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CSC 595

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Spring 2025

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Gradient Descent for Logistic Regression

Gradient Descent Implementation

Training logistic regression

Find $\vec{w}, b$

Given new $\vec{x}$, output $f_{\vec{w},b}(\vec{x}) = \frac{1}{1+e^{-(\vec{w}\cdot\vec{x}+b)}}$

$$P(y = 1|\vec{x}; \vec{w}, b)$$

Gradient Descent Implementation

Gradient descent

cost

$$J(\vec{w}, b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) + (1 - y^{(i)})\log\left(1 - f_{\vec{w},b}(\vec{x}^{(i)})\right)\right]$$

repeat {

$$w_j = w_j - \alpha\frac{\partial}{\partial w_j}J(\vec{w}, b)$$

$$b = b - \alpha\frac{\partial}{\partial b}J(\vec{w}, b)$$

}

Gradient Descent Implementation

Gradient descent

cost

$$J(\vec{\mathrm{w}}, b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right]$$

repeat {

$j = 1 \ldots n$

$$w_j = w_j - \alpha \overbrace{\frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b)}$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{\mathrm{w}}, b)$$

}

Gradient Descent Implementation

Gradient descent

cost

$$J(\vec{w},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{w},b}(\vec{x}^{(i)})\right)\right]$$

repeat {

$j=1\ldots n$

$$w_j = w_j - \alpha\frac{\partial}{\partial w_j}J(\vec{w},b) \qquad \frac{\partial}{\partial w_j}J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)x_j^{(i)}$$

$$b = b - \alpha\frac{\partial}{\partial b}J(\vec{w},b)$$

}

Gradient Descent Implementation

Gradient descent

cost

$$J(\vec{w}, b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) + (1 - y^{(i)})\log\left(1 - f_{\vec{w},b}(\vec{x}^{(i)})\right)\right]$$

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$j = 1 \ldots n$

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{w}, b) \qquad \frac{\partial}{\partial w_j} J(\vec{w}, b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)x_j^{(i)}$$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{w}, b) \qquad \frac{\partial}{\partial b} J(\vec{w}, b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)$$

} simultaneous updates

Gradient Descent Implementation

Gradient descent for logistic regression

repeat {

looks like linear regression!

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) x_j^{(i)} \right]$$

$$b = b - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) \right]$$

} simultaneous updates

Gradient Descent Implementation

Gradient descent for logistic regression

repeat {

looks like linear regression!

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) x_j^{(i)} \right]$$

$$b = b - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) \right]$$

} simultaneous updates

Linear regression $\quad f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

Logistic regression $\quad f_{\vec{w},b}(\vec{x}) = \dfrac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$

Gradient Descent Implementation

Gradient descent for logistic regression

repeat {

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$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) x_j^{(i)} \right]$$

$$b = b - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) \right]$$

} simultaneous updates

Same concepts:

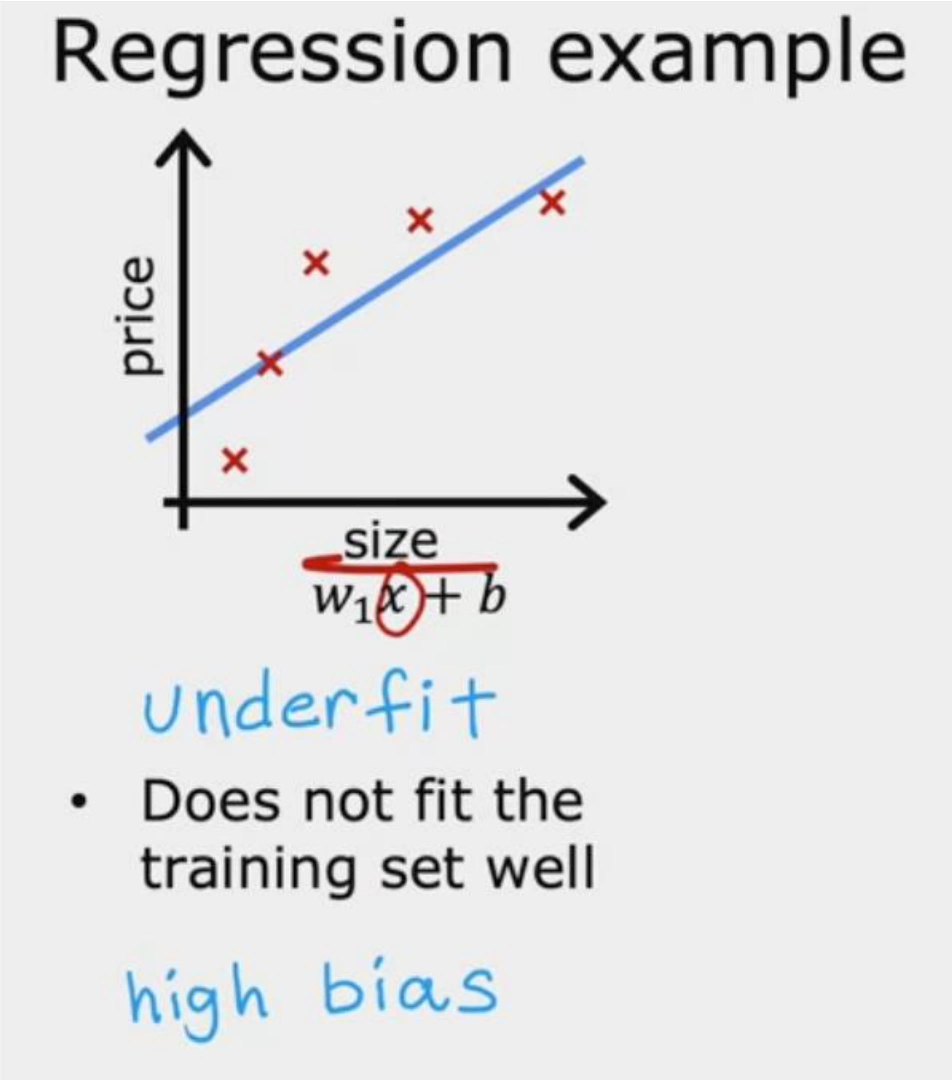
- Monitor gradient descent (learning curve)
- Vectorized implementation
- Feature scaling

