Introduction
to
Differentiation

In an equation such as $\mathbf{y = 3x^2 + 2x - 5}$

y is said to be a function of x and may be written as $y = f(x)$.

An equation written in the form $f(x) = 3x^2 + 2x - 5$,

So, the value of $f(x)$ when $x = 0$ is denoted by $f(0)$,

and the value of $f(x)$ when $x = 2$ is denoted by $f(2)$, and so on. Thus, when

$$\mathbf{f(x) = 3x^2 + 2x - 5,}$$

$$\mathbf{f(0) = 3(0)^2 + 2(0) - 5 = -5}$$

$$\textbf{and } \mathbf{f(2) = 3(2)^2 + 2(2) - 5 = 11}$$

Problem 1. If $f(x) = 4x^2 - 3x + 2$,

Find $f(0)$, $f(3)$, $f(-1)$ and $f(3) - f(-1)$

$$f(x) = 4x^2 - 3x + 2$$

$$f(0) = 4(0)^2 - 3(0) + 2 = 2$$

$$f(3) = 4(3)^2 - 3(3) + 2 = 36 - 9 + 2 = 29$$

$$f(-1) = 4(-1)^2 - 3(-1) + 2 = 4 + 3 + 2 = 9$$

$$f(3) - f(-1) = 29 - 9 = 20$$

Problem 2. $f(x) = 5x^2 + x - 7$

determine $f(-2)$ and $f(2) \div f(1)$

$$f(-2) = 5(-2)^2 + (-2) - 7 = 20 - 2 - 7 = 11$$

$$f(2) = 5(2)^2 + 2 - 7 = 15$$

$$f(1) = 5(1)^2 + 1 - 7 = -1$$

$$f(2) \div f(1) = 15 / -1 = -15$$

Differentiation of $y = a x^n$ by the general rule

From differentiation by first principles, a general rule for differentiating **$a x^n$** emerges where *a* and *n* are any constants. This rule is

if $y = a x^n$

then $dy / dx = a n x^{n-1}$

or

if $f(x) = a x^n$

Then $f'(x) = a n x^{n-1}$

For example

(a) if $y = 3x^2$ then $dy / dx = 6x$

Revision of some laws of indices

$\mathbf{\dfrac{1}{x^a} = x^{-a}}$ For example, $\dfrac{1}{x^2} = x^{-2}$ and $x^{-5} = \dfrac{1}{x^5}$

$\mathbf{\sqrt{x} = x^{\frac{1}{2}}}$ For example, $\sqrt{5} = 5^{\frac{1}{2}}$ and

$16^{\frac{1}{2}} = \sqrt{16} = \pm 4$ and $\dfrac{1}{\sqrt{x}} = \dfrac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$

$\mathbf{\sqrt[a]{x^b} = x^{\frac{b}{a}}}$ For example, $\sqrt[3]{x^5} = x^{\frac{5}{3}}$ and $x^{\frac{4}{3}} = \sqrt[3]{x^4}$

and $\dfrac{1}{\sqrt[3]{x^7}} = \dfrac{1}{x^{\frac{7}{3}}} = x^{-\frac{7}{3}}$

$\mathbf{x^0 = 1}$ For example, $7^0 = 1$ and $43.5^0 = 1$

Problem 3. Differentiate the following with respect to *x*:

$$y = 4\,X^7 \qquad (a\,x^n)$$

$$dy/dx = a\,n\,x^{n-1} = (4)\,(7)\,x^{7-1} = 28\,x^6$$

Problem 4. Differentiate the following with respect to *x*:

$$y = 3\,/\,X^2$$

$y = 3\,/\,x^2 = 3\,x^{-2}$, so $a = 3$ and $n = -2$ in the general rule.

$$dy\,/\,dx = a\,n\,x^{n-1} = (3)\,(-2)\,x^{-2-1} = -6\,x^{-3} = -6\,x^{-3}$$

Problem 4. Differentiate the following with respect to x:

$$y = 5\sqrt{x}$$

$y = 5\sqrt{x} = 5x^{\frac{1}{2}}$, hence $a = 5$ and $n = \frac{1}{2}$ in the general rule.

$$\frac{\mathbf{dy}}{\mathbf{dx}} = anx^{n-1} = (5)\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$$

$$= \frac{5}{2}x^{-\frac{1}{2}} = \frac{5}{2x^{\frac{1}{2}}} = \mathbf{\frac{5}{2\sqrt{x}}}$$

Problem 5. Differentiate $\boldsymbol{y = 4}$

$y = 4$ may be written as $y = 4x^0$; in the general rule $a = 4$ and $n = 0$. Hence,

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = (4)(0)x^{0-1} = \mathbf{0}$$

