Numerical Analysis and Applications—HW1

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◻ Question1

The centered finite-difference approximation of the 1st derivative:

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2).$$

1° Find $f(x_{i+1})$ and $f(x_{i-1})$'s taylor Series.

$$\Rightarrow f(x_{i+1}) = f(x_i) + f'(x_i)h + f''(x_i)h^2/2! + f'''h^3/3! + \cdots$$

$$f(x_{i-1}) = f(x_i) - f'(x_i)h + f''(x_i)h^2/2! - f'''h^3/3! + \cdots$$

2° $f(x_i+1) - f(x_i-1)$

$$= 2f'(x_i)h + \underline{2f'''(x_i)h^3/3! + 2f^{(5)}(x_i)h^5/5! + \cdots}$$

3° $f(x_i+1) - f(x_i-1)\ /2h$

$$= f'(x_i) + \underline{f'''(x_i)h^2/3! + f^{(5)}(x_i)h^4/5! + \cdots}$$

$$\Rightarrow f'(x_i) = f(x_i+1) - f(x_i-1) / 2h + O(h^2)$$

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◻ Question2

a.

In order to find the roots of this function between -2.5 and 2.5 using graphical methods, I plotted the graph and zoomed in to observe the intersection points with the y-axis. I obtained six solutions, approximately -2.061, -1.780, -1.004, -0.3100, 0.2904, and 1.0200.

I believe that this equation there has solutions outside the range of x between -2.5 and 2.5. From the graph, we can see that f(-2.5) > 0. **If there exists some x < -2.5 such that f(x) < 0, then this equation still has other solutions.** We know that the value of the cosine function is between -1 and 1. If x is negative, then the value of 10cos(3pix/2)e^-x is negative. If **abs(10cos(3pi*x/2)*e^-x) is greater than (x^6-1), then f(x) < 0.**

b.

Both the Bisection and False Position methods converged and found the solution to be 0.304. The curves of relative error revealed that the False Position method had a faster convergence rate, taking only six iterations to obtain the solution, while the Bisection method required 15 iterations to obtain the solution.

c.

The error curves for another root in bracket [0.5, 2]

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a.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

b. c. d. HW1_3.py

e.

By plugging in values from 0 to 90 degrees into the simulator function and iterating 500 times each time, we can plot the results. From the relationship between the angle and Sxy, we can roughly estimate that θopt is around 32.0.

f. derivative.py

Plotting the first derivative function and observing the intersection with the y-axis using graphical method, we can estimate that θopt is around 32.2 degrees.

g. golden-section.py

By iterating 25 times using the golden-section method, we obtained θopt to be 32.1689 degrees. We also plotted a graph showing the relationship between the number of iterations and the worst-case error.

SWIPE

h.

By using arccos(e^(-t/T1)), we obtained the optimization of θ to be 32.16896, which is very close to the value obtained by the golden-section method, which is 32.16903. I believe that to improve efficiency, we can use **graphical method to estimate the approximate range of the solution**, and use that to **narrow down the initial value for the golden-section method**, thereby reducing the number of iterations needed.

