_{\text{model}}/2}{MS_{\text{Error}}} = \frac{270{,}241.6/2}{119.5} = 1130.61$$

to be compared to $F_{0.05,2,10}$.

Puromycin Data (Ex 12.6, page 410)

To test H_0: $\theta_1 = 0$, use approximate t statistic

$$t_0 = \frac{\hat{\theta}_1}{\mathrm{se}\left(\hat{\theta}_1\right)} = \frac{212.7}{6.9471} = 30.62$$

To test H_0: $\theta_2 = 0$, use approximate t statistic

$$t_0 = \frac{\hat{\theta}_2}{\mathrm{se}\left(\hat{\theta}_2\right)} = \frac{0.0641}{0.00828} = 7.74$$

Both are compared to $t_{0.025,10}$.

Puromycin Data (Ex 12.6, page 410)

Approximate 95% CIs:

$$\hat{\theta}_1 - t_{0.025,10}\mathrm{se}\left(\hat{\theta}_1\right) \le \theta_1 \le \hat{\theta}_1 + t_{0.025,10}\mathrm{se}\left(\hat{\theta}_1\right)$$

$$212.7 - 2.228(6.9471) \le \theta_1 \le 212.7 + 2.228(6.9471)$$

$$197.2 \le \theta_1 \le 228.2$$

$$\hat{\theta}_2 - t_{0.025,10}\mathrm{se}\left(\hat{\theta}_2\right) \le \theta_2 \le \hat{\theta}_2 + t_{0.025,10}\mathrm{se}\left(\hat{\theta}_2\right)$$

$$0.0641 - 2.228(0.00828) \le \theta_2 \le 0.0641 + 2.228(0.00828)$$

$$0.0457 \le \theta_2 \le 0.0825$$

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