California State University, Dominguez Hills
Department of Computer Science
CSC 595

Professor: Dr. Benyamin Ahmadnia
bahmadniayebosari@csudh.edu

Spring 2025

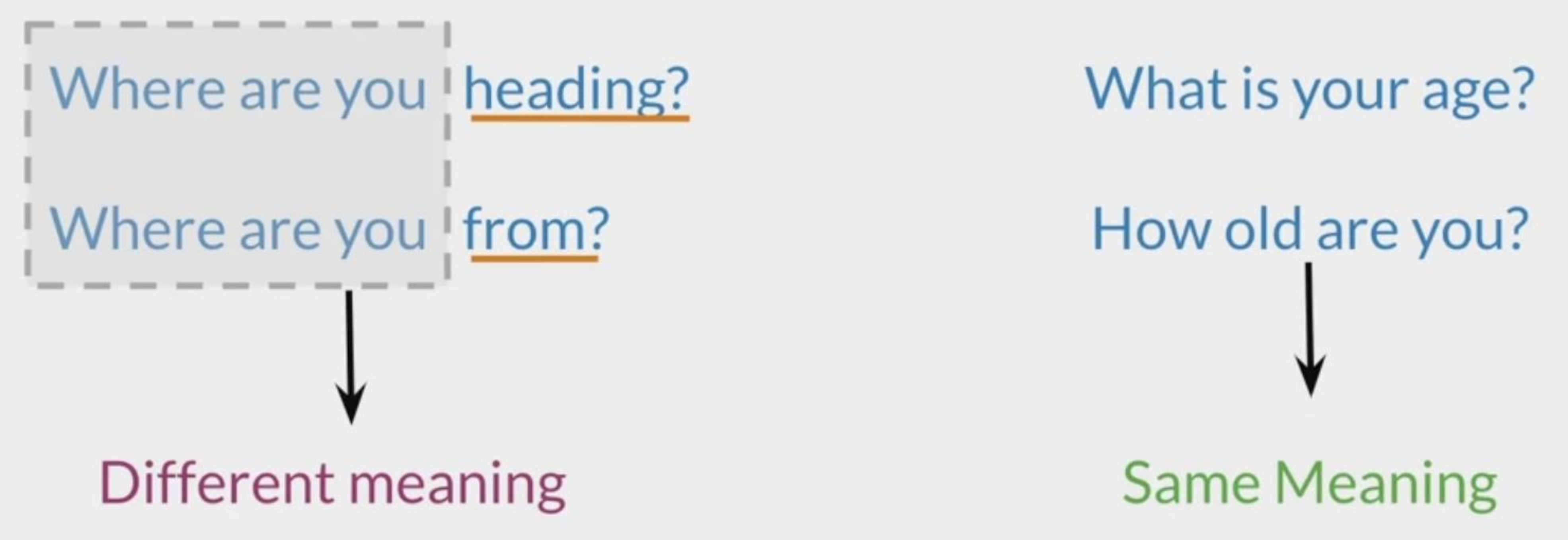
Copyright Notice

Table of Contents

Lecture 3

Vector Space Models

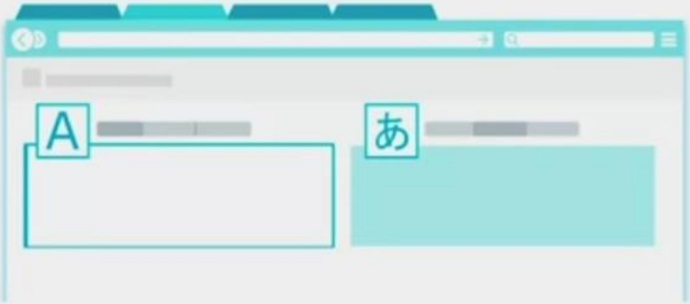
Why Learn Vector Space?



Vector Space Models Applications

- You eat *cereal* from a *bowl*
- You *buy* something and someone else *sells* it

Information Extraction



Machine Translation

Chatbots

Word-by-Word Design

Number of times they *occur together within a certain distance k*

k=2

I like simple data

I prefer simple raw data

	simple
data	2

Word-by-Word Design

Number of times they *occur together within a certain distance k*

I like simple data

I prefer simple raw data

k=2

	simple	raw	like	I
data	2	1	1	0

n

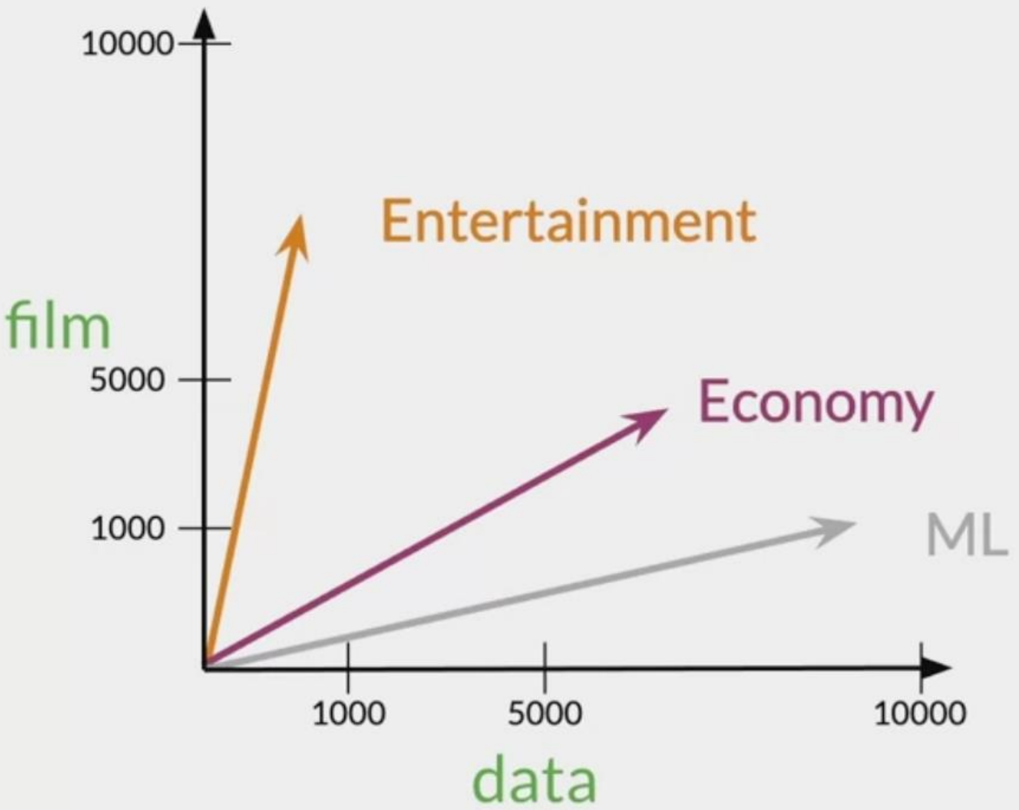
Word-by-Document Design

Number of times a word *occurs within a certain category*

Entertainment | Economy | Machine Learning

	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

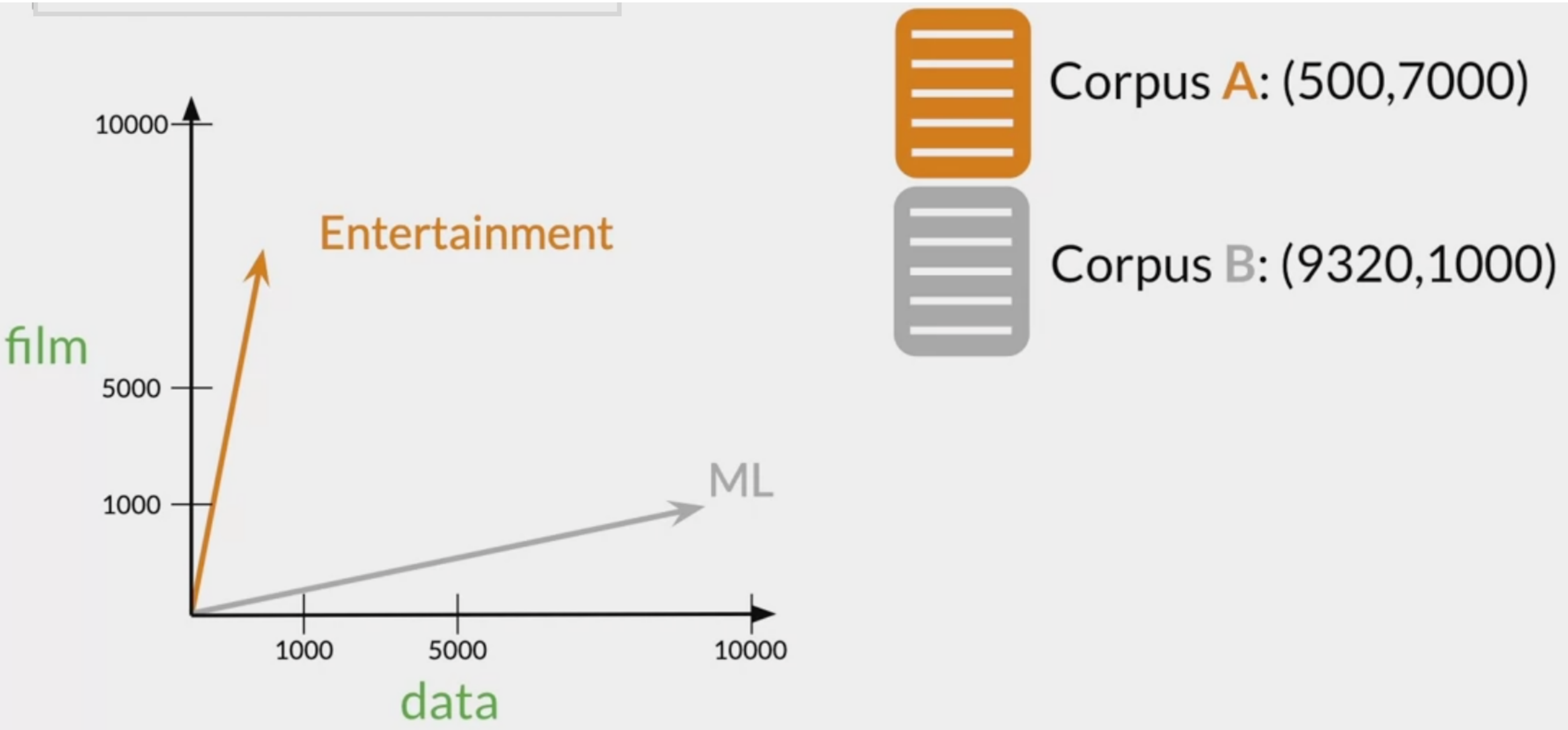
Vector Space



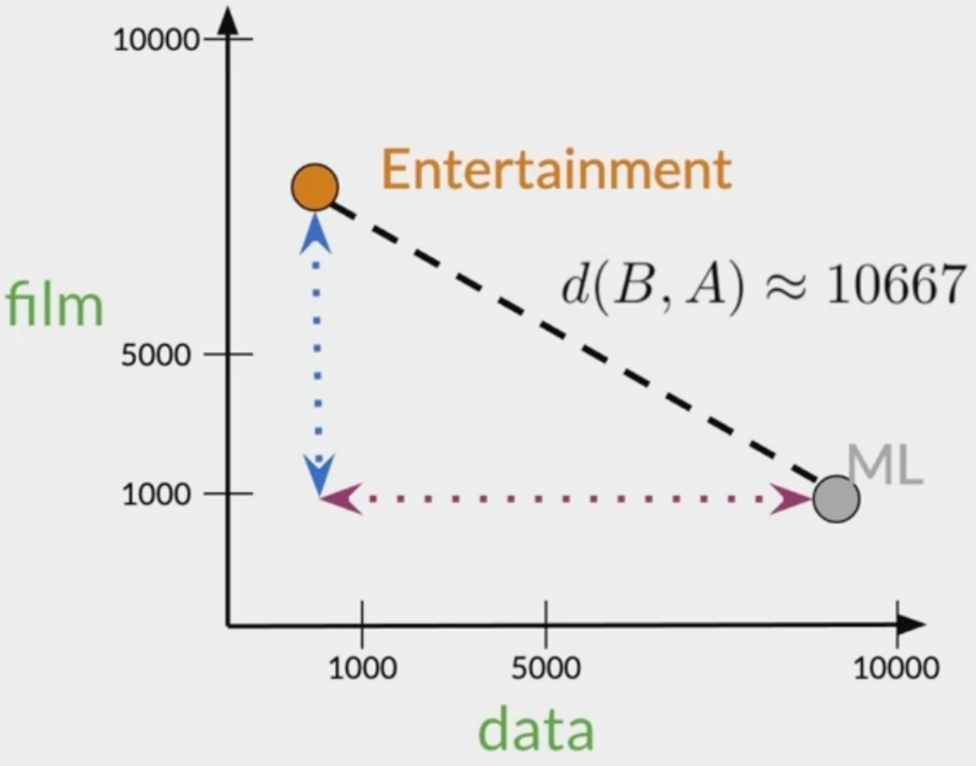
	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Measures of "similarity:"
Angle
Distance

Euclidean Distance



Euclidean Distance



Corpus A: (500,7000)

Corpus B: (9320,1000)

$$d(B, A) = \sqrt{\underline{(B_1 - A_1)^2} + \underline{(B_2 - A_2)^2}}$$

$$c^2 = a^2 + b^2$$

$$d(B, A) = \sqrt{(-8820)^2 + (6000)^2}$$

Euclidean Distance for N-Dimensional Vectors

	data	boba ($\vec{w}$)	ice-cream ($\vec{v}$)
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1-0)^2 + (6-4)^2 + (8-6)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^{n} (v_i - w_i)^2} \longrightarrow \text{Norm of } (\vec{v} - \vec{w})$$

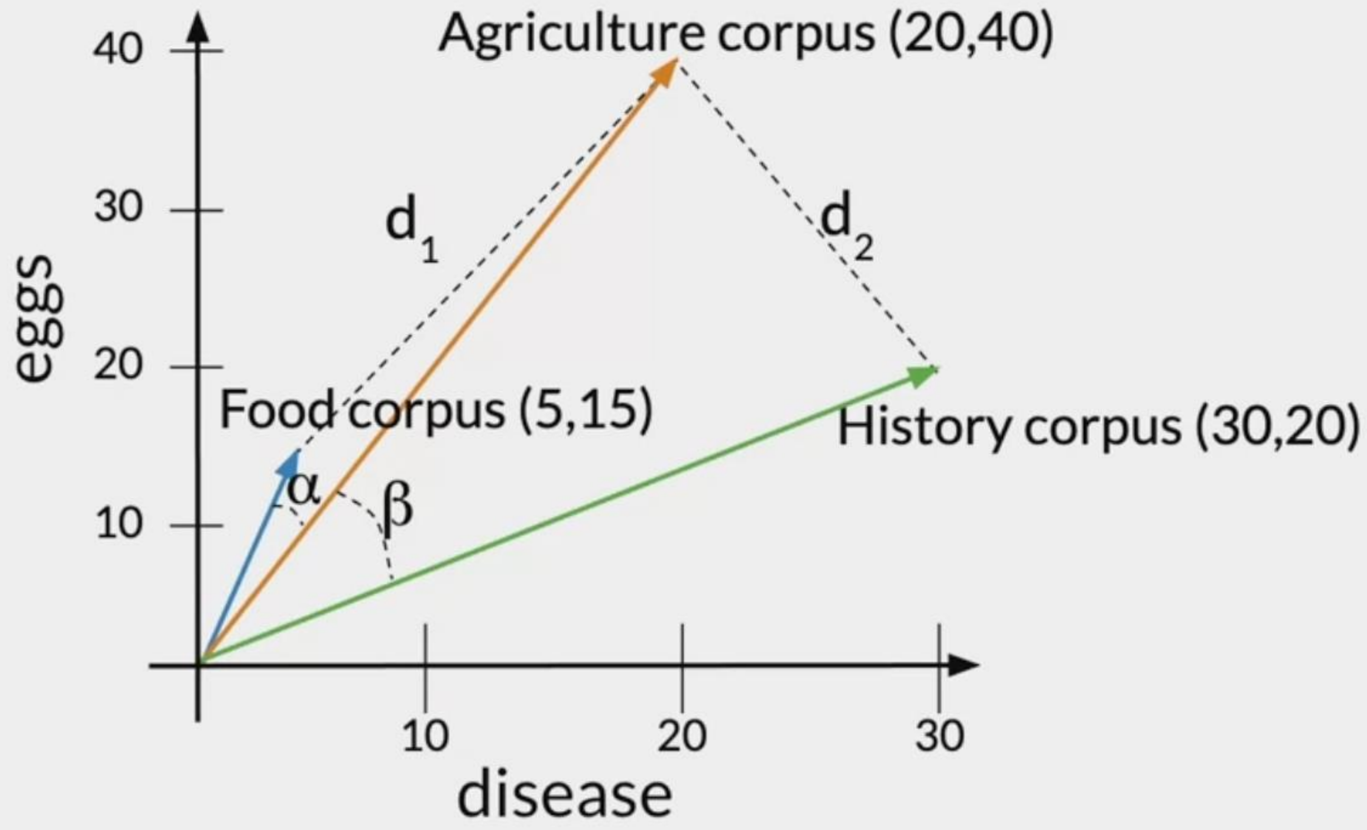
Euclidean Distance in Python

```python
# Create numpy vectors v and w
v = np.array([1, 6, 8])
w = np.array([0, 4, 6])

# Calculate the Euclidean distance d
d = np.linalg.norm(v-w)
# Print the result
print("The Euclidean distance between v and w is: ", d)
```

```
The Euclidean distance between v and w is: 3
```

Euclidean Distance in Python



Euclidean distance: $d_2 < d_1$

Angles comparison: $\beta > \alpha$

The cosine of the angle between the vectors

Previous Definition

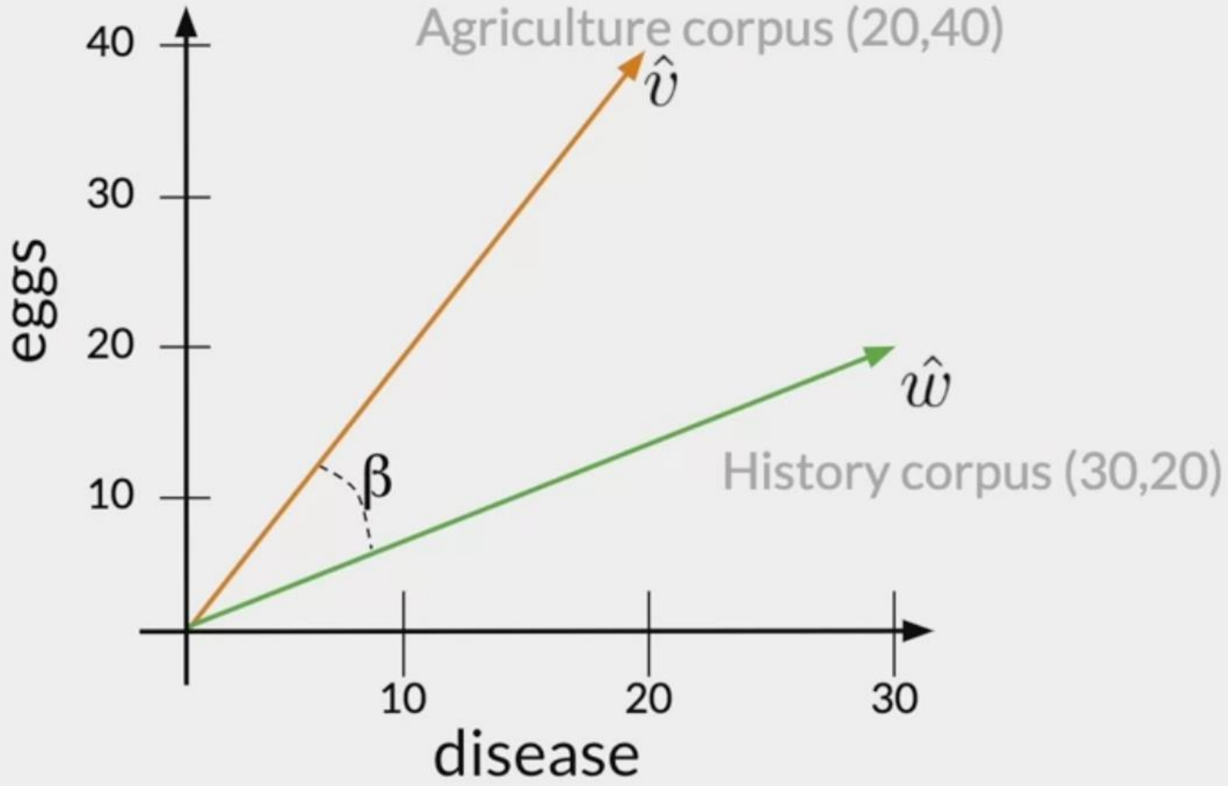
Vector norm

$$\|\vec{v}\| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

Dot product

$$\vec{v}.\vec{w} = \sum_{i=1}^{n} v_i.w_i$$

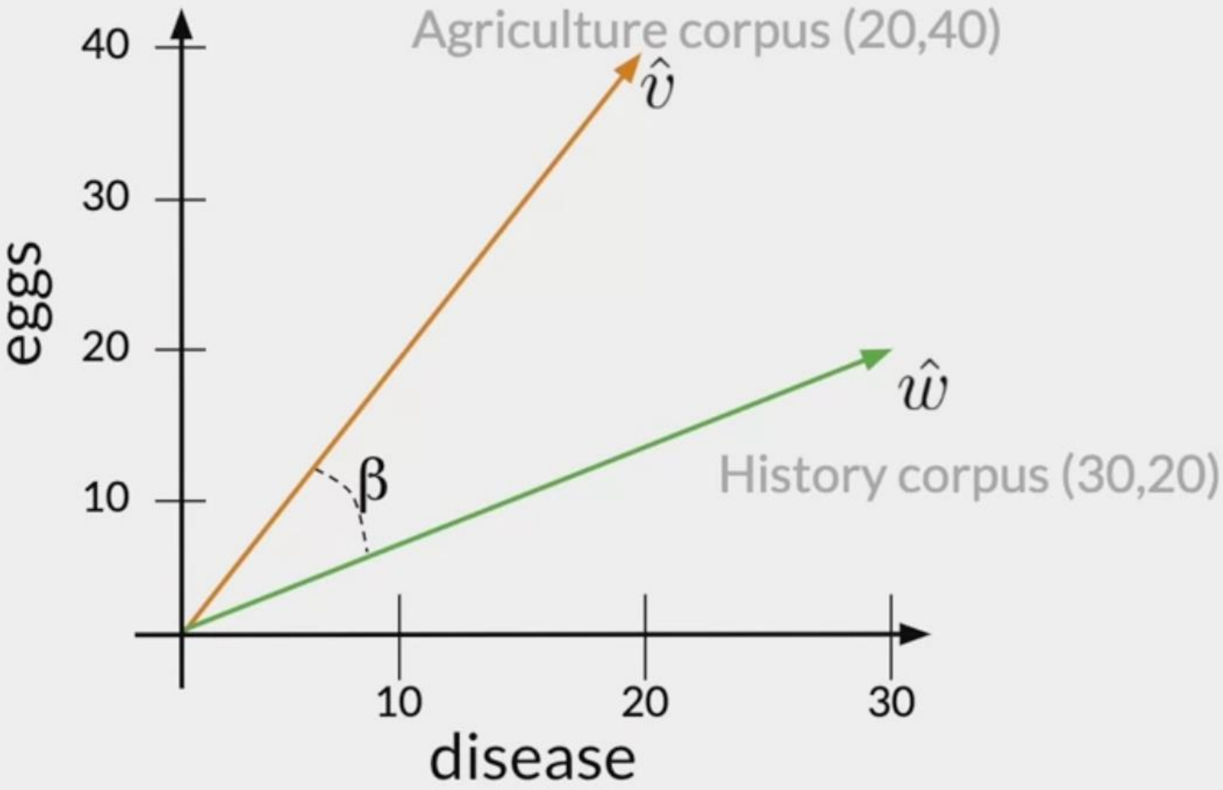
Cosine Similarity



$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}}$$

Cosine Similarity



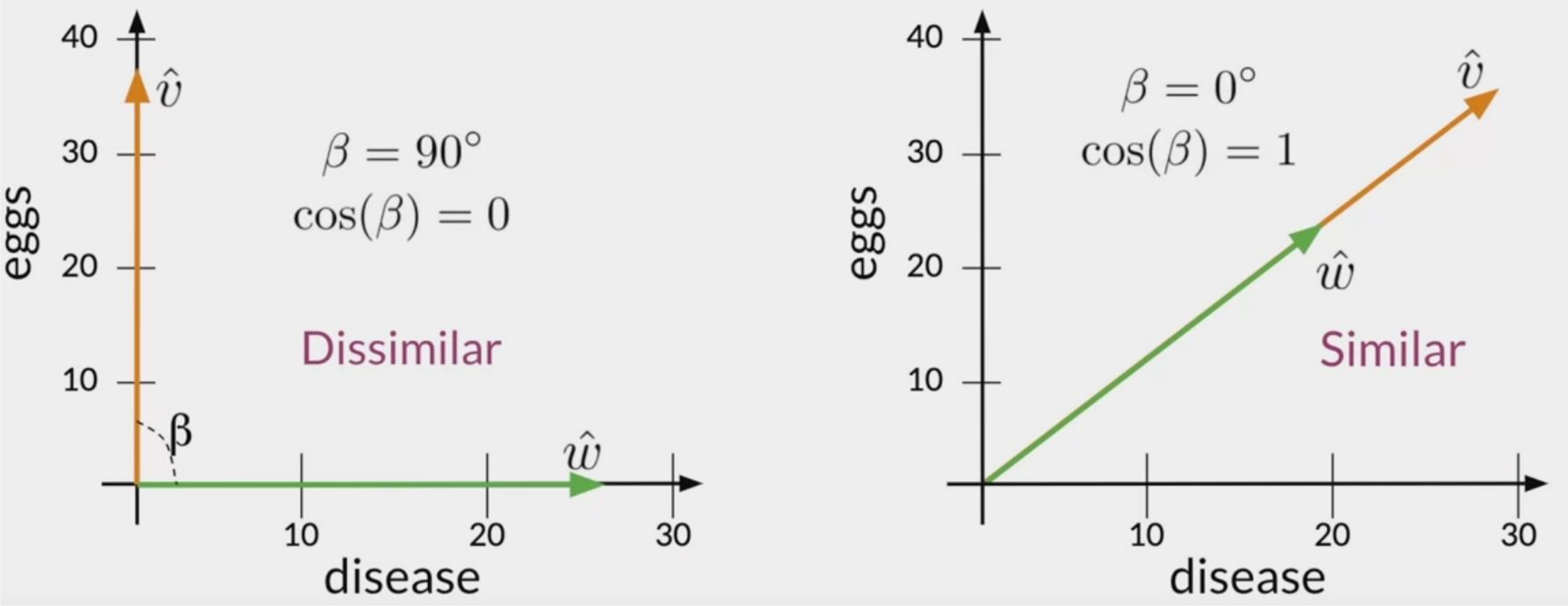
$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}}$$

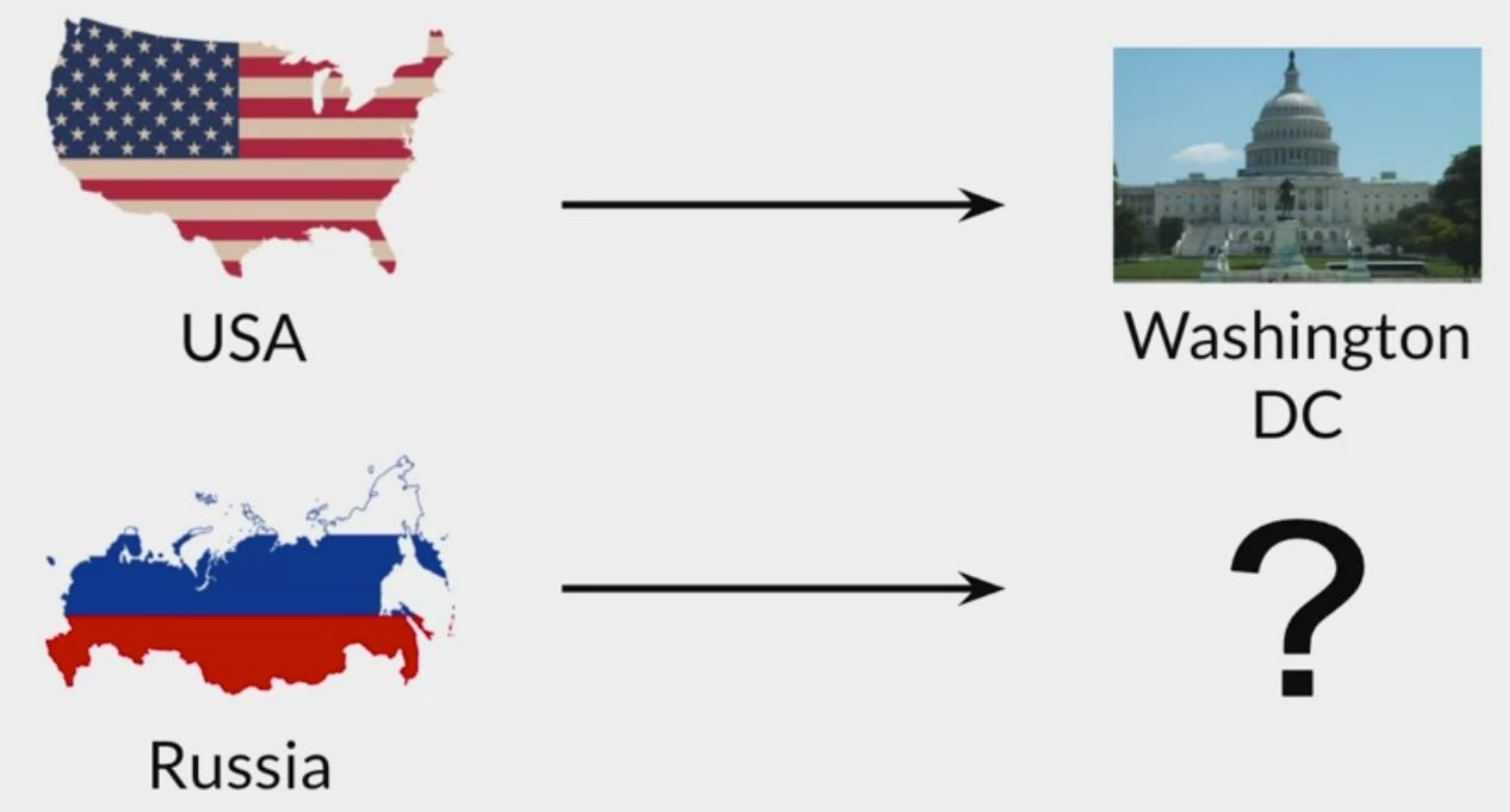
$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}}$$

$$= 0.87$$

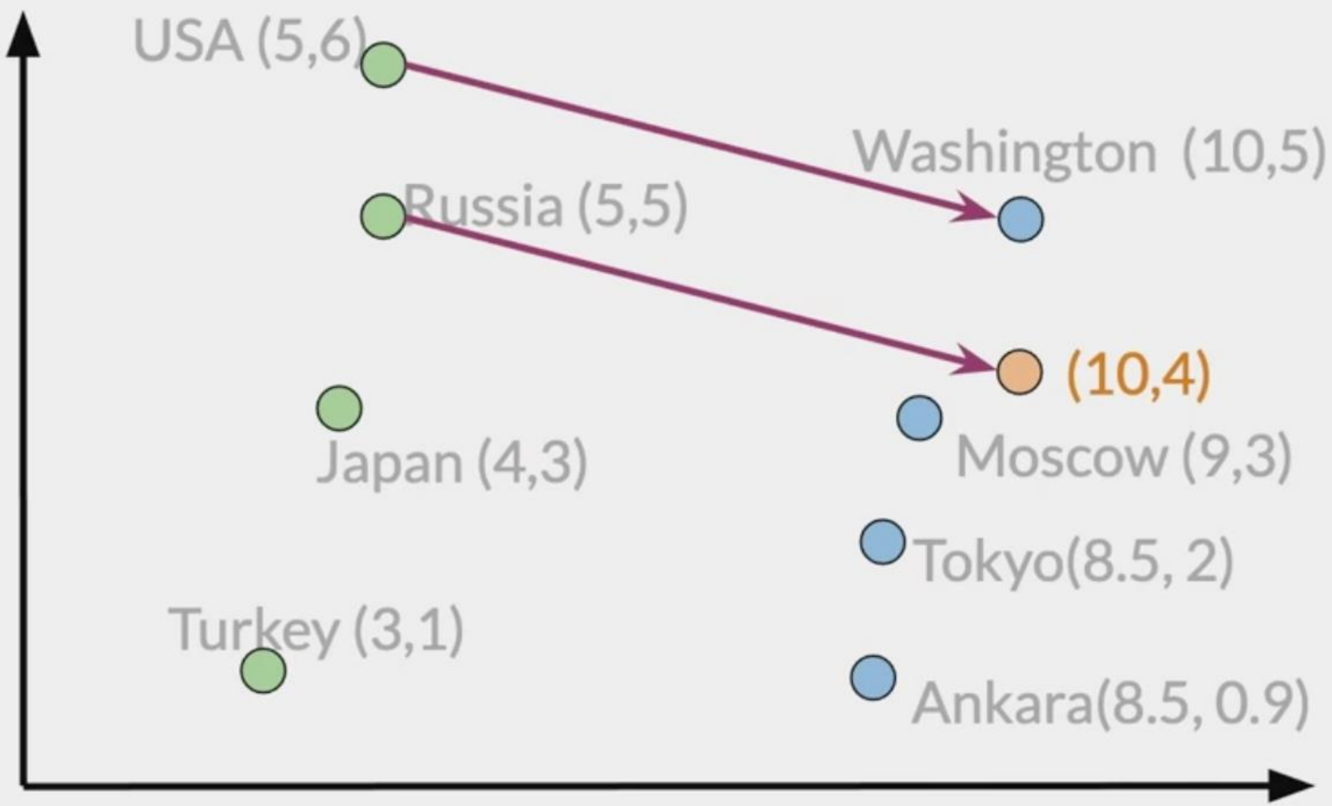
Cosine Similarity



Manipulating Word Vectors



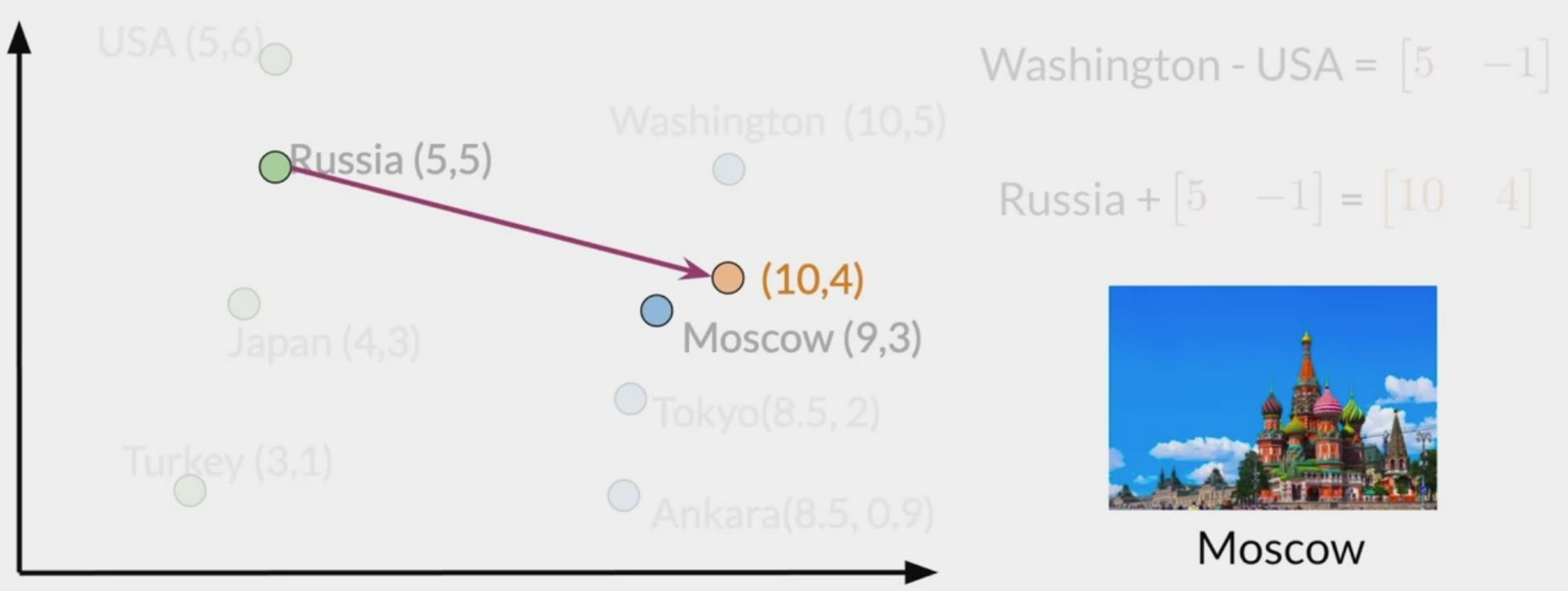
Manipulating Word Vectors



Washington - USA = $\begin{bmatrix} 5 & -1 \end{bmatrix}$

Russia + $\begin{bmatrix} 5 & -1 \end{bmatrix}$ = $\begin{bmatrix} 10 & 4 \end{bmatrix}$

Manipulating Word Vectors



Visualization of Word Vectors

$d > 2$

oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

How can you visualize if your representation captures these relationships?

oil & gas

town & city

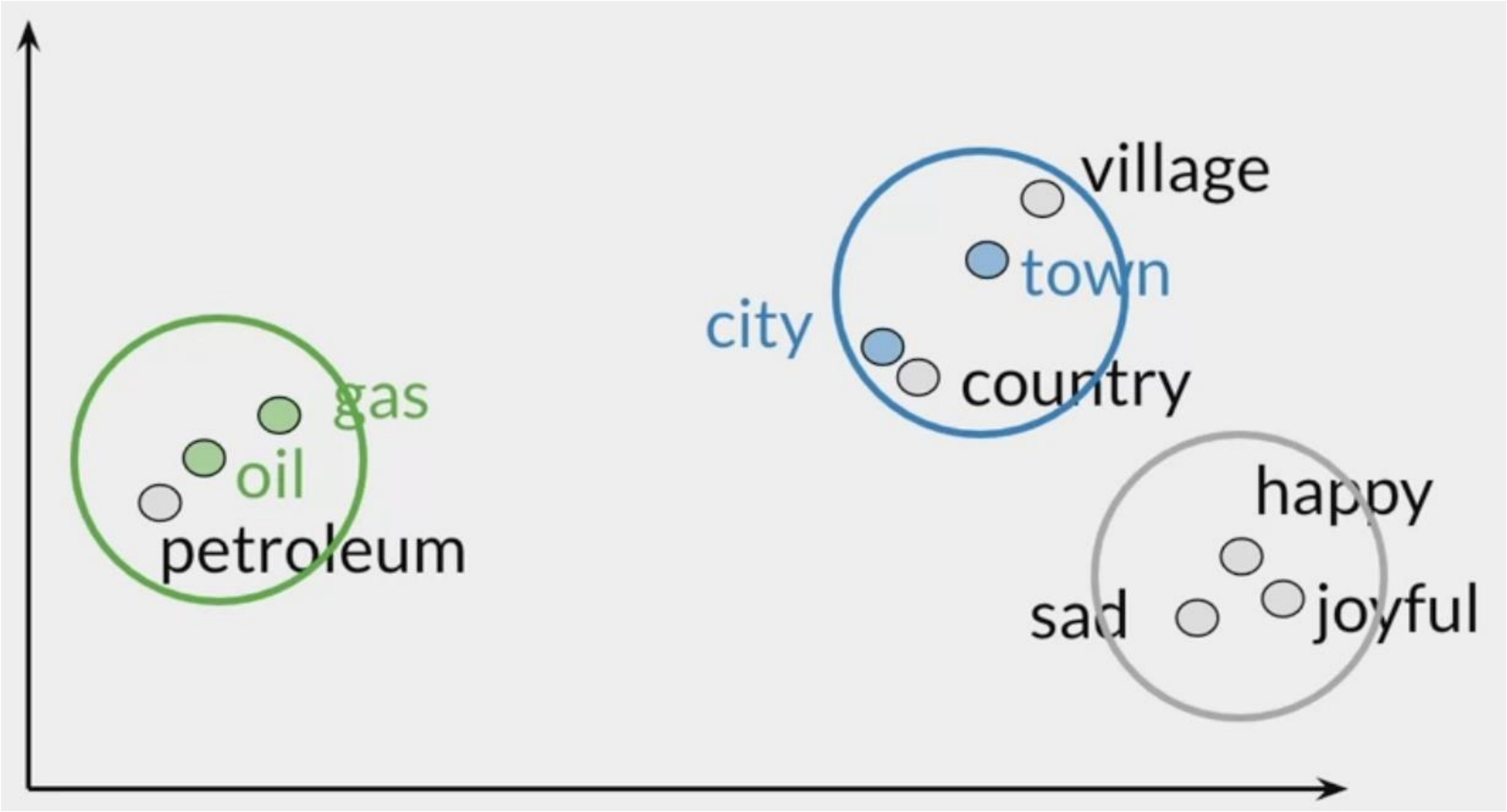
Visualization of Word Vectors

	$d > 2$		
oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

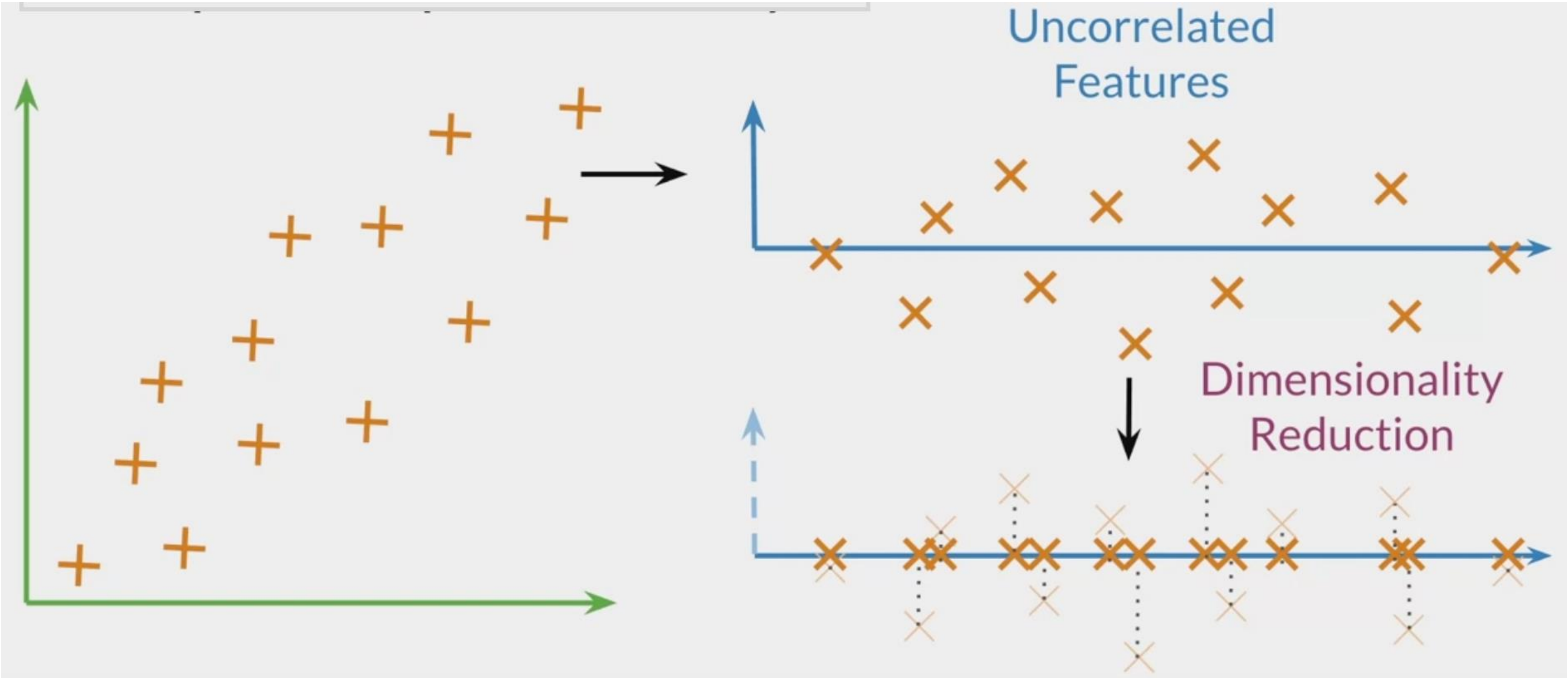
$\xrightarrow{\text{PCA}}$

	$d = 2$	
oil	2.30	21.2
gas	1.56	19.3
city	13.4	34.1
town	15.6	29.8

Visualization of Word Vectors



Principal Component Analysis