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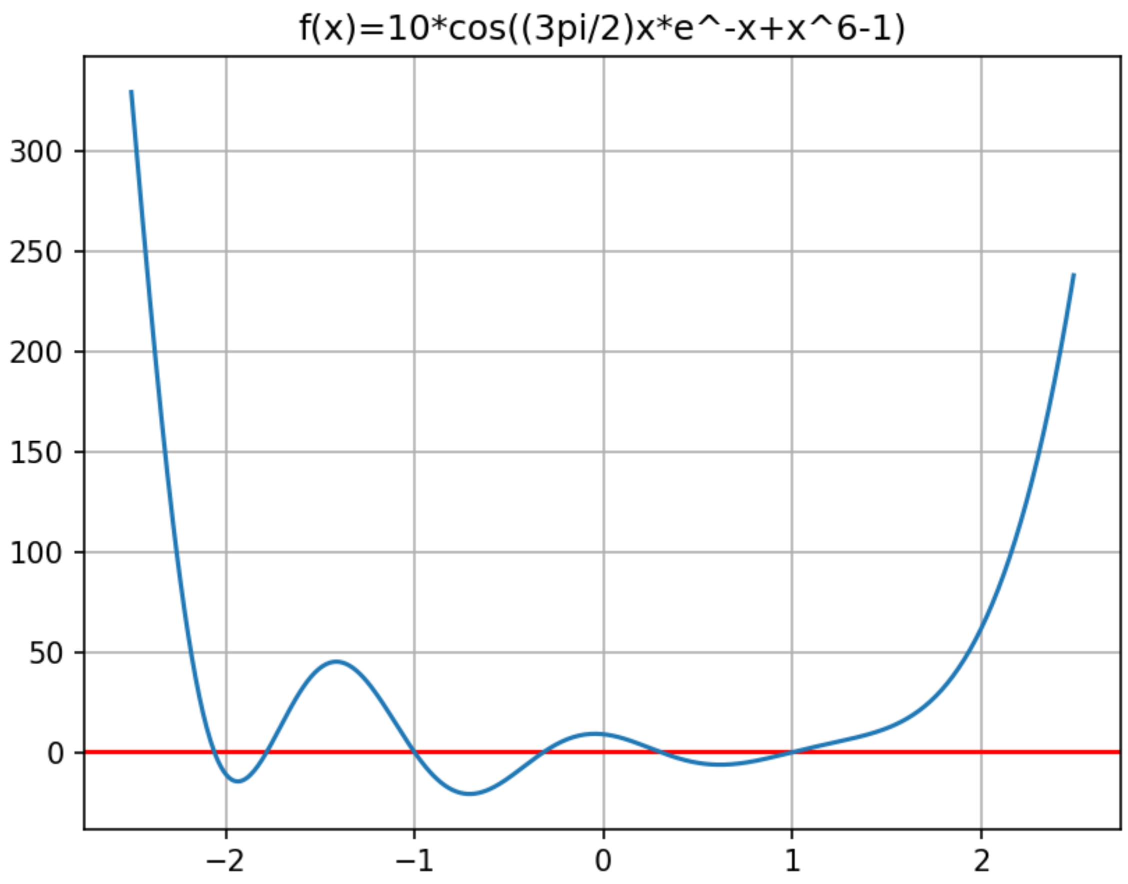
a.



