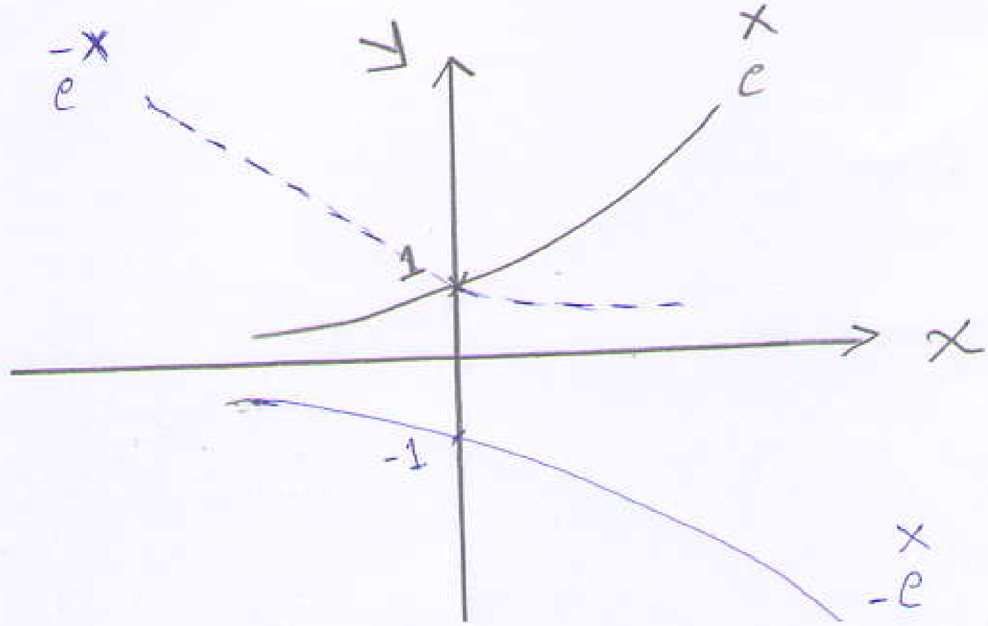
4 محاضرة

Exponential Function Advanced

Ex: sketch $y = e^{x}$

then sketch $y = -e^{x}$

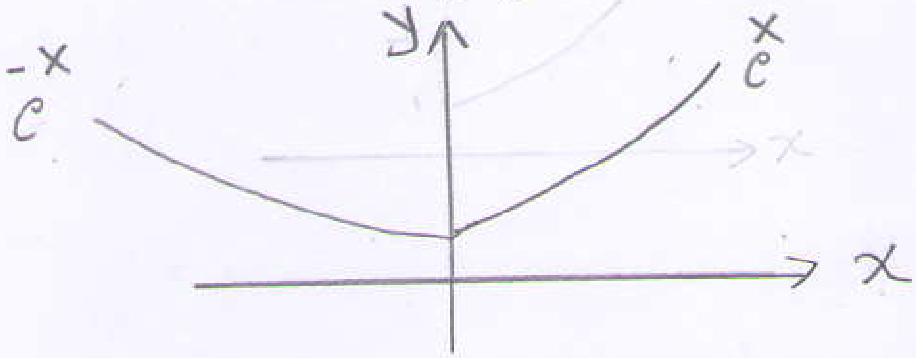


(-ve) لـ ⇐ multiply لو عملت Function ×

سوف يحدث reflection

Ex: sketch $y = e^{|x|}$

