Notes on BIOMEDE 211, or: Circuits, Systems, & Signals in Biomedical Engineering

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February 5, 2019

Contents

0.1 How can I print off and use this document?

Frankly, in just about any way thats useful to you. I am going to try something here, where I will try to make more or less the entirety of the notes associated with the Winter 2019 semester of BIOMEDE 211, Circuits, Systems, and Signals in Biomedical Engineering, to you, dear reader.
Please dont plagiarize this. If you were raised right, you ought to know what that is. If youd like my judgment on any sort of action, my opinions can be laid bare.
The first assignment I am giving you (worth 4% of your grade and which must be completed by the end of the semester) is to figure out where this document is located online, download it, print it off, sign your name to it, and get it to me. If you know who I am, I would expect a competent engineer to find that without much to-do about it. Start with Google, go from there. Further, for those in the class, BIOMEDE 211, Winter 2019, you must join Github and make at least four substantive contributions to this repository. The term all you engineers (and lawyers) cant wait to parse is substantive to which I will always enter a judgment which I deem final in this class, and I am ever in favor of beneficence over stricture. So, just help out the class in a way you think is helpful and watch those around you do the same. Failure to contribute to this living document by the end of the semester for those in this class will result in a loss of up to 4% of one's total grade outright.

0.2 How to contribute to GitHub

Follow these general steps to propose a change to this online document:

1. Create a GitHub account

   This should be rather self-explanatory. Use your e-mail account and verify it to be able to edit. You should proceed with the following steps while logged onto your account.

2. Find Dr. Belmont's GitHub page and go to the biomede-211-w19 repository ("repo"). Then click on the biomede-211-w19.tex file.

3. Edit the file

   You will find a small pencil icon on the right side of the page. Click on this to create your own branch ("forking"), and edit the file as you wish.

4. Propose file change

   After making your changes, you should scroll to the bottom of the page, find the message box that says, 'Propose file change', and fill it out. The first line should say what you have updated and can be explained in the description.

5. Create pull request

   After finishing your file, you will be brought to a page that displays what you have modified on the original document. Press the green 'Create pull request' button to let Dr. Belmont know that you want to create a change. Once he has approved via his own GitHub account, your changes should now be in the updated master branch!

0.3 Who comprises this class and how can they be reached?

0.3.1 The Captain at the helm

Barry Belmont
Wednesdays 11:00 a.m. — 1:00 p.m., 2130 LBME
belmont@umich.edu

0.3.2 The A-Team

Annabelle St. Pierre
Wednesdays 5:00 p.m. — 6:30 p.m., UGLI basement
astpierr@umich.edu

Alice Tracey
Wednesdays 4:00 p.m. — 5:00 p.m., UGLI basement
atracey@umich.edu

0.3.3 You, yourselves

In this class, we will be learning a lot from each other. You are encouraged to learn from one another. You are encouraged to talk to one another. You are are encouraged to share ideas and at times data. You are not encouraged and are hereby expressly forbidden to submit the work of another as your own. If you get help from others, you will put their name on it somewhere. Too much of this and you are committing plagiarism, not enough and you are committing fraud. Please be honest and let's all learn together.

0.4 The policies of this class

1. There will be six homework assignments, each one is worth 8 percent of final grade

   Name must be in upper right corner of the page

   A pdf version must also be submitted to canvas by the time the assignment is due

   Failure to comply will result in a significant deduction in homework grade

   Homework 1 is due in class on Thursday, January 31, 2019

2.

3.

Part I

Circuits

Chapter 1

I. Potential, current, energy, conservation

01/10/2019

1.1 What is electricity?

1. A form of energy resulting from the existence of charged particles
2. The physical phenomena arising from the existence, presence, and motion of charged particles
3. Rather ill-defined in common vernacular we will generally avoid its use

1.2 Charge

1. Charge is the property of matter that causes it to experience a force when placed in an electromagnetic field; measured in coulombs (C)
2. Charges are found in nature in discrete, integral multiples of electronic charge: e = -1.602 x 10^{-19} C (the charge of one electron)
3. **How many electrons are needed to form one coulomb?** (What is the weight of all those electrons?)
4. One byte is eight bits. Bits are essentially a single electron stored in a transistor. **If we were to take all the electrons from one**

terabyte of well distributed information (equal number of ones and zeros), how many coulombs would we have?