Linear regression $f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

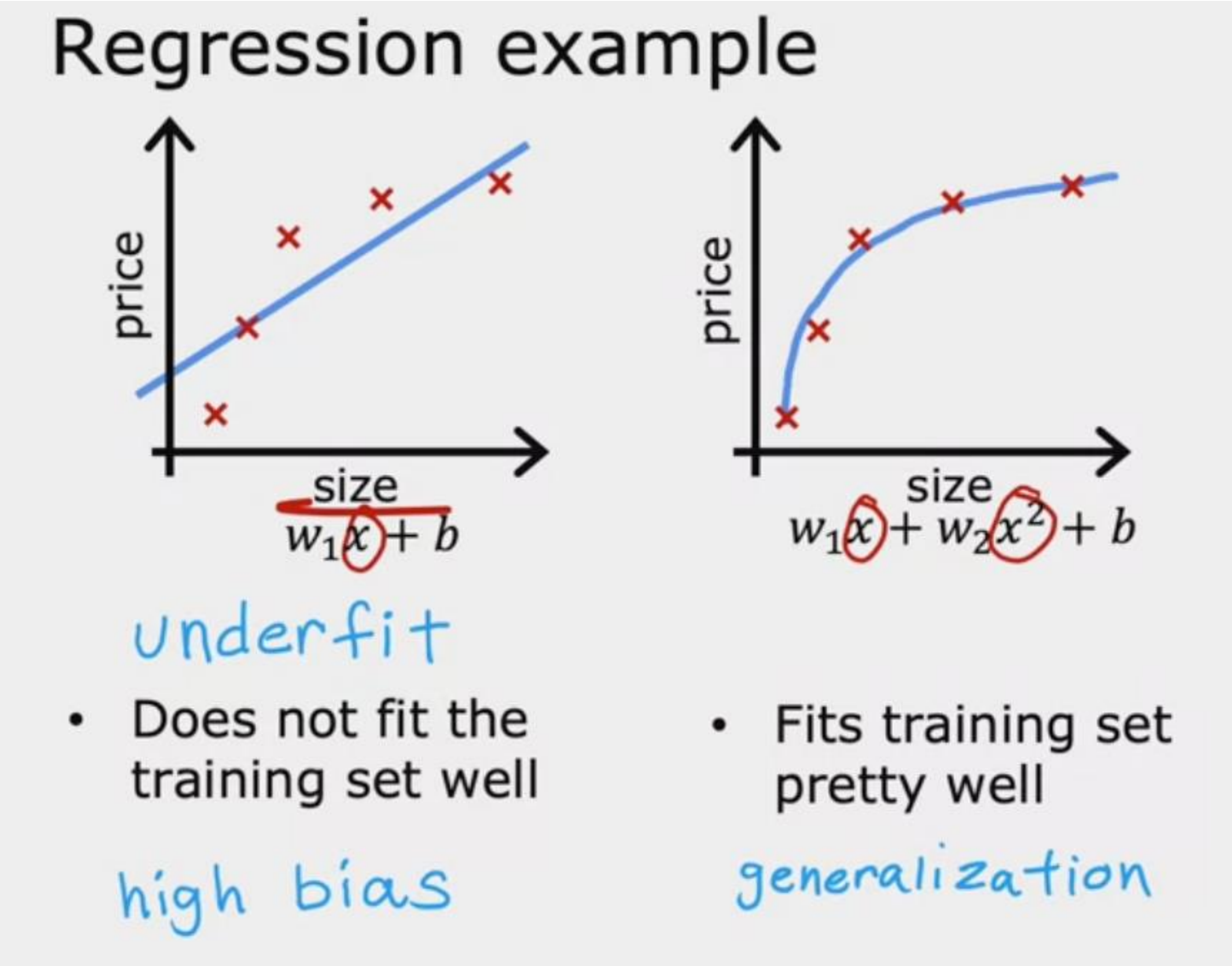
Logistic regression $f_{\vec{w},b}(\vec{x}) = \dfrac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$