$$y = e^{|x|} = \begin{cases} e^{x} & x \geq 0 \\ e^{-x} & x < 0 \end{cases}$$



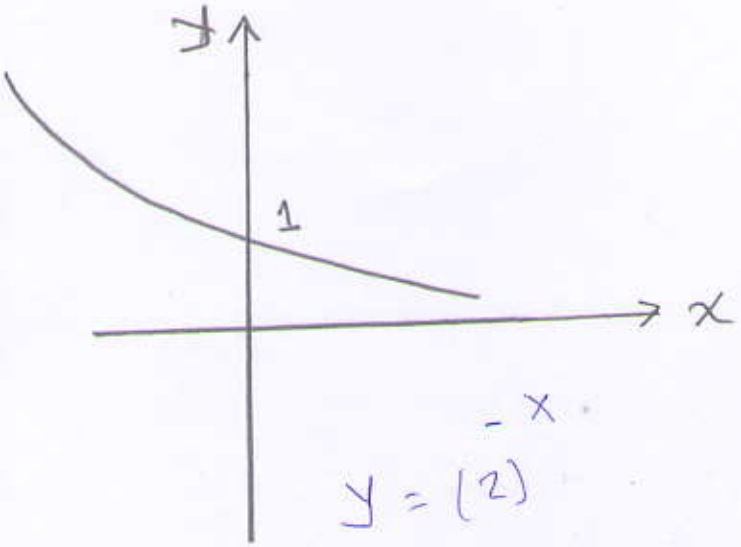
D : IR

r : [1, ∞[

Exo: $y = 3^x$

يجب ان يكون الـ base اكبر من الواحد

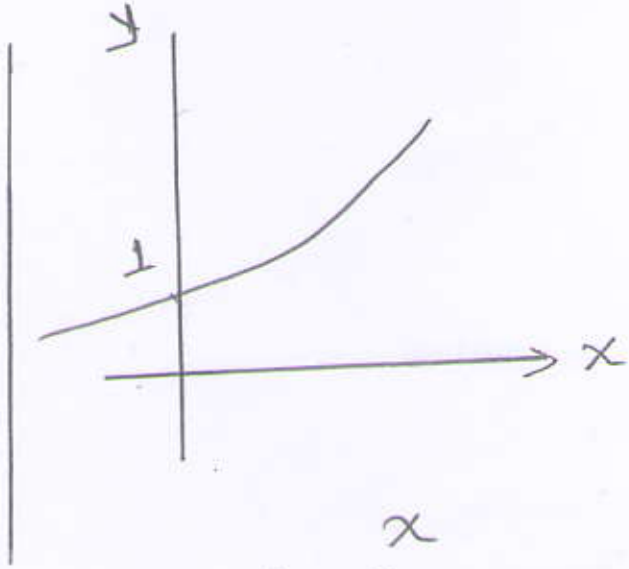
$y = \left(\frac{1}{2}\right)^x = (2)^{-x}$