1.3 Current

1. The time rate of change of charge charges (charged particles) in motion; measured in amperes; defined mathematically as

$$i := dq/dt \tag{1.1}$$

   where i is current, q is charge, and t is time

2. Conversely, the total charge transferred over time can be expressed as

$$Q := \int_{t_0}^{t} idt \tag{1.2}$$

3. 1 ampere is equal to 1 coulomb/second

4. Direct current, “DC”, is current that remains constant with time

5. Alternating current, “AC”, is current that varies sinusoidally with time

1.3.1 The directionality of current

Ultimately, the direction in which we say ”current” flows is largely arbitrary. As arbitrary as choosing one type of charge and calling it “positive” and another “negative”. The reason it doesn’t matter is that the only consequence of having chosen a “wrong direction” for the current in a given analysis is that we have to switch the sign of the value. Thus, 3 amps in one direction is *the exact same thing* as -3 in the opposite direction.

1. Thanks to Benjamin Franklin we say that current is

   i. **Positive in the direction in which positively charged particles flow** and

   ii. **Negative in the direction in which negatively charged particles**

   iii. We also now know that current results primarily from the movement of negatively charged particles (electrons) and therefore our convention is wrong in one sense, though convenient and entrenched enough that were not liable to change it in our life time (besides, the math comes out the same, and the actual flow of electrons will only matter to us in a few special circumstances, diodes)

1.3.2 The at times deadly serious nature of current

Much of the point of learning this material here is its eventual application by our hands or by the hands of those we work with. Before we put any of this stuff in our hands, we should probably know what is and is not safe.

1. 1 mA, you will feel
2. 10 mA, you will really feel
3. 100 mA, you will likely die
4. 1000 mA, you will definitely die

1.3.3 The "speed" of current

A possible misconception is that the electrons inside a wire travels at the speed of light. The speed of current is actually relatively slow. If one were to imagine an electron starting at the wire next to a light switch in an average classroom, it would take a very long period of time for it to travel to the light itself. The light's immediate reaction to a switch is due to a "hose effect"; the electrons inside the wire push other electrons in the direction opposite to the [conventional] current. This cascade of electrons is what happens close to the speed of light, not the electron movement itself.

1. **The *signal* of electrical current (that is electromagnetic radiation) travels anywhere between about 50-99% the speed of light** (dependent on a number of conditions) depending upon the material through which it travels (based on a dielectric behavior known as permittivity)
2. **The drift velocity of electrons** within a copper wire is 25 μm/s, so how does anything ever turn on?
3. **The hose effect** - The electrons at the light switch will almost certainly never pass through a light bulb, but they will move around and bump into their neighbors which bump into their neighbors which bump into their neighbors, etc., until it causes the electrons nearest the light to pass through. This is how water at a spigot is able to push water at the end of a hose.

1.4 Potential (difference)

1. The amount of work needed to move a unit of (positive) charge from a reference point to another point [without producing an acceleration]).

2. Potential is measured in "volts" and is often called "voltage". In this class we will endeavor to avoid such a term as it can be very confusing to talk about potential as if there were such a *thing* as voltage.

3. Defined as
$$v := \frac{dw}{dq} \tag{1.3}$$

4. Potential describes the *potential* to do something. Increasing the potential is akin to increasing the height of a cliff. The height does not *do* anything other than increase what can be done on the drop. If potential is the cliff's height, charge would be pebbles you'd drop off the side, and current describes how fast those pebble fall.

5. In this class, and for the vast vast majority of electrical engineering work, we care about the *difference* in potential. One element held at 100 billion volts and another held at 100 billion + 1 volts has a *potential difference* of 1 V, which is less than a single AA battery.