The Problem of Overfitting

The Problem of Overfitting



The Problem of Overfitting



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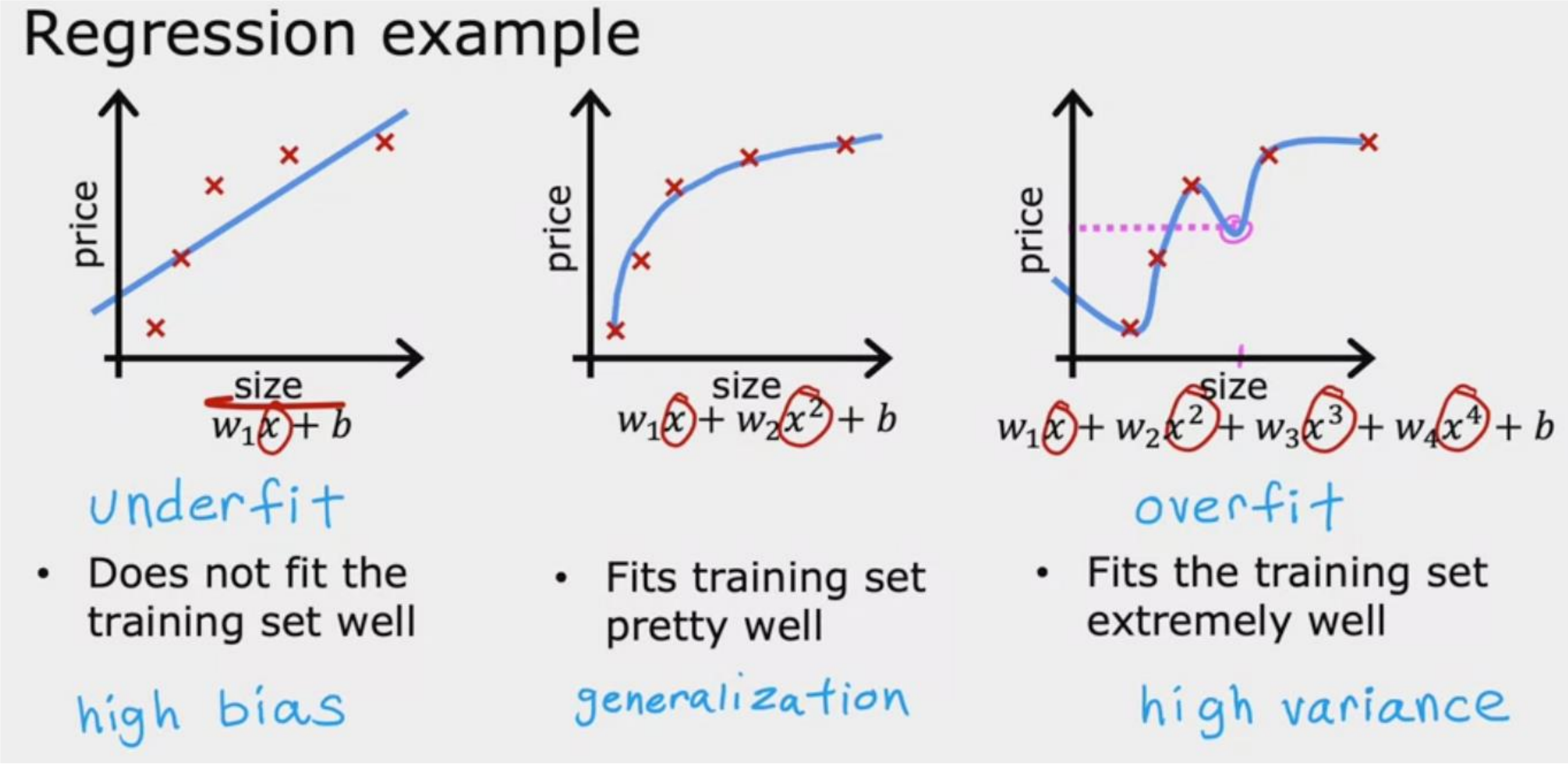
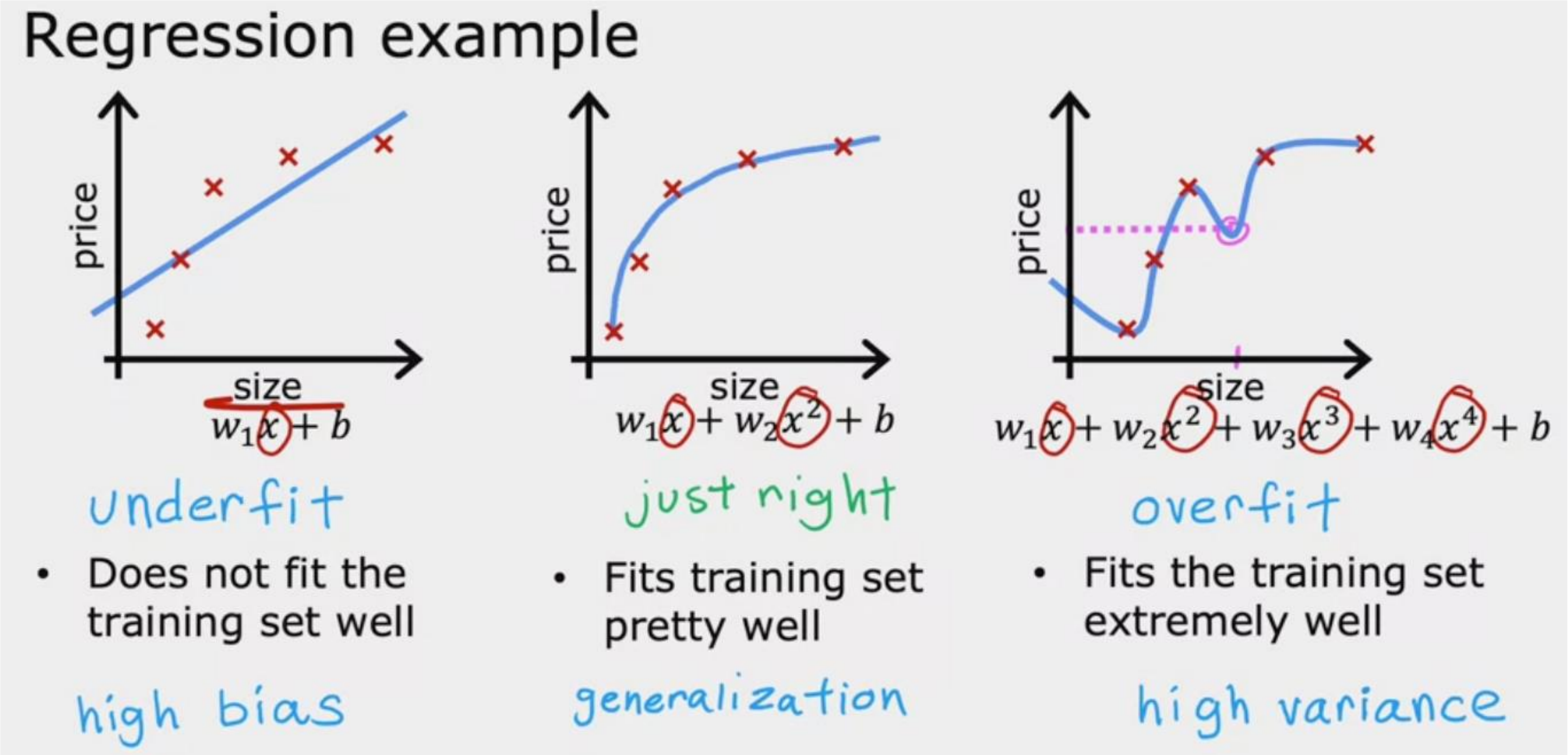
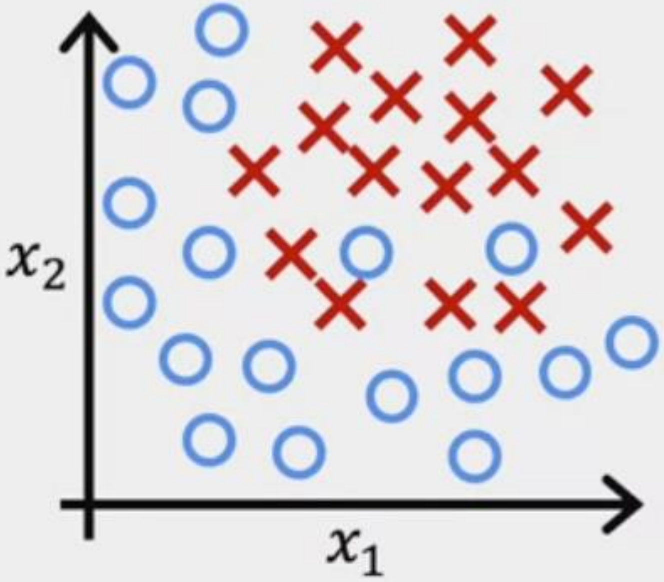