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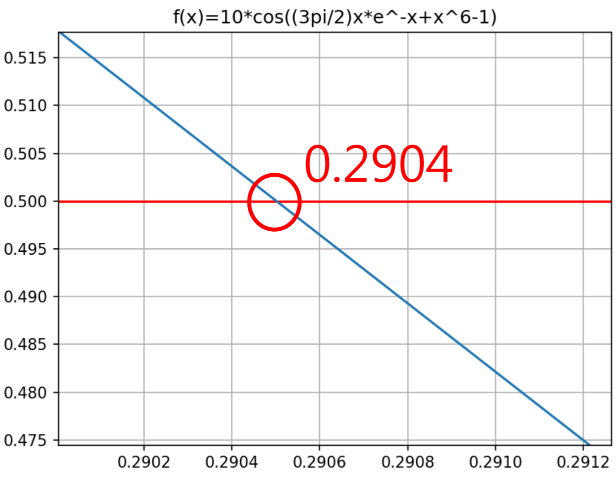
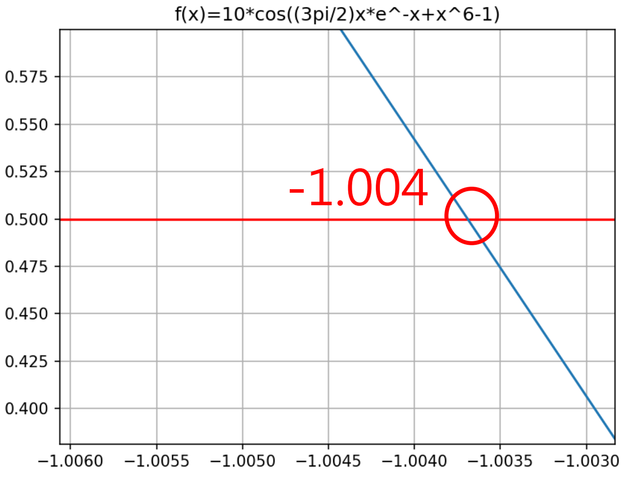
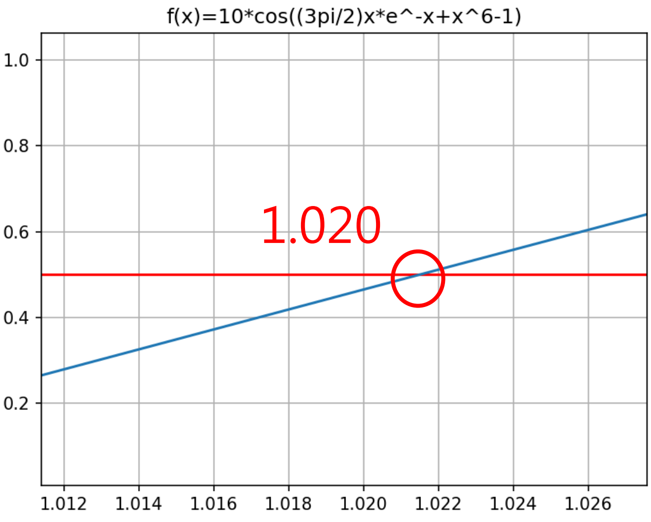
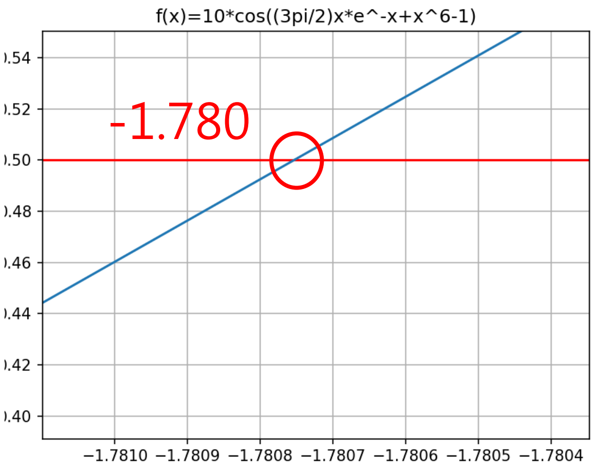
a.