6. Voltage can also be thought of as how badly the current "wants" to flow, while current is the actual flow of charges per second. Since charge flows but not the voltage, voltage can exist without current - a single charge induces a voltage. On the other hand, current can't exist without voltage since having a current means that charges are flowing, and if charges are flowing there is a potential difference across the charges.

7. Some typical voltages to be aware of

   Consumer level batteries (AA, AAA): 1.5 V (DC); 9 V (DC)

   Car batteries: 12 V (DC)

   The "mains" (levels provided by power companies to consumers): 110-120 V (AC) and 220-240 V (AC) in America

   Power transmission lines: 110-1200 kV (AC), transformers are used to step up and down the potential before used by consumers

1.5 Power

1. The time rate of expending or absorbing energy.
2. Quantifies the rate of energy transfer.
3. Mathematically:
$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \tag{1.4}$$
4. Measured in watts: 1 W $= 1 \ \frac{\text{J}}{\text{s}} = 1 \ \frac{\text{N}\cdot\text{m}}{\text{s}} = 1 \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3} = 1 \text{ V} \cdot 1 \text{ A}$
5. **Passive sign convention**: If current enters through the positive terminal of an element, $p = +vi$; if current enters through the negative terminal of an element, $p = -vi$.

1.6 Energy

1. The capacity to do work.
2. Measured in joules.
3. $E = \int \frac{dw}{dt} dt \rightarrow$ power x time
4. $\text{J} = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \text{N} \cdot \text{m} = \text{Pa} \cdot \text{m}^3 = \text{W} \cdot \text{s} = \text{C} \cdot \text{V}$

1.7 Conservation

Here, as elsewhere, things will be conserved. In electrical circuits there are two laws of conservation that will matter most for us:

1. **The Conservation of Mass.** The conservation of mass means that no mass can be added to or removed from a circuit without being accounted for. Put differently, in a closed system (the type we will concern ourselves with here) no mass is added or removed.

   In electrical circuits, the mass we care the most about are the charges whipping around. Thus, for us, *the amount of charge within a circuit must remain constant.*

2. **The Conservation of Energy.** The conservation of energy means that no energy can be added to or removed from a circuit without

> being accounted for. Put differently, in a closed system (the type we will concern ourselves with here) no energy is added or removed.
>
> In electrical circuits, the energy we care the most about is the potential provided by sources and depleted by other elements in the circuits. Thus, for us, *the sum of potentials within a circuit must equal zero.*

In evaluating circuits, the main focus of the first third of this class, it will be the application of these two conservative laws that will enable us to "solve" them. That is, by understanding (1) how energy is generated and used and (2) how charges move around in closed loops ("circuits") we will be able to predict the behavior of the myriad electrical systems which may cross our paths.

1.8 Worksheet

1.8.1 Problem 1, constant charge through a cross-section

How much charge passes through a cross-section of a conductor in 60 seconds if a DC current value is measured at 0.1 mA? **Solution**

1.8.2 Problem 2, arbitrary charge through a cross-section

Determine the total charge entering a terminal between $t = 0$ seconds and $t = 10$ seconds if the current (in amps) passing through is

$$i(t) = \frac{1}{\sqrt{5t + 2}}. \tag{1.5}$$

Solution

1.8.3 Problem 3, a "tera"ble puzzle

Approximately how much current is necessary to transmit one terabyte of information in an hour? **Solution**

1.8.4 Problem 4, power necessary to run a pacemaker

A cardiac pacemaker will provide approximately 5,000 J of energy over 5 years. Determine the capacity of a 5 V lithium battery necessary to drive this pacing such that only 40% of its energy is spent over that time. **Solution**

1.8.5 Problem 5, energy needed to excite a neuron

A colleague of yours has been in their lab ginning up new neurons. You, as their resident electrical expert, are tasked with determining the energy consumed by the cell. If the current and voltage variations are found to be functions of time ($t \geq 0$)

$$i(t) = 3t \tag{1.6}$$

$$v(t) = 10e^{6t} \tag{1.7}$$

determine the energy consumed between 0 and 2 ms. **Solution**

1.8.6 Problem 6, a thump to the chest

(a) A typical defibrillator delivers 200-1000 V in less than 10 ms. How much current is needed to deliver 120, 240, and 360 Joules?
(b) A human heart ways about 300 grams. From approximately how high of a cliff would one have to drop a heart such that the impact was equivalent to the energy delivered to someone's chest from a defibrillator? **Solution**