$y = (2)^{-x}$

$D_f : \mathbb{R}$

$r_f :]0, \infty[$



$y = 3^x$

$D_f : \mathbb{R}$

$r =]0, \infty[$

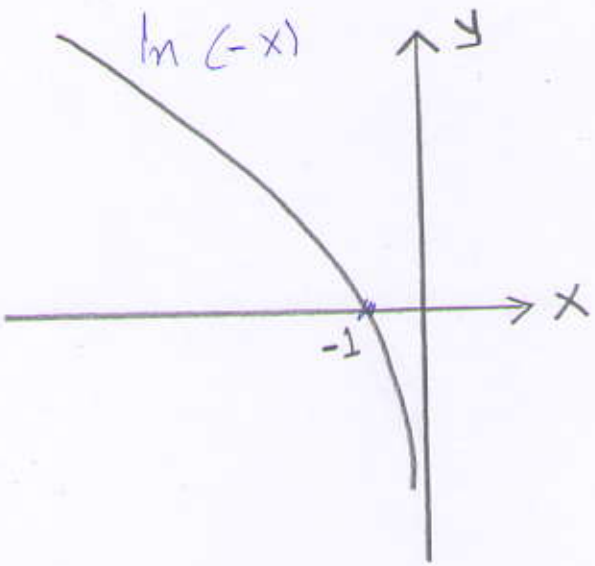
Logarithmic Function

Standard form

$$y = \log_a x$$

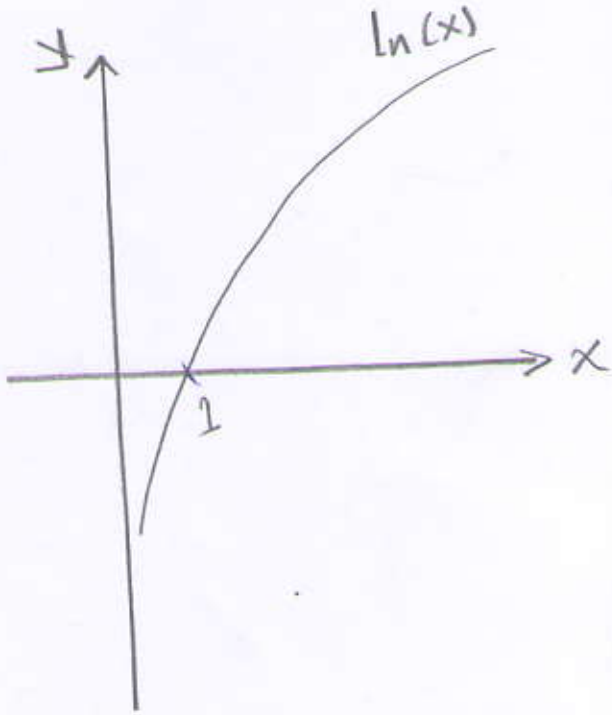
if $a = e \Rightarrow$ the log. is called natural Logarithm

$$y = \log_e x \equiv \ln(x)$$



$D_f =]-\infty, 0[$

$r_g = \mathbb{R}$



$D_f =]0, \infty[$

$r_g = \mathbb{R}$

(لا يوجد لوغاريتم لرقم سالب)

Notes :

$\ln(1) = 0$ $\qquad$ $\ln(e) = 1$

$\ln(0) = -\infty$ $\qquad$ $\ln(\infty) = \infty$

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* $\ln p + \ln q = \ln(pq)$

* $\ln p - \ln q = \ln\left(\frac{p}{q}\right)$

* $r \ln p = \ln p^r$ والعكس صحيح

* Logarithmic function is the inverse of exponential function

$e^{\ln(f(x))} = f(x)$, $\ln\left(e^{f(x)}\right) = f(x)$

* So To remove exponTial from equation Take ln To both sides

Ex: sketch $y = 1 + 2 \ln(1-x)$

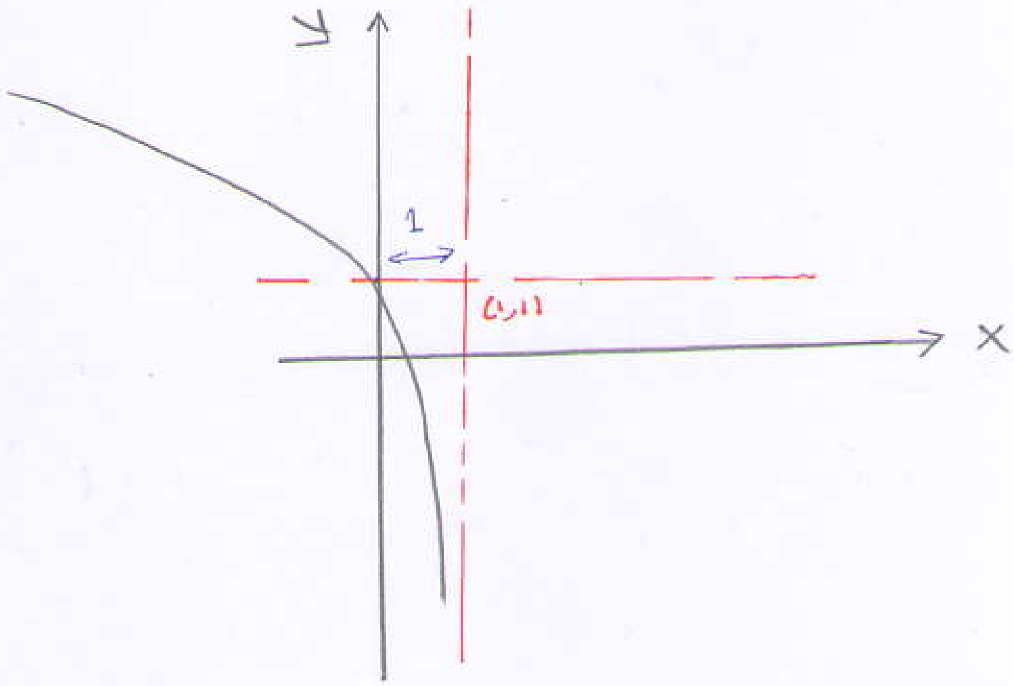
$y - 1 = 2 \ln(1-x)$ (-ve) اذا كان الباب ,