The Problem of Overfitting



The Problem of Overfitting

Classification



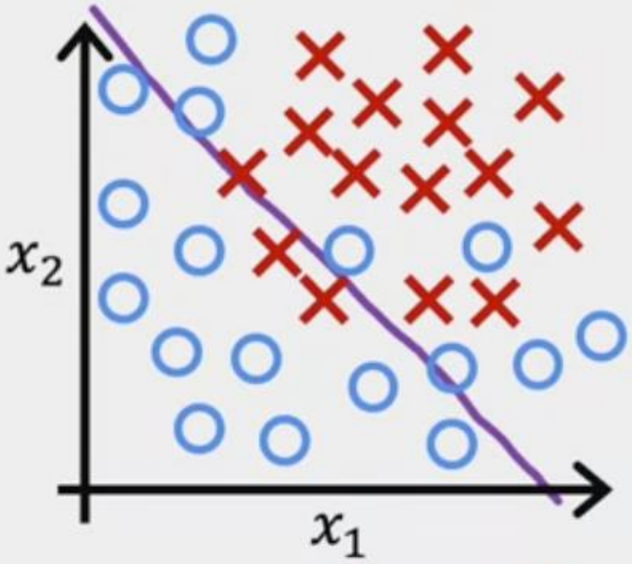
$$z = w_1 x_1 + w_2 x_2 + b$$

$$f_{\vec{w},b}(\vec{x}) = g(z)$$

g is the sigmoid function

The Problem of Overfitting

Classification



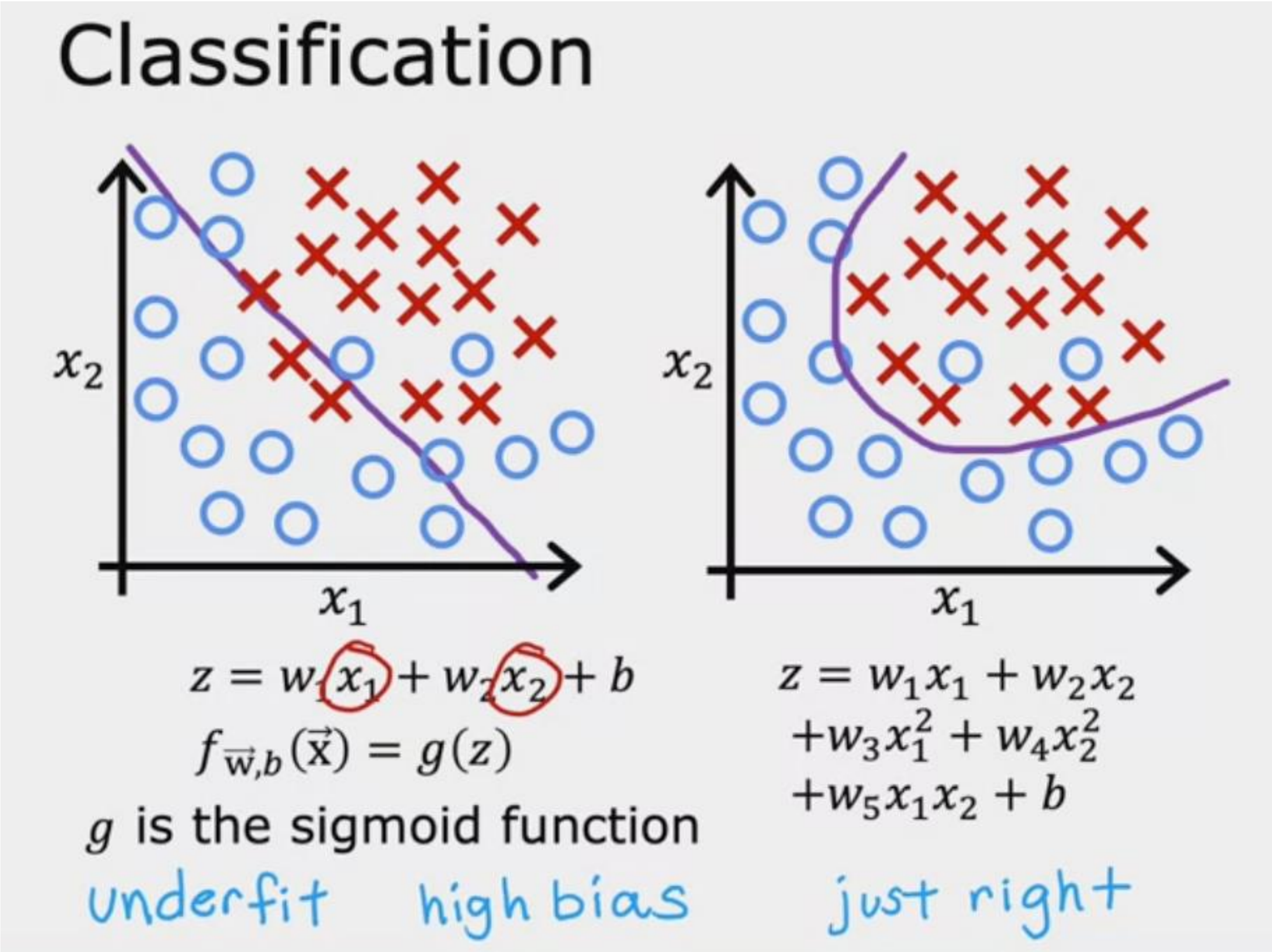
$$z = w_1 x_1 + w_2 x_2 + b$$

$$f_{\vec{w},b}(\vec{x}) = g(z)$$

g is the sigmoid function

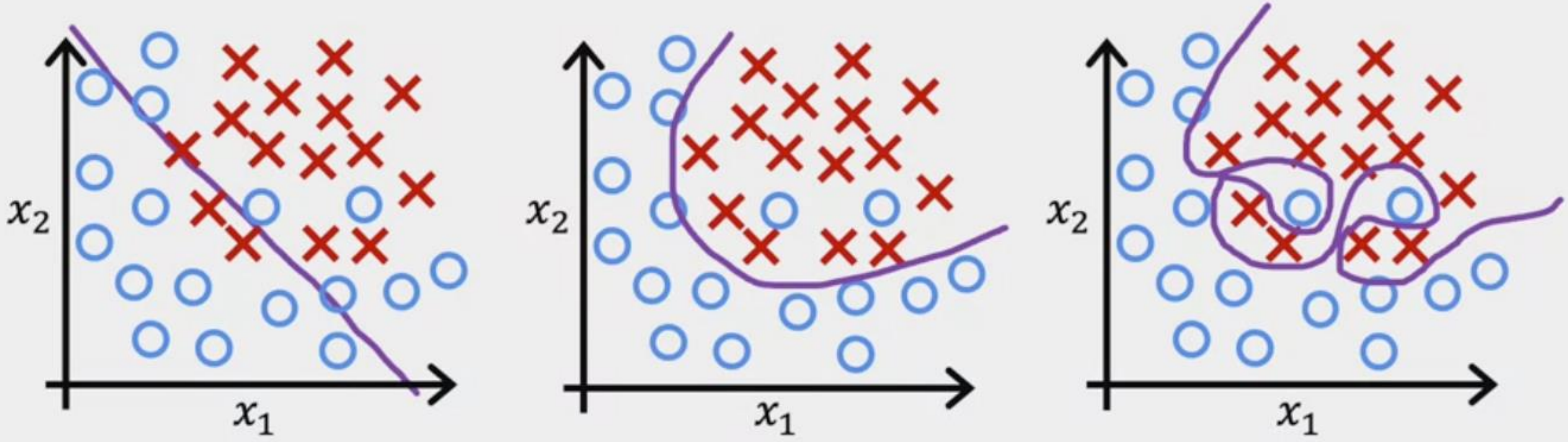
underfit high bias

The Problem of Overfitting



The Problem of Overfitting

Classification



$z = w_1x_1 + w_2x_2 + b$

$f_{\vec{w},b}(\vec{x}) = g(z)$

g is the sigmoid function

underfit high bias

$z = w_1x_1 + w_2x_2$
$+w_3x_1^2 + w_4x_2^2$
$+w_5x_1x_2 + b$

just right

$z = w_1x_1 + w_2x_2$
$+w_3x_1^2x_2 + w_4x_1^2x_2^2$
$+w_5x_1^2x_2^3 + w_6x_1^3x_2$
$+ \cdots + b$

Evaluate Yourself!