Chapter 2

An introduction: II. Circuit elements

01/15/2019

2.1 Active v. passive

1. Active elements are capable of generating energy while passive components cannot

2. **Active**: generators, batteries, operational amplifiers, "sources"

3. **Passive**: resistors, capacitors, inductors, i.e., most circuit elements

2.2 Ohm's Law and what it means

Ohm's Law is concerned with the relationship between voltage, or potential difference, and current across a conductor. The potential difference across a conductor is proportional to the current flowing thorugh the conductor with the proportionality constant being denoted as R, or resistance. This can be expressed as:

$$V := iR \tag{2.1}$$

This essentially states that the drop in potential across the conductor, or resistor, is equivalent to the current flowing through the conductor and its resistance. When considering impedance, the equation can be modified to state:

$$V := iZ \tag{2.2}$$

2.3 Sources

1. **An ideal independent source** is an active element that provides a specified value of potential or current, regardless of other circuit elements.

   Batteries and power supplies may be approximated as ideal potential sources.

2. **An ideal dependent (or controlled) source** is an active element in which the source quantity is controlled by another quantity (such as potential, current, temperature, measured resistance, etc.).

3. **An ideal potential source** will produce any current required to ensure that the terminal voltage stated is satisfied.

4. **An ideal current source** will produce any voltage required to ensure that the terminal current as stated is satisfied

5. Symbols

   Voltage-controlled voltage source, VCVS

   Current-controlled voltage source, CCVS

   Voltage-controlled current source, VCCS

   Current-controlled current source, CCCS

2.4 Resistors

Resistors are electrical (circuit) elements that resist the flow of electric charge (current); passive two-terminal components that implement a defined/"constant" resistance; meant to reduce current flow and change potential

2.4.1 Resistance, R

1. **Resistance** is the physical property describing an element's ability to resist current and is most often represented by R

2. Resistance is measured in "ohms", Ω, which is equivalent to 1 V/A

3. Resistance is one half of a broader physical phenomenon known as **"impedance"** - the property describing an element's ability to *impede* current. Impedance is typically represented by Z, which we'll explore more thorough in a bit.

2.4.2 Resistivity, ρ

1. The resistance of an element (such as a resistor) depends on three things:

   Resistivity, ρ, of the material comprising the element, which is the *material*'s ability to resist the flow of charges; measured in ohm-meters

   Length, l, of the element; measured in meters

   Area, A, of the cross-section of the element; measured in m^2

   Such that $R = \rho \frac{l}{A}$

   What units are we left with?

   What are the effects of length and area?

2. **Materials with low resistivity** are generally called (and treated as) "conductors" as they are able to more effectively *conduct* the motion of electrical charges than materials with high resistivity

3. **Materials with very high resistivity** are generally used as "insulators" as they prevent the flow of current through them and thus *insulate* the current within prescribed bounds, such as with a copper wire with plastic wrapped around it.

Here is a link to a video that further explains the concepts of resistivity and resistance: `<https://www.youtube.com/watch?v=4rsswT_Rv1M>`.

2.4.3 Conductance

1. The inverse of resistance is conductance, G, which describes the ability of an element to conduct current

2. Measured in Siemens

3. Allows us express Ohm's law slightly differently, $i = Gv$, which says that the current generated through an element by a potential is directly proportional to some constant, namely conductance.

4. The material specific property **conductivity**, σ is measured in S/m

2.5 Capacitors

1. Passive two-terminal components that store energy in an electric field; introduces capacitance to a circuit.

2. Can be thought of as two conductive plates sandwiching a "dielectric" material. Essentially it is two "conductors" separated by a "non-conductive region".