$1 - x = 0.0$, $y - 1 = 0.0$

$x = 1$ $y = 1$ New origin $(1, 1)$

D: $]-\infty, 1]$

r: $\mathbb{R}$



Ex: on the same axes sketch

$y = \ln x$, $y = 2 \ln x$ and $y = \ln(2x)$

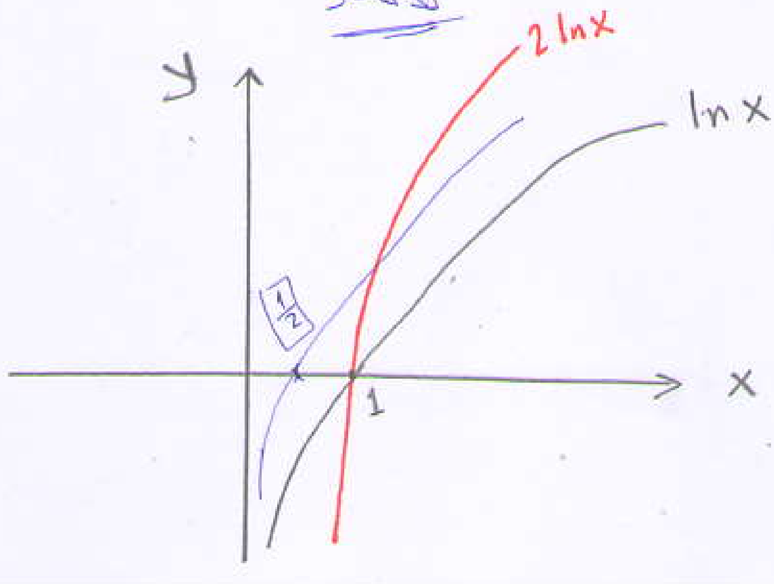
sol

To get intersection with x axes put $y = 0$

$0 = \ln(2x)$

$e^0 = 2x = 1$

$x = \frac{1}{2}$



<u>Generally</u> : $y = \ln(ax)$

$$0 = \ln(ax) \Rightarrow ax = e^0 = 1$$

$$\boxed{x = \frac{1}{a}}$$

[:نقطة التقاطع مع محور x هي مقلوب الـ coefficient بتاع x]

<u>Ex:</u> sketch $y + 1 = 3\ln(2x - 4)$ (+ve)

