A computer algorithm for reconstructing a scene from two projections

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8点算法

A simple algorithm for computing the three-dimensional structure of a scene from a correlated pair of perspective projections is described here, when the spatial relationship between the two projections is unknown. This problem is relevant not only to photographic surveying[1] but also to binocular vision[2], where the non-visual information available to the observer about the orientation and focal length of each eye is much less accurate than the optical information supplied by the retinal images

Nature, 1981

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

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$$(x_r, y_r, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} x_l \\ y_l \\ 1 \end{pmatrix} = 0$$

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$(x_r x_l, x_r y_l, x_r, y_r x_l, y_r y_l, y_r, x_l, y_l, 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$(x_r x_l, x_r y_l, x_r, y_r x_l, y_r y_l, y_r, x_l, y_l, 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

两个视点之间的一对对应点

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$\begin{pmatrix} x_{r1}x_{l1}, x_{r1}y_{l1}, x_{r1}, y_{r1}x_{l1}, y_{r1}y_{l1}, y_{r1}, x_{l1}, y_{l1}, 1 \\ \vdots \\ x_{rn}x_{ln}, x_{rn}y_{ln}, x_{rn}, y_{rn}x_{ln}, y_{rn}y_{ln}, y_{rn}, x_{ln}, y_{ln}, 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = \mathbf{0}$$

n对对应点

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$\begin{pmatrix} x_{r1}x_{l1}, x_{r1}y_{l1}, x_{r1}, y_{r1}x_{l1}, y_{r1}y_{l1}, y_{r1}, x_{l1}, y_{l1}, 1 \\ \vdots \\ x_{rn}x_{ln}, x_{rn}y_{ln}, x_{rn}, y_{rn}x_{ln}, y_{rn}y_{ln}, y_{rn}, x_{ln}, y_{ln}, 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = \mathbf{0}$$

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$\begin{pmatrix} x_{r1}x_{l1}, x_{r1}y_{l1}, x_{r1}, y_{r1}x_{l1}, y_{r1}y_{l1}, y_{r1}, x_{l1}, y_{l1}, 1 \\ \vdots \\ x_{rn}x_{ln}, x_{rn}y_{ln}, x_{rn}, y_{rn}x_{ln}, y_{rn}y_{ln}, y_{rn}, x_{ln}, y_{ln}, 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = \mathbf{0}$$

矩阵秩为8

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$\begin{pmatrix} x_{r1}x_{l1}, x_{r1}y_{l1}, x_{r1}, y_{r1}x_{l1}, y_{r1}y_{l1}, y_{r1}, x_{l1}, y_{l1}, 1 \\ \vdots \\ x_{rn}x_{ln}, x_{rn}y_{ln}, x_{rn}, y_{rn}x_{ln}, y_{rn}y_{ln}, y_{rn}, x_{ln}, y_{ln}, 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = \mathbf{0}$$

矩阵秩为8

8对对应点可得一组非零解

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$\begin{pmatrix} x_{r1}x_{l1}, x_{r1}y_{l1}, x_{r1}, y_{r1}x_{l1}, y_{r1}y_{l1}, y_{r1}, x_{l1}, y_{l1}, 1 \\ \vdots \\ x_{rn}x_{ln}, x_{rn}y_{ln}, x_{rn}, y_{rn}x_{ln}, y_{rn}y_{ln}, y_{rn}, x_{ln}, y_{ln}, 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = \mathbf{0}$$

矩阵秩为8

8对对应点可得一组非零解

解是零空间

8点算法

$$\mathbf{p}_r^{\mathrm{T}}\mathbf{F}\mathbf{p}_l = 0$$

$$\begin{pmatrix} x_{r1}x_{l1}, x_{r1}y_{l1}, x_{r1}, y_{r1}x_{l1}, y_{r1}y_{l1}, y_{r1}, x_{l1}, y_{l1}, 1 \\ \vdots \\ x_{rn}x_{ln}, x_{rn}y_{ln}, x_{rn}, y_{rn}x_{ln}, y_{rn}y_{ln}, y_{rn}, x_{ln}, y_{ln}, 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = \mathbf{0}$$

矩阵秩为8

使用八个以上点并用齐次最小二乘法求解