3. When a capacitor is attached across a source, an electric field develops across the dielectric causing a net positive charge to collect on one conductor and a net negative charge to collect on the other.

4. We can define the capacitance of an element mathematically as

$$C = Q/V \tag{2.3}$$

   where C is capacitance in farads, Q is positive or negative charge on each conductor, and V is the potential between them

5. We can also represent capacitance by the voltage-based rate of charge accumulation: $C = dQ/dV$.

2.5.1 Its time varying behavior

Unlike resistors, capacitors have a *time-varying* element to that. That is, since $C = Q/V$, $V = Q/C$.

If we then recall Equation **??**, we can write the time-dependent potential relationship of a capacitor

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + V(t_0) \tag{2.4}$$

We can also recall[1] Equation **??**, and represent the time-dependent current relationship as

$$I(t) = \frac{dQ(t)}{dt} = C\frac{dV(t)}{dt} \tag{2.5}$$

[1]We could also take the derivative of the equation preceding this one and do a little rearrangement. As it turns out, these physical relationships are rather codified and thus can be gotten out by any number of means.

2.5.2 Charge accumulation

1. While charges accumulate on a capacitor, no current flows *through* the capacitor.
2. **Well, then why use them? After awhile won't the current just stop?** Yes, indeed it will – in a DC circuit!
3. The capacitor will become "charged" over time, eventually reaching the same potential as that established across it, e.g., by a source. Since potential only ever travels down potential gradients, if the capacitor and the source (say, a battery) are at the same potential, no current will flow.
4. Thus, a fully charged capacitor will act as an "open" circuit, while an uncharged capacitor will act as a "short" circuit.

2.5.3 A simple example

If we consider Ohm's law for a simple RC circuit (one in which a source, a resistor, and a capacitor are in series), we can describe the system by

$$V_0 = v_R(t) + v_C(t) \tag{2.6}$$

$$V_0 = i(t)R + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau \tag{2.7}$$

Taking the derivative of both sides:

$$0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t) \tag{2.8}$$

$$0 = RC\frac{di(t)}{dt} + i(t) \tag{2.9}$$

$$i(t) = \frac{V_0}{R} \cdot e^{-t/RC} \tag{2.10}$$

$$v(t) = V_0\left(1 - e^{-t/RC}\right) \tag{2.11}$$

$$Q(t) = C \cdot V_0\left(1 - e^{-t/RC}\right) \tag{2.12}$$

2.6 Inductors

1. Passive two-terminal components that store energy in a magnetic field

2. Can be thought of as an insulated wire wound into a coil around a core (which may either be filled with a material or left open to the environment)

3. Behavior can be modeled as $L = \frac{\Phi}{I}$, where L is the inductance, Φ is the magnetic flux generated by a current, I.

4. By Faraday's law of induction, voltage induced by a change in magnetic flux through a circuit is

$$v = \frac{d\Phi}{dt} \tag{2.13}$$

which we can rewrite as

$$v = \frac{d}{dt}(Li) = L\frac{di}{dt} \tag{2.14}$$

5. In this class, at this level, and for most biomedical applications you're liable to experience in your tenure, you will not work extensively with inductors. However, you should be able to recall at least this much at a moment's notice to be able to ascertain a system's behavior.

2.7 Impedance

1. The measure of opposition a circuit element presents to a current when a potential is applied. (It is measured in ohms.)

2. It is "complex" in two sense of the term. First, the actual phenomenon itself comprises complex numbers; that is, there is both a "real" and an "imaginary" component.

   The real component is known as resistance, R

   The imaginary component is known as reactance, X

   Impedance can be represented as a combination of either

   Resistance and reactance: $\mathbf{Z} = R + \jmath X$, where $\mathbf{Z}$ is impedance, R is resistance, and X is reactance, or

   Magnitude and phase: $\mathbf{Z} = |Z|e^{\jmath\theta}$, where $|Z|$ is the magnitude of the impedance vector, $\mathbf{Z}$, and θ is the phase of said vector (i.e., the delay between current and potential). Phase, θ is equivalent to $\tan^{-1}(X/R)$

3. Impedance is also complex in the sense that it is complicated. The impedance of an object is a factor of many parameters including permittivity, geometry, quantum states, thermal stability, etc. Let us not view this sort of complexity as an impediment to our understanding of impedance.