$2x - 4 = 0.0$

$\boxed{x = 2}$

$y + 1 = 0.0$

$\boxed{y = -1}$

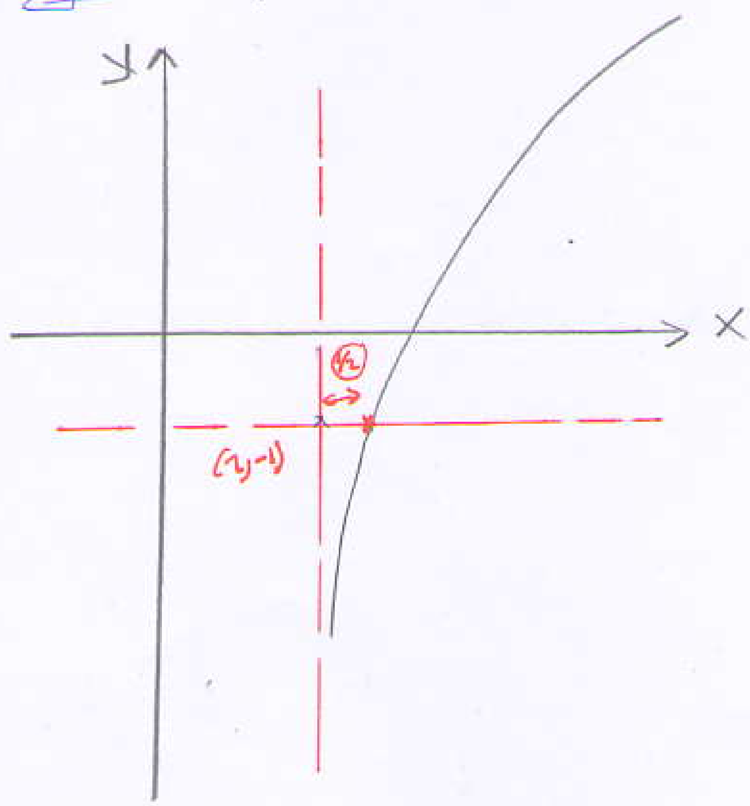
New origin $(2, -1)$

x يجب معرفة بعد الـ curve على محور x.

مقلوب الـ coeff. : $\Leftarrow$ $\frac{1}{2}$

$D :]2, \infty[$

$r : \mathbb{R}$



$$\log_a x = \frac{\ln x}{\ln a} = A \ln x$$

$$A = \frac{1}{\ln a}$$

$$\log_3 x = \frac{1}{\ln 3} \ln(x)$$

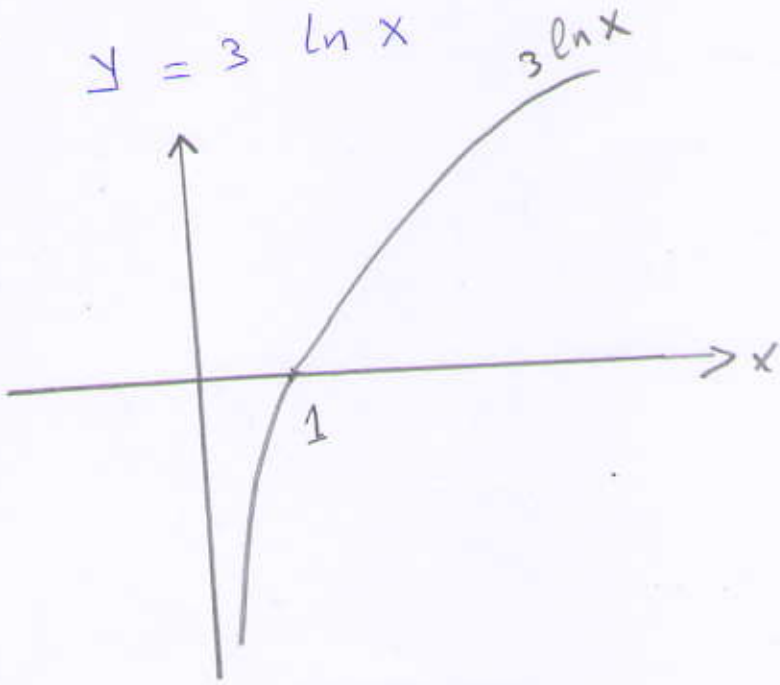
$$\log_2 x = \frac{1}{\ln 2} \ln(x)$$

Ex° Sketch $y = \ln x^3$

$y = 3 \ln x$

$D_f =]0, +\infty[$

$r_f = \mathbb{R}$



Ex° Solve for x : $e^{x-1} = 3$

Take ln

$x - 1 = \ln 3$

$x = 1 + \ln 3$

Ex° $\ln(2x-1) = 5$

$2x - 1 = e^5$

$x = \frac{1 + e^5}{2}$