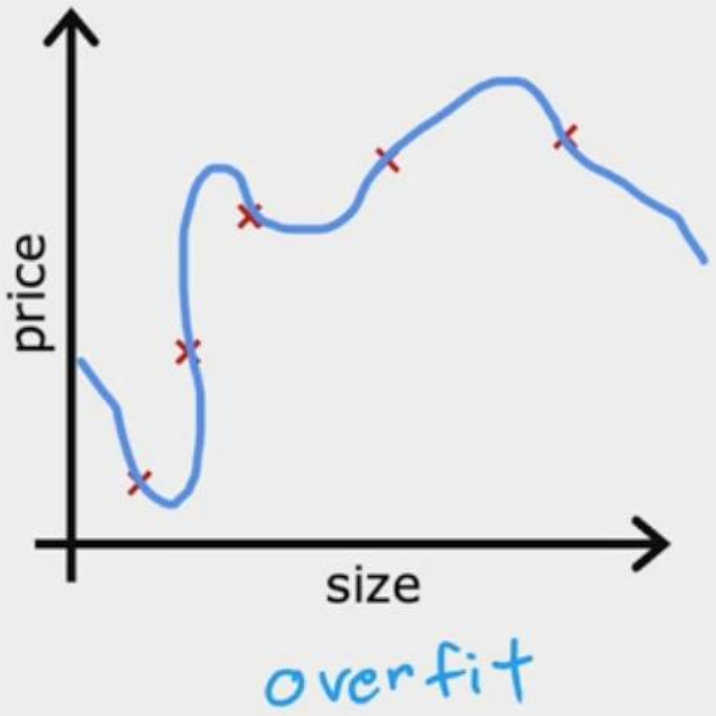
Our goal when creating a model is to be able to use the model to predict outcomes correctly for **new examples**. A model which does this is said to **generalize** well.

When a model fits the training data well but does not work well with new examples that are not in the training set, this is an example of:

- ○ Underfitting (high bias)
- ○ None of the above
- ○ Overfitting (high variance)
- ○ A model that generalizes well (neither high variance nor high bias)

The Problem of Overfitting

Collect more training examples



The Problem of Overfitting

Select features to include/exclude

size	bedrooms	floors	age	avg income	...	distance to coffee shop	price
x_1	x_2	x_3	x_4	x_5		x_{100}	y