4. The inverse of impedance is **admittance**, Y, and comprises a real component, **conductance**, G, (which is the inverse of resistance) and an imaginary component, **susceptance**, B (which is the inverse of reactance). (It is measured in Siemens.)

   $\mathbf{Y} = G + \jmath B$

2.7.1 A quick note on "imaginary" numbers

The term "imaginary" is an unfortunate name for an excellent mathematical tool. All the imaginary operator – in this class represented by $\jmath = \sqrt{-1}$ – is is a type of number "orthogonal" to our "real" numbers. Imaginary numbers are no less "real" than real numbers. Unfortunately, they aren't necessarily the most intuitive to our little mammalian brains and thus we must be trained to work with them. However, as we will see in this class, they can be quite useful.

2.8 Equivalent impedance

1. It will often be more convenient to think about the impedance which a component burdens a system with (or the conductance which it affords) rather than its resistance. **Therefore, we need to begin to think in terms of equivalent impedances as we start to evaluate circuits.**

2. Recall Ohm's law

   Resistors, $v = iR \qquad \rightarrow Z_{eq,R} = R$

   Capacitors, $v = \frac{1}{C}\int i dt \qquad \rightarrow Z_{eq,C} = \frac{1}{\jmath\omega C}$

   Inductors, $v = L\frac{di}{dt} \qquad \rightarrow \jmath\omega L$

   I want to plant a flag here for you to notice the relationship between the $\jmath\omega$ terms from the capacitor and inductor and the corresponding derivative and integral forms of current in the Ohm's law representation. This will become very important once we get into the Laplace and Fourier transforms.

3. We must also recognize that few will be the circuits comprising but a single element. As such, we should know how to find the equivalent impedance of many elements.

2.8.1 Impedances in general

Series

$$Z_{eq,series} = Z_1 + Z_2 + Z_3 + ... \tag{2.15}$$

Parallel

$$\frac{1}{Z_{eq,parallel}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + ... \tag{2.16}$$

A special case to remember. When dealing with only two elements:

$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \tag{2.17}$$

2.8.2 Resistors

Series

$$R_{eq,series} = R_1 + R_2 + R_3 + ... \tag{2.18}$$

Parallel

$$\frac{1}{R_{eq,parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \tag{2.19}$$

2.8.3 Capacitors

Series

$$\frac{1}{C_{eq,series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... \tag{2.20}$$

Parallel

$$C_{eq,parallel} = C_1 + C_2 + C_3 + ... \tag{2.21}$$

2.8.4 Delta-Wye (Δ-Y) transformations

Going from Delta to Wye

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \tag{2.22}$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} \tag{2.23}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \tag{2.24}$$

Going from Wye to Delta

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1} \tag{2.25}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2} \tag{2.26}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3} \tag{2.27}$$

$$\tag{2.28}$$

2.8.5 A few examples

Example 1 Find the equivalent resistance, if a resistor $R_1 = 10$ kΩ is connected in parallel to $R_2 = 3.3$ kΩ.

Solution. $R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{(10)(3.3)}{10+3.3} = 2.48$ kΩ

Example 2 Find the equivalent resistance of three parallel-connected resistors of equal value. If $R = R_1 = R_2 = R_3 = 10$ kΩ, what's R_{eq}?