Problem 6. Differentiate $\boldsymbol{y = 7x}$

Since $y = 7x$, i.e. $y = 7x^1$, in the general rule $a = 7$ and $n = 1$. Hence,

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = (7)(1)x^{1-1} = 7x^0 = \mathbf{7} \qquad \text{since } x^0 = 1$$

Problem 7. Differentiate
$$y = \frac{2}{3}x^4 - \frac{4}{x^3} + 9$$

$$y = \frac{2}{3}x^4 - \frac{4}{x^3} + 9$$

$$y = \frac{2}{3}x^4 - 4x^{-3} + 9$$

$$\frac{dy}{dx} = \left(\frac{2}{3}\right)(4)x^{4-1} - (4)(-3)x^{-3-1} + 0$$

$$= \frac{8}{3}x^3 + 12x^{-4}$$

Problem 8. If $f(t) = 4t + \dfrac{1}{\sqrt{t^3}}$ find $f'(t)$

$$f(t) = 4t + \frac{1}{\sqrt{t^3}} = 4t + \frac{1}{t^{\frac{3}{2}}} = 4t^{1} + t^{-\frac{3}{2}}$$

$$f'(t) = (4)(1)t^{1-1} + \left(-\frac{3}{2}\right)t^{-\frac{3}{2}-1}$$

$$= 4t^{0} - \frac{3}{2}t^{-\frac{5}{2}}$$

$$f'(t) = 4 - \frac{3}{2t^{\frac{5}{2}}} = \mathbf{4 - \frac{3}{2\sqrt{t^5}}}$$

Problem 9. Determine $\frac{dy}{dx}$ given $y = \frac{3x^2 - 5x}{2x}$

$$y = \frac{3x^2 - 5x}{2x} = \frac{3x^2}{2x} - \frac{5x}{2x} = \frac{3}{2}x - \frac{5}{2}$$

C = 0

$$\frac{dy}{dx} = \frac{3}{2} \text{ or } \mathbf{1.5}$$

Problem 10.

$$y = \frac{2}{5}x^3 - \frac{4}{x^3} + 4\sqrt{x^5} + 7$$

$$y = \frac{2}{5}x^3 - 4x^{-3} + 4x^{\frac{5}{2}} + 7$$

$$\frac{dy}{dx} = \left(\frac{2}{5}\right)(3)x^{3-1} - (4)(-3)x^{-3-1} + (4)\left(\frac{5}{2}\right)x^{\frac{5}{2}-1} + 0$$

$$= \frac{6}{5}x^2 + 12x^{-4} + 10x^{\frac{3}{2}}$$

$$\boldsymbol{\frac{dy}{dx} = \frac{6}{5}x^2 + \frac{12}{x^4} + 10\sqrt{x^3}}$$

Problem 11. Differentiate $y = \dfrac{(x+2)^2}{x} = \dfrac{(x+2)(x+2)}{x}$

$$y = \frac{(x+2)^2}{x} = \frac{x^2+4x+4}{x} = \frac{x^2}{x} + \frac{4x}{x} + \frac{4}{x}$$

$$y = x^1 + 4 + 4x^{-1}$$

$$\frac{dy}{dx} = 1x^{1-1} + 0 + (4)(-1)x^{-1-1}$$

$$= x^0 - 4x^{-2} = \mathbf{1 - \frac{4}{x^2}} \quad (\text{since } x^0 = 1)$$

Problem 12. Find the gradient of the curve descried by the following equation:

$$y = 2x^2 - \frac{3}{x} \text{ at } x = 2$$

$$y = 2x^2 - \frac{3}{x} = 2x^2 - 3x^{-1}$$

$$\text{Gradient} = \frac{dy}{dx} = (2)(2)x^{2-1} - (3)(-1)x^{-1-1}$$

$$= 4x + 3x^{-2} = 4x + \frac{3}{x^2}$$

When $x = 2$,

$$\textbf{gradient} = 4x + \frac{3}{x^2} = 4(2) + \frac{3}{(2)^2} = 8 + \frac{3}{4} = \mathbf{8.75}$$

Problem 13. Find the gradient of the curve

$$y = 3x^4 - 2x^2 + 5x - 2 \text{ at the points } (0, -2) \text{ and } (1, 4)$$

$$\textbf{gradient} = \frac{dy}{dx} = 12x^3 - 4x + 5$$

At the point (0, –2), $x = 0$, thus

the **gradient** $= 12(0)^3 - 4(0) + 5 = \mathbf{5}$

At the point (1, 4), $x = 1$, thus

the **gradient** $= 12(1)^3 - 4(1) + 5 = \mathbf{13}$

Thank You