all features

+

insufficient data

↓

overfit

selected features

size
bedrooms
age
just right
feature selection

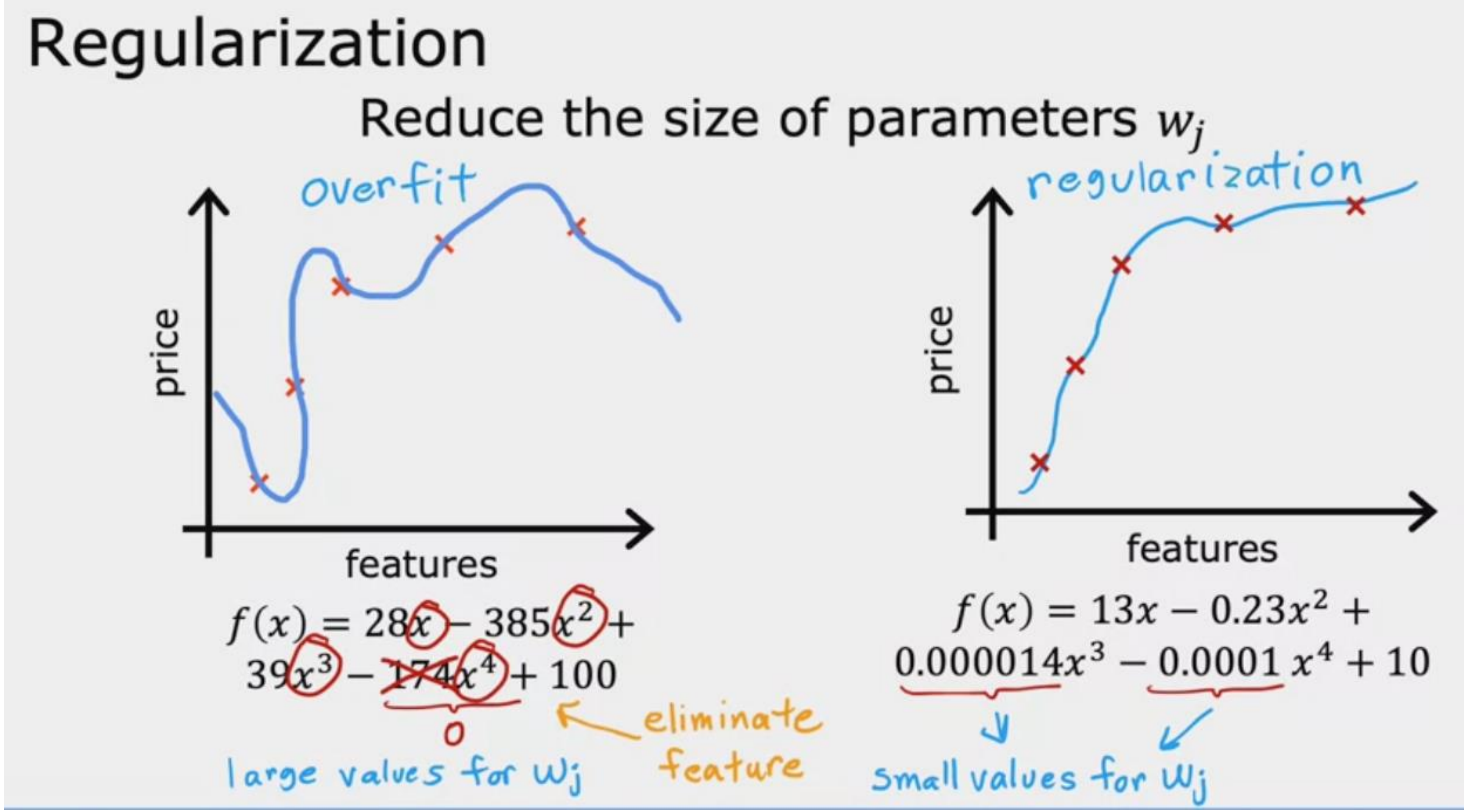
course 2

disadvantage

↓

useful features could be lost

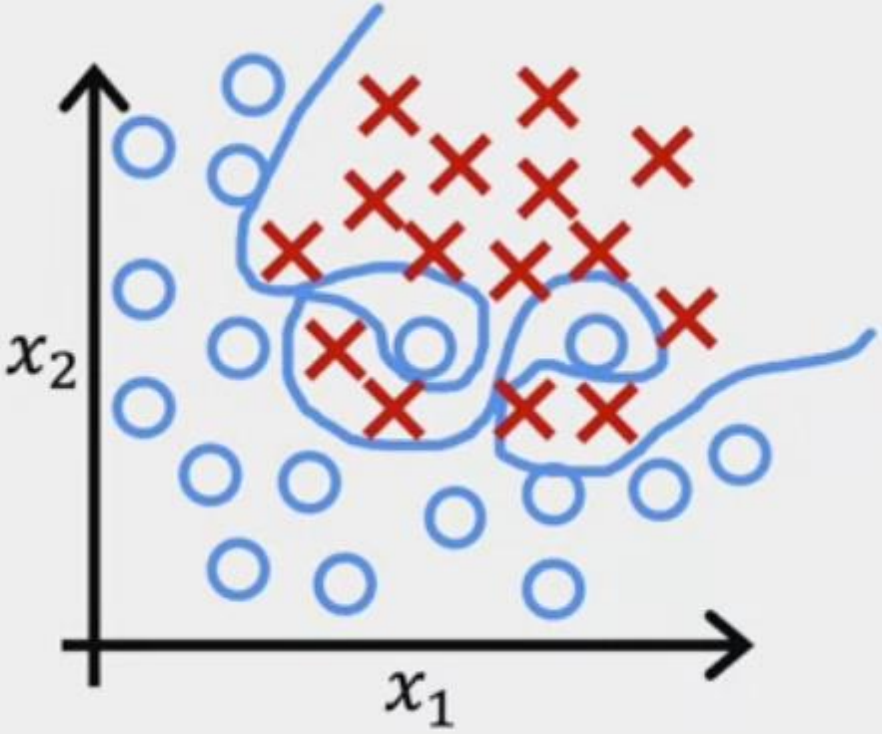
The Problem of Overfitting



Regularized Logistic regression

Regularized Logistic Regression

Regularized logistic regression

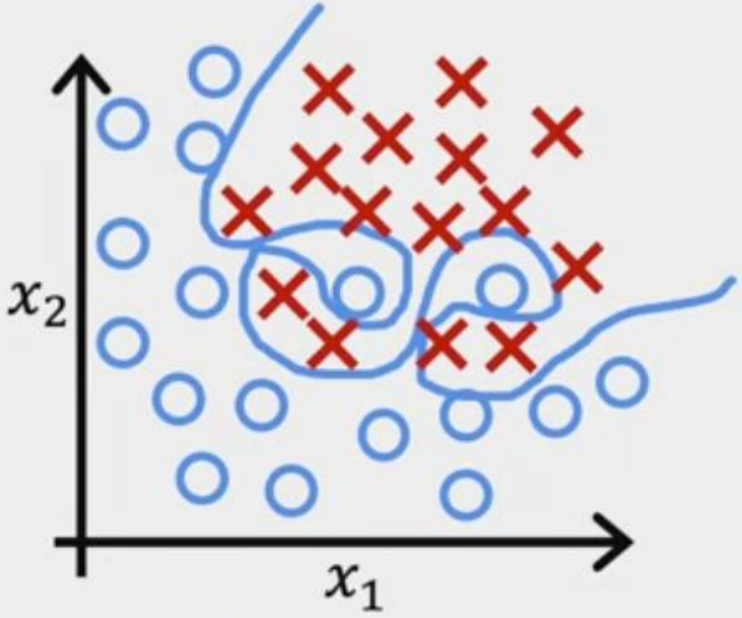


$$z = w_1x_1 + w_2x_2 + w_3x_1^2x_2 + w_4x_1^2x_2^2 + w_5x_1^2x_2^3 + \cdots + b$$

$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1+e^{-z}}$$

Regularized Logistic Regression

Regularized logistic regression



$$z = w_1 x_1 + w_2 x_2 + w_3 x_1^2 x_2 + w_4 x_1^2 x_2^2 + w_5 x_1^2 x_2^3 + \cdots + b$$

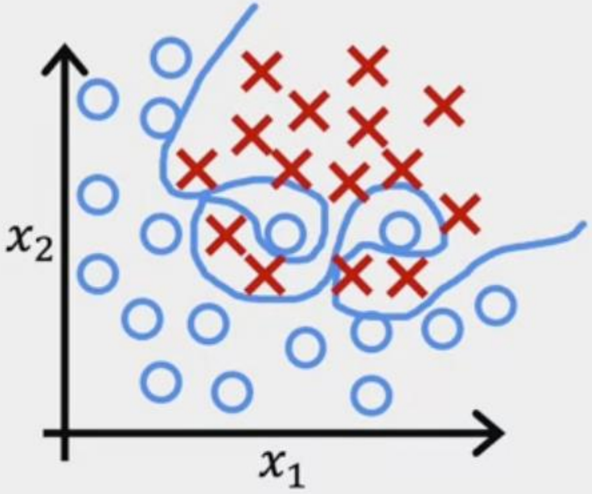
$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1 + e^{-z}}$$

Cost function

$$J(\vec{\mathrm{w}}, b) = -\frac{1}{m} \sum_{i=1}^{m} \left[y^{(i)} \log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) \right) + (1 - y^{(i)}) \log\left(1 - f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) \right) \right]$$

Regularized Logistic Regression

Regularized logistic regression



$$z = w_1x_1 + w_2x_2 + w_3x_1^2x_2 + w_4x_1^2x_2^2 + w_5x_1^2x_2^3 + \cdots + b$$

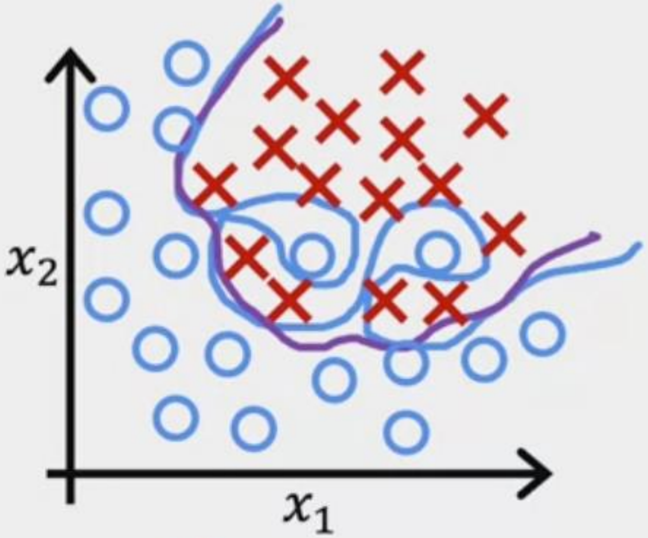
$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1+e^{-z}}$$

Cost function

$$J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

Regularized Logistic Regression

Regularized logistic regression



$$z = w_1x_1 + w_2x_2 + w_3x_1^2x_2 + w_4x_1^2x_2^2 + w_5x_1^2x_2^3 + \cdots + b$$

$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1+e^{-z}}$$

Cost function

$$\underbrace{J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2}_{\min_{\vec{w},b} J(\vec{\mathrm{w}},b) \;\rightarrow\; w_j \downarrow}$$

Regularized Logistic Regression

Regularized logistic regression

$$J(\vec{\mathrm{w}}, b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1 - y^{(i)})\log\left(1 - f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$\min_{\vec{w},b}$

Gradient descent

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b)$$

$j = 1 \ldots n$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{\mathrm{w}}, b)$$

}

Regularized Logistic Regression

Regularized logistic regression

$$J(\vec{\mathrm{w}}, b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1 - y^{(i)})\log\left(1 - f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$\min\limits_{\vec{w},b}$

Gradient descent

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{\mathrm{w}}, b) \qquad = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right) x_j^{(i)}$$

$j = 1 \ldots n$

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{\mathrm{w}}, b) \qquad = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right)$$

}

Regularized Logistic Regression

Regularized logistic regression

$$J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$\min_{\vec{w},b}$

Gradient descent

repeat {

$$w_j = w_j - \alpha\frac{\partial}{\partial w_j}J(\vec{\mathrm{w}},b)$$

$j = 1 \ldots n$

$$b = b - \alpha\frac{\partial}{\partial b}J(\vec{\mathrm{w}},b)$$

}

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right)$$

Regularized Logistic Regression

Regularized logistic regression

$$J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$\min_{\vec{\mathrm{w}},b}$

Gradient descent

Looks same as for linear regression!

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(\vec{\mathrm{w}},b) \quad = \frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

$j = 1 \ldots n$

logistic regression

$$b = b - \alpha \frac{\partial}{\partial b} J(\vec{\mathrm{w}},b) \quad = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)}) - y^{(i)}\right)$$

}

don't have to regularize