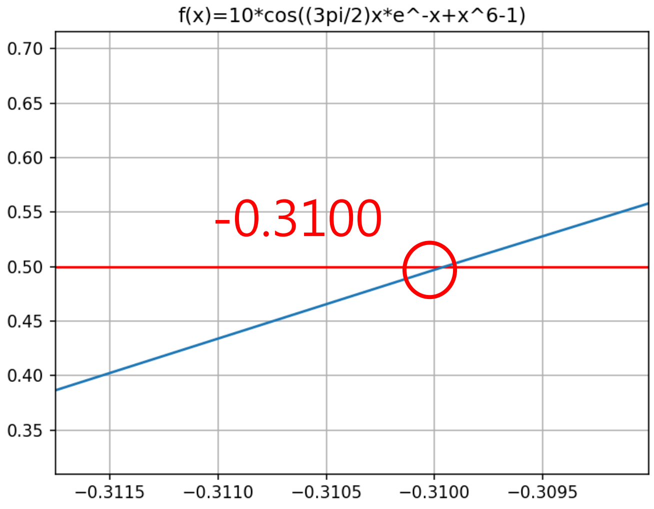
Zoom In

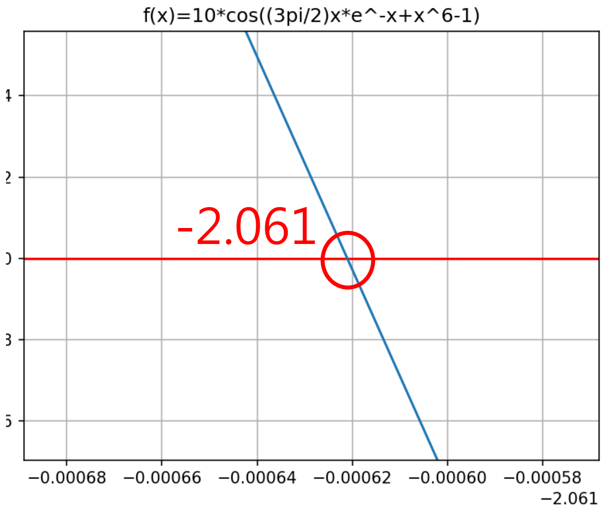












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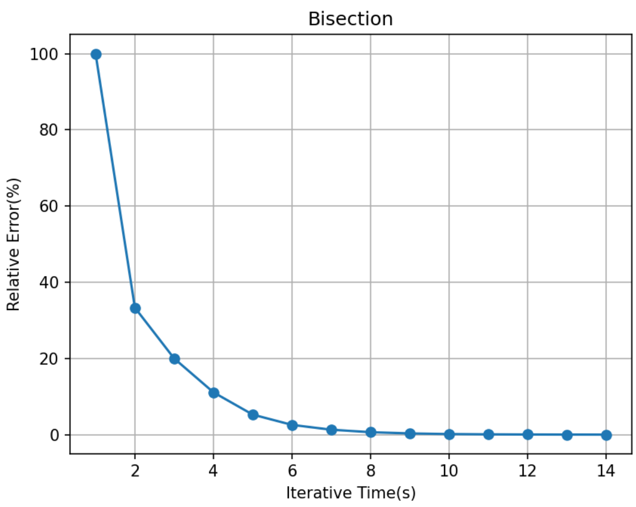
b.

Root: 0.304

Bisection

	Xl	Xu	Xr	Error
1.	0.250000	0.500000	0.250000	100.000000
2.	0.250000	0.375000	0.375000	33.333333
3.	0.250000	0.312500	0.312500	20.000000
4.	0.281250	0.312500	0.281250	11.111111
5.	0.296875	0.312500	0.296875	5.263158
6.	0.296875	0.304688	0.304688	2.564103
7.	0.300781	0.304688	0.300781	1.298701
8.	0.302734	0.304688	0.302734	0.645161
9.	0.303711	0.304688	0.303711	0.321543
10.	0.304199	0.304688	0.304199	0.160514
11.	0.304443	0.304688	0.304443	0.080192
12.	0.304443	0.304565	0.304565	0.040080
13.	0.304443	0.304504	0.304504	0.020044
14.	0.304474	0.304504	0.304474	0.010023
15.	0.304489	0.304504	0.304489	0.005011

0.3044891357421875



Relative Error

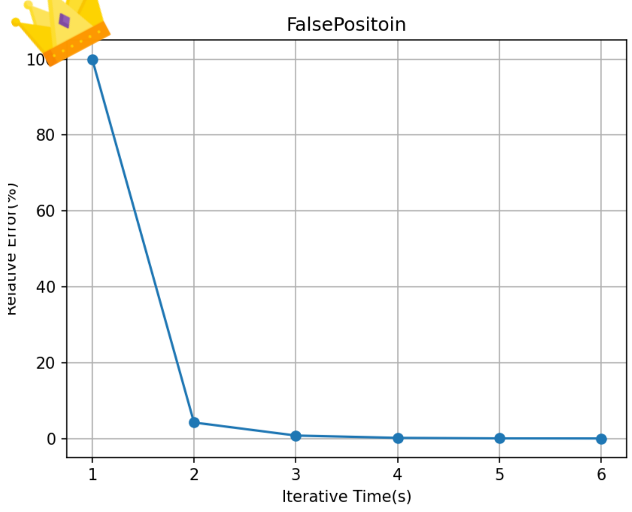
Root: 0.304

False Position

	Xl	Xu	Xr	Error
1.	0.000000	0.315276	0.315276	100.000000
2.	0.000000	0.302538	0.302538	4.210537
3.	0.000000	0.304888	0.304888	0.770666
4.	0.000000	0.304416	0.304416	0.154940
5.	0.000000	0.304509	0.304509	0.030628
6.	0.000000	0.304491	0.304491	0.006075

0.3044906074355459

Less number of iterations!



Relative Error

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c.

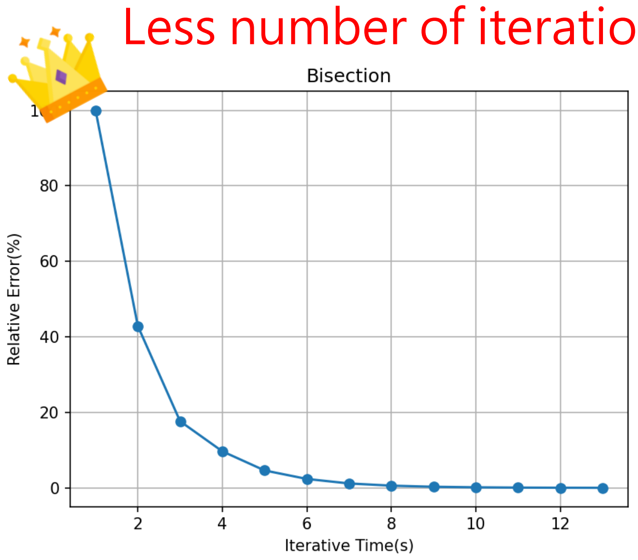
Root:0.9999

Bisection

	Xl	Xu	Xr	Error
1.	0.500000	1.250000	1.250000	100.000000
2.	0.875000	1.250000	0.875000	42.857143
3.	0.875000	1.062500	1.062500	17.647059
4.	0.968750	1.062500	0.968750	9.677419
5.	0.968750	1.015625	1.015625	4.615385
6.	0.992188	1.015625	0.992188	2.362205
7.	0.992188	1.003906	1.003906	1.167315
8.	0.998047	1.003906	0.998047	0.587084
9.	0.998047	1.000977	1.000977	0.292683
10.	0.999512	1.000977	0.999512	0.146556
11.	0.999512	1.000244	1.000244	0.073224
12.	0.999878	1.000244	0.999878	0.036626
13.	0.999878	1.000061	1.000061	0.018309
14.	0.999969	1.000061	0.999969	0.009156

0.999969482421875

Less number of iterations!



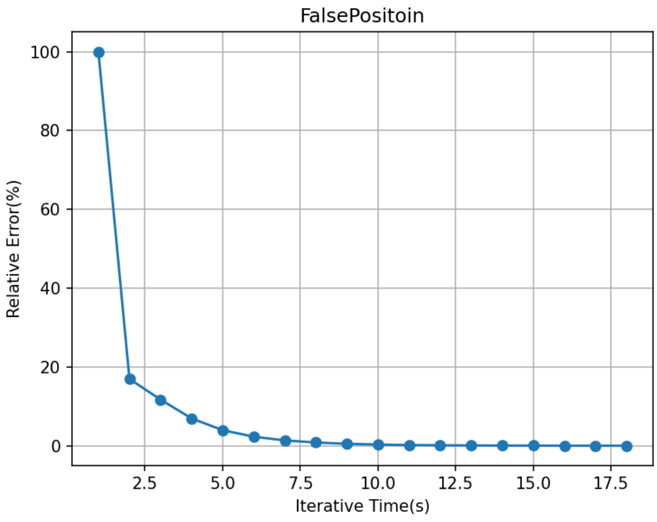
Relative Error

Root:0.9998

False Position

	Xl	Xu	Xr	Error
1.	0.618198	2.000000	0.618198	100.000000
2.	0.744348	2.000000	0.744348	16.947705
3.	0.843161	2.000000	0.843161	11.719313
4.	0.905976	2.000000	0.905976	6.933503
5.	0.943338	2.000000	0.943338	3.960540
6.	0.965531	2.000000	0.965531	2.298554
7.	0.978875	2.000000	0.978875	1.363180
8.	0.986987	2.000000	0.986987	0.821922
9.	0.991958	2.000000	0.991958	0.501105
10.	0.995019	2.000000	0.995019	0.307698
11.	0.996911	2.000000	0.996911	0.189792
12.	0.998083	2.000000	0.998083	0.117398
13.	0.998810	2.000000	0.998810	0.072746
14.	0.999261	2.000000	0.999261	0.045127
15.	0.999541	2.000000	0.999541	0.028013
16.	0.999715	2.000000	0.999715	0.017396
17.	0.999823	2.000000	0.999823	0.010806
18.	0.999890	2.000000	0.999890	0.006714