Solution. Recall, Equation **??**

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow 3R_{eq} = R \rightarrow R_{eq} = \frac{R}{3} \rightarrow R_{eq} = \frac{10k}{3} = 3.33k\Omega \tag{2.29}$$

Example 3 Four resistors are connected in parallel. $R_1 = 10$ kΩ, $R_2 = 1$ kΩ, $R_3 = 5$ kΩ, and $R_4 = 3$ kΩ. Calculate their equivalent resistance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \tag{2.30}$$

$$\frac{1}{R_{eq}} = \frac{1}{10k} + \frac{1}{1k} + \frac{1}{5k} + \frac{1}{3k} \tag{2.31}$$

$$= 612.3\ \Omega \tag{2.32}$$

2.9 Grounds

1. A reference point in an electrical circuit from which potentials are measured
2. A common return path within a circuit

2.10 Conductors

1. Allow from the transmission of electrical energy
2. Serve to connect circuit elements
3. Also known as wires and traces
4. Within circuit schematics we must be mindful of “junctions” and “jumps” in conductors

2.11 Operational amplifiers (”Op-amps”)

1. Active components that deliver the amplified difference between its inverting and non-inverting terminals
2. Will be discussed at length in the next class and along with resistors, capacitors, and sources, will be among the primary circuit components we work with
3. Allow us to model mathematical functions; any mathematical function that can be represented by a differential equation can be replicated with an op-amp

2.12 Diodes

Two-terminal circuit elements that allow current to flow only in one direction

2.13 Switches

Make/break/change circuit paths (thereby diverting current or removing potential)

1. Single pole, single throw, SPST

2. Single pole, double throw, SPDT
3. Double pole, single throw, DPST
4. Double pole, double throw, DPDT

2.14 Transistors

2.15 Transformers

1. Transfer electrical energy between circuits using induction
2. Allows for the effective transmission of power and the stepping up/down of potential
3. Crucial for the transmission, distribution, and utilization of AC

2.16 Worksheet

2.16.1 Problem 1, expressing power in ohms

Utilizing Ohm's law, express units of power to include ohms.

Solution

2.16.2 Problem 2, a couple toaster based problems

A toaster draws 2 A at 120 V. What is its resistance?

Solution

How much current is drawn by a toaster with a resistance of 10 Ω at 110 V?

Solution

2.16.3 Problem 3, currently conducting power

In the circuit shown, calculate the current, i, the conductance, G, and the power, p.

Solution

2.16.4 Problem 4, conductance of a sodium channel

Conductance (G/A) of a sodium channel of a cell membrane at a specific time is 10 mS/cm^2. If the channel length as 100 nm, what is its conductivity?

Solution

2.16.5 Problem 5, resistance of a simple tissue

Determine the resistance of a homogenous and isotropic tissue with a cross-sectional area which can be described by the functions $y = 8 - x^2$ from $x = -2$ cm to $x = +2$ cm, a length of 10 cm (parallel to the z-axis), and a resistivity of 80 Ωm.

Solution

Chapter 3

An introduction: III. Operational amplifiers

01/17/2019

3.1 Some details

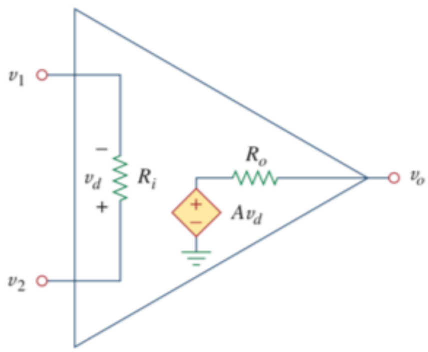


Figure 5.4
The equivalent circuit of the nonideal op amp.

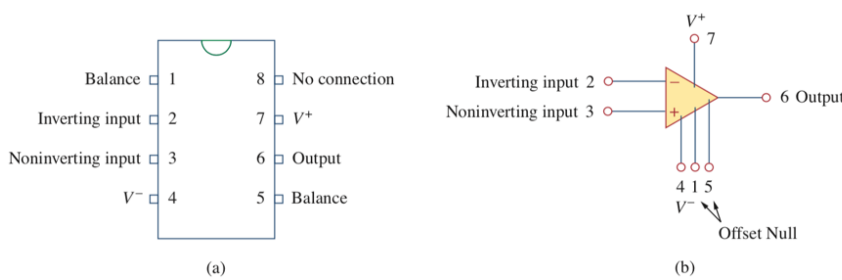


Figure 5.2
A typical op amp: (a) pin configuration, (b) circuit symbol.

1. