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Module 6 Discussion

1. Two different treatments are of interest. The first treatment has two levels ( versus ). The second treatment has two levels ( versus ). Denote by the targeted response variable.
   1. Construct a multiple linear regression model to estimate the difference in the expected value of between and , the difference in the expected value of between and , and the difference in the expected value of among the four treatment combinations , , , .

Ans:

Here, we have a regression model of the form,

where

This form of the model is going beyond the first order terms by including the interaction between and with .

The first part asks us to estimate the difference in the expected value of between and . This can be expressed as follows:

Since in equation (1), we are including an interaction term, then it will include the impact of . Therefore, if for example the observation belongs to treatment , then the resulting expected value would be and simply if the observation instead belonged to treatment .

The second part asks us to estimate the difference in the expected value of between and . This can be expressed as follows:

Similarly, like before, since in equation (1), we are including an interaction term, then it will include the impact of . Therefore, if for example the observation belongs to treatment , then the resulting expected value would be and simply if the observation instead belonged to treatment .

The third part asks us to estimate the difference in the expected value of among the four treatment combinations , , , . These different models can be seen below in Table 1.

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Table The above table shows the different resulting multiple linear regression models based on the combinations of treatment levels from equation (1).

In such a scenario, there would be a total of different combinations. These different scenarios can be seen below in Table 2. It is possible to add more to for example show , but that will be ignored for simplicity since the results are roughly the same.

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Table The above table shows the difference in expected value for the various combinations of treatment levels.

* 1. Construct an analysis of variance model to estimate the difference in the expected value of between and , the difference in the expected value of between and , and the difference in the expected value of among the four treatment combinations , , , .

Ans:

Let the two-way analysis-of-variance model be represented as

where is the effect of level of the first treatment, is the effect of level of the second treatment type, is an interaction effect between the two treatment types, and is an random-error component. In this problem corresponds with levels and , while corresponds with levels and respectively.

The first part asks us to estimate the difference in the expected value of between and . This can be expressed as follows:

Like in part a), this result is dependent on the second treatment due to the interaction term. Therefore, depending on if the observation is given treatment or the result will vary by the index where or respectively.

The second part asks us to estimate the difference in the expected value of between and . This can be expressed as follows:

Like before, this result is dependent on the second treatment due to the interaction term. Therefore, depending on if the observation is given treatment or the result will vary by the index where or respectively.

The third part asks us to estimate the difference in the expected value of among the four treatment combinations , , , . Similar to part a), there will be a total of different combinations. These different scenarios can be seen below in Table 3. It is possible to add more to for example show , but again that will be ignored for simplicity since the results are roughly the same.

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Table 3 The above table shows the difference in expected value for the various combinations of treatment levels.