0.9998898116773632



Relative Error

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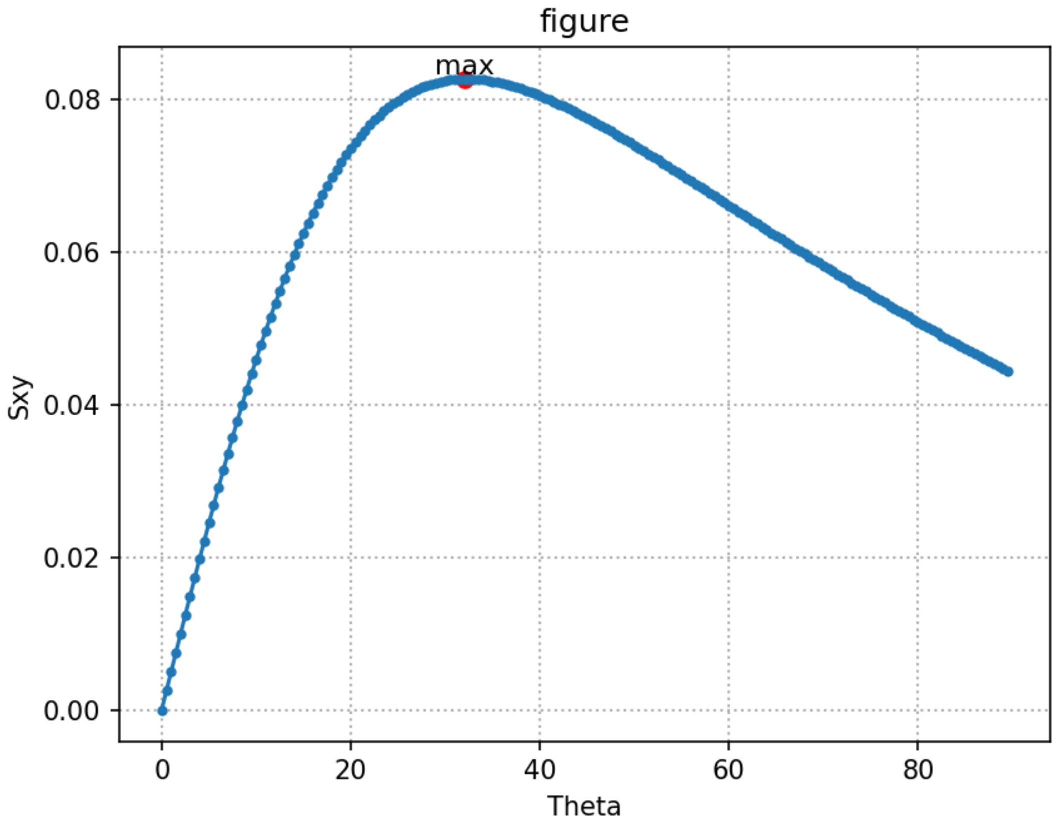
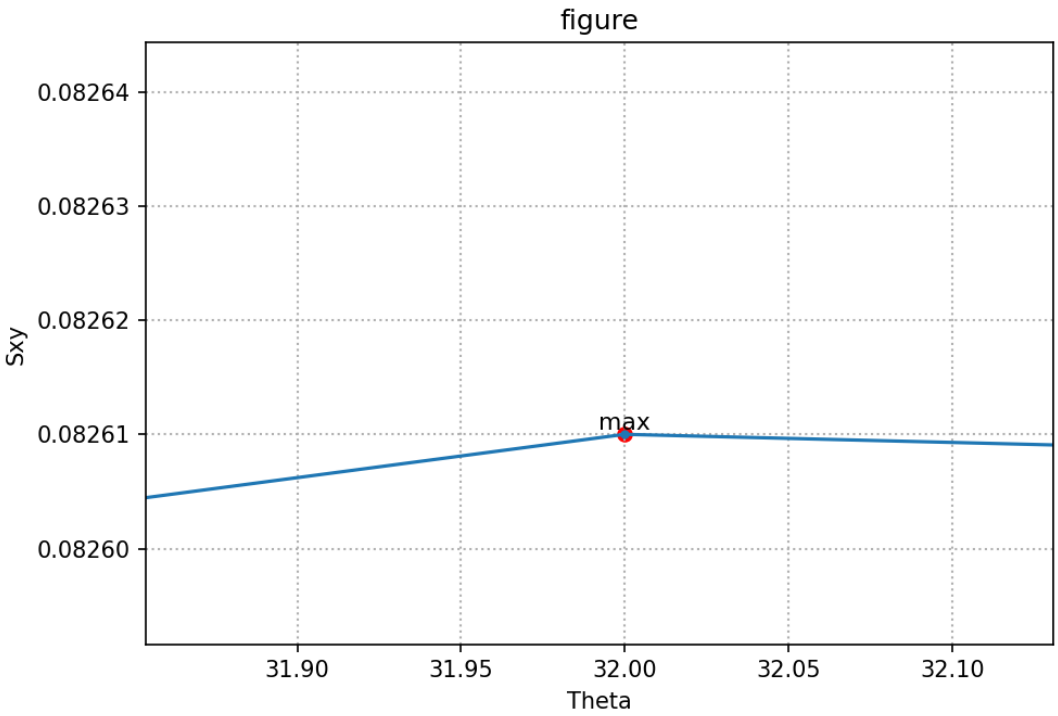
e.



