test\_2\_rmd

### Problem 3  
df <- data.frame(  
 y=c(7,8,5,4,2,10,9,10,8,8),  
 x1=c(9,6,10,8,5,7,6,5,5,4),  
 x2=rep(c(1,-1), each=5)  
)  
y <- df[,1]  
n <- nrow(df)  
ones <- rep(1, n)  
x1 <- df[,2]; x2 <- df[,3]  
X <- cbind(ones, x1, x2, x1\*x2)  
  
### part (a)  
beta\_hat\_calc <- function(X, y) {  
 beta\_hat <- solve(t(X) %\*% X) %\*% t(X) %\*% y  
 return(beta\_hat)  
}  
H\_calc <- function(X) {  
 H <- X %\*% solve(t(X) %\*% X) %\*% t(X)  
 return(H)  
}  
y\_hat\_calc <- function(H, y) {  
 y\_hat <- H %\*% y  
 return(y\_hat)  
}  
e\_calc <- function(y, y\_hat) {  
 e <- y - y\_hat  
 return(e)  
}  
  
X\_red <- cbind(ones, x1)  
beta\_hat\_full <- beta\_hat\_calc(X = X, y = y)  
beta\_hat\_red <- beta\_hat\_calc(X = X\_red, y = y)  
  
SS\_R\_calc <- function(beta\_hat, X, y) {  
 n <- length(y)  
 SS\_R <- (t(beta\_hat) %\*% t(X) %\*% y) - ((sum(y)^2) / n)  
 return(SS\_R)  
}  
SS\_Res\_calc <- function(y, beta\_hat, X) {  
 SS\_Res <- (t(y) %\*% y) - (t(beta\_hat) %\*% t(X) %\*% y)  
 return(SS\_Res)  
}  
  
SS\_Res\_full <- SS\_Res\_calc(y = y, beta\_hat = beta\_hat, X = X)  
k <- 3; p <- k + 1  
r <- 2  
MS\_Res <- SS\_Res\_full / (n - p)  
SS\_R\_full <- SS\_R\_calc(beta\_hat = beta\_hat, X = X, y = y)  
SS\_R\_red <- SS\_R\_calc(beta\_hat = beta\_hat\_red, X = X\_red, y = y)  
  
F\_0 <- ((SS\_R\_full - SS\_R\_red) / r) / MS\_Res  
alpha <- 0.05  
qf(p = (1 - alpha), df1 = k, df2 = (n - k - 1))  
pf(q = F\_0, df1 = r, df2 = (n - k - 1), lower.tail = FALSE)  
  
### part (b)  
2 \* beta\_hat[3] + 10 \* beta\_hat[4] # -4.38551  
  
z\_value <- qnorm(alpha/2, lower.tail = FALSE)  
x\_01 <- c(1, 5, 1, 5)  
x\_02 <- c(1, 5, -1, -5)  
y\_hat\_01 <- t(x\_01) %\*% beta\_hat  
y\_hat\_02 <- t(x\_02) %\*% beta\_hat  
sigma\_squared <- 2  
  
CI\_bound <- z\_value \* sqrt(sigma\_squared \* t(x\_01 - x\_02) %\*%  
 solve(t(X) %\*% X) %\*%  
 (x\_01 - x\_02))  
  
(y\_hat\_01 - y\_hat\_02) + CI\_bound  
(y\_hat\_01 - y\_hat\_02) - CI\_bound  
  
### part (c)  
PI\_bound <- z\_value \* sqrt(sigma\_squared \*  
 (2 +  
 t(x\_01 - x\_02) %\*%  
 solve(t(X) %\*% X) %\*%  
 (x\_01 - x\_02)))  
  
(y\_hat\_01 - y\_hat\_02) + PI\_bound  
(y\_hat\_01 - y\_hat\_02) - PI\_bound  
  
### part (d)  
X\_d <- X[,c(1,3)]  
beta\_hat\_d <- beta\_hat\_calc(X = X\_d, y = y)  
H\_d <- H\_calc(X = X\_d)  
y\_hat\_d <- y\_hat\_calc(H = H\_d, y = y)  
e\_d <- e\_calc(y = y, y\_hat = y\_hat\_d)  
  
e\_d[8]  
  
residual\_variance <- sigma\_squared \* (diag(n) - H\_d)  
residual\_variance[8,8]