Assignment: maths

Instructions:

- Attempt all questions.
- Some questions may have multiple parts.

Maths Assignment

Long Answer Questions

If a and b are positive real numbers such that $a^2 + b^2 = 1$, then find the maximum value of $a^m b^n$ for $m, n \geq 0$ and $m + n = 4$, and prove that your answer is indeed the maximum value.

Let $P(x) = x^3 + ax^2 + bx + c$ be a cubic polynomial with real coefficients such that $P(0) = -1$, $P(1) = 0$, and $P(-1) = 2$. Find the values of a, b, and c and then factorize the given polynomial.

A line L is parallel to the line $3x + 4y + 5 = 0$. Find the equation of the line L that passes through the point $(1, 2)$ and is at a distance of 3 units from the line $3x + 4y + 20 = 0$.

In a triangle ABC, it is given that the length of the medians AD, BE, and CF are 7, 9, and 11 respectively. Find the lengths of the sides AB, BC, and CA of the triangle.

Let $f(x)$ be a real-valued function defined on the interval $[0, 1]$ such that $f(x) + f(1 - x) = 1$ for all x in $[0, 1]$. Find the value of $\int_0^1 f(x) dx$.

If x, y, and z are real numbers such that $x^2 + y^2 + z^2 = 1$ and $x + y + z = 0$, find the maximum value of $xy + yz + zx$ and prove that your answer is indeed the maximum value.