Zoom In



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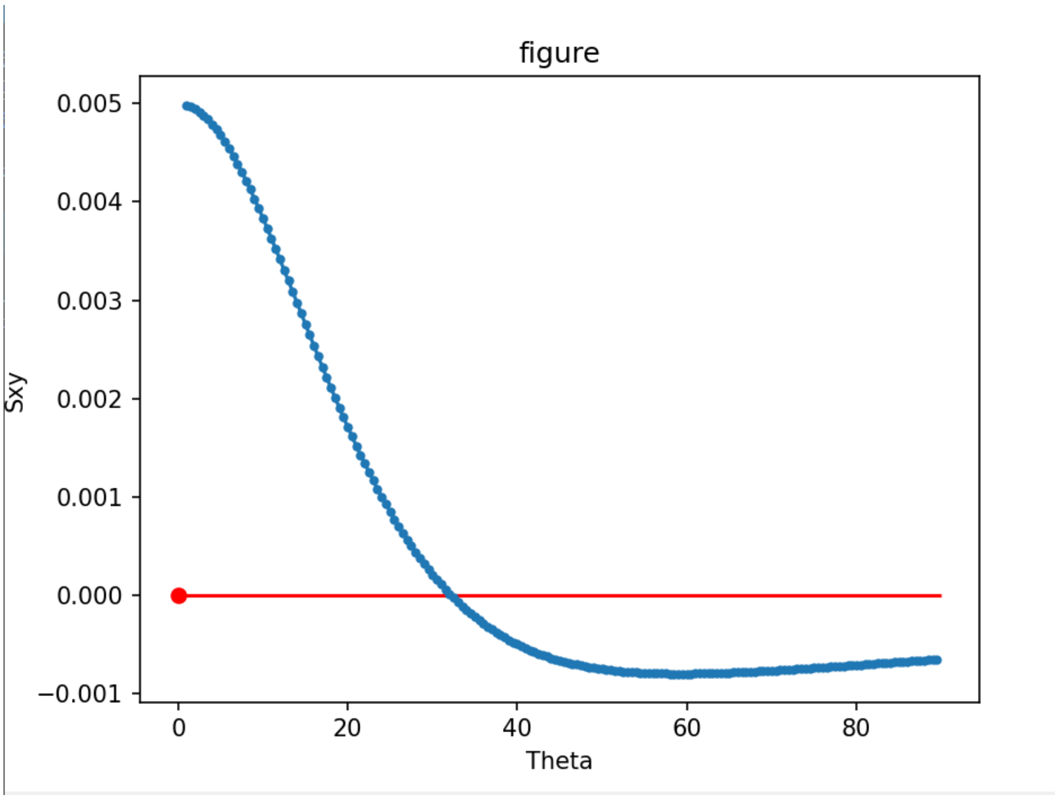
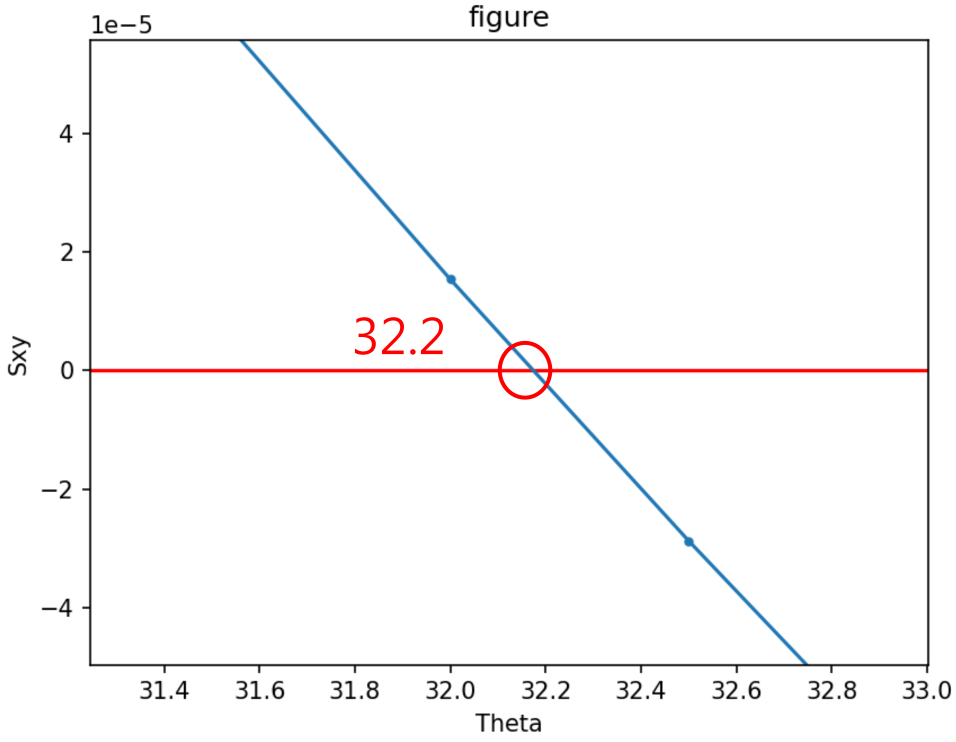
f.



Zoom In



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g.

```
Iter    Xl    Xu    Error
 1,0.000000,55.623059,61.803399
 2,21.246118,55.623059,38.196601
 3,21.246118,42.492236,23.606798
 4,29.361413,42.492236,14.589803
 5,29.361413,37.476708,9.016994
 6,29.361413,34.376941,5.901699
 7,31.277174,34.376941,3.647451
 8,31.277174,33.192935,2.254249
 9,31.277174,32.461180,1.412887
10,31.729424,32.461180,0.873212
11,32.008930,32.461180,0.536778
12,32.008930,32.288436,0.331747
13,32.115692,32.288436,0.205031
14,32.115692,32.222453,0.126716
15,32.115692,32.181674,0.078376
16,32.140895,32.181674,0.048439
17,32.156471,32.181674,0.029928
18,32.156471,32.172047,0.018497
19,32.162421,32.172047,0.011432
20,32.166098,32.172047,0.007065
21,32.166098,32.169775,0.004366
22,32.167502,32.169775,0.002698
23,32.168370,32.169775,0.001668
24,32.168370,32.169238,0.001031
25,32.168702,32.169238,0.000637
32.168969979171436
```

θopt $\boxed{32.168969979171436}$

Relationship graph