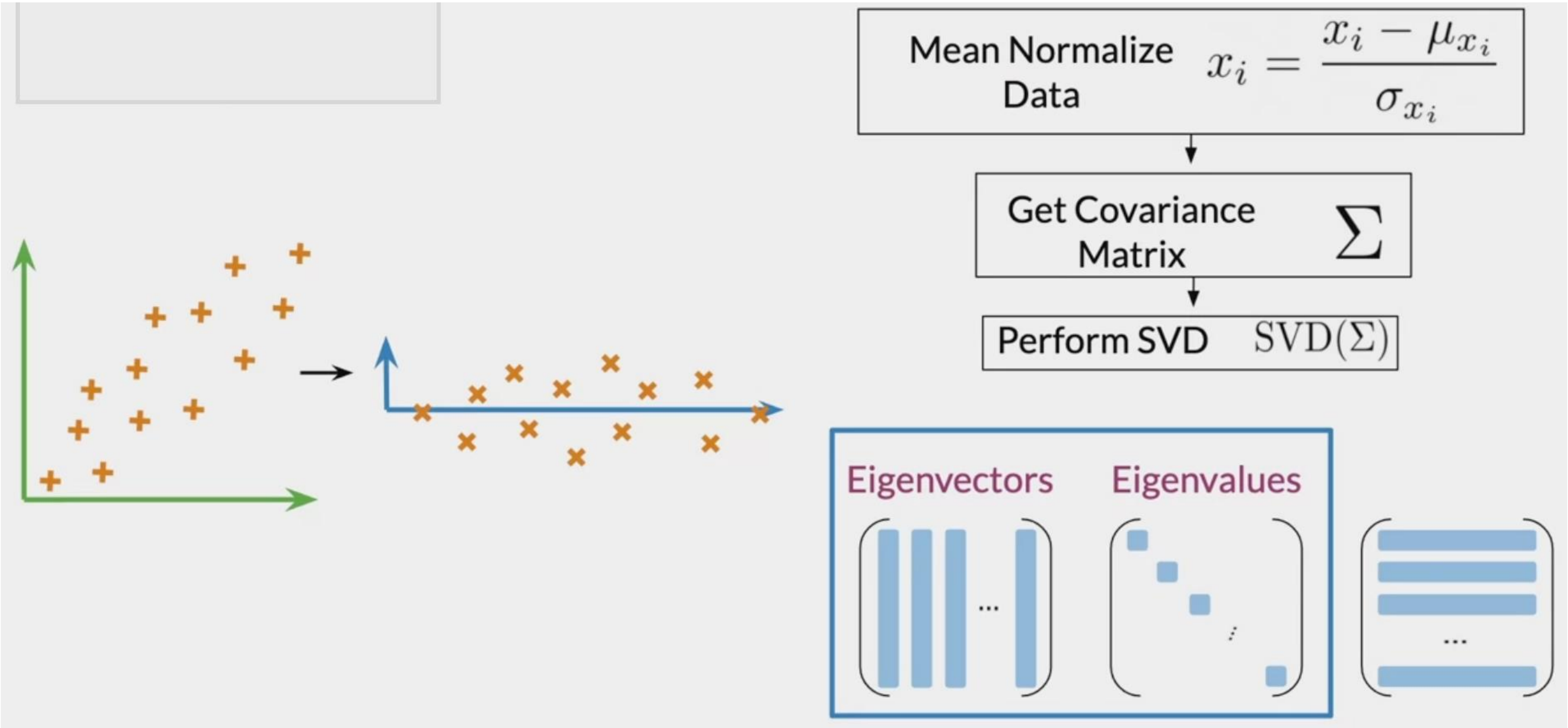


PCA Algorithm

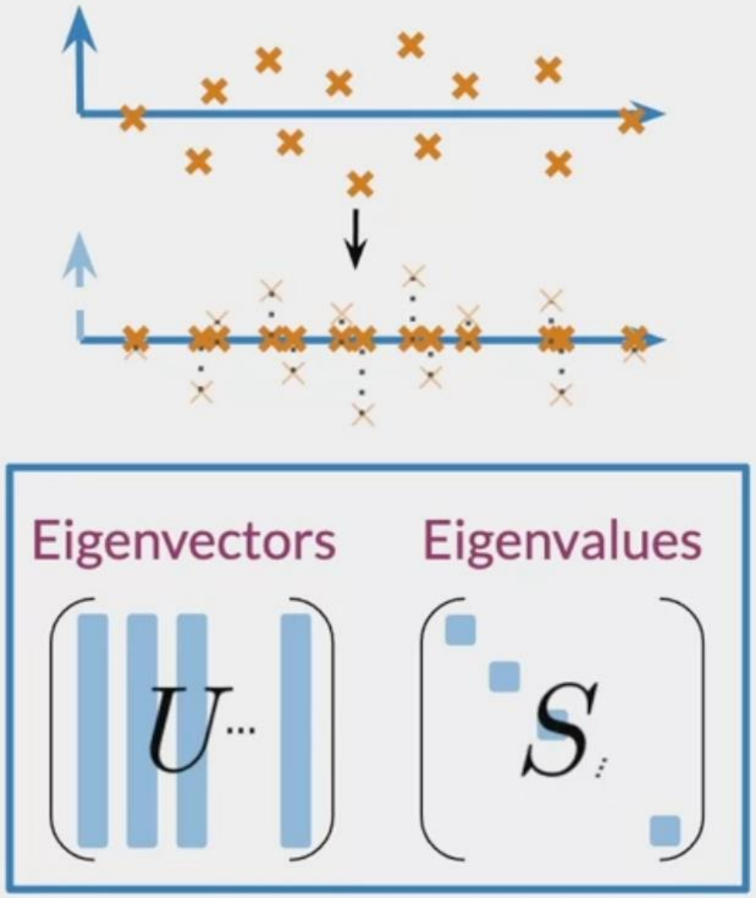
Eigenvector: Uncorrelated features for your data

Eigenvalue: the amount of information retained by each feature

PCA Algorithm



PCA Algorithm



Dot Product to Project Data $X' = XU[:, 0:2]$

Percentage of Retained Variance $\dfrac{\sum_{i=0}^{1} S_{ii}}{\sum_{j=0}^{d} S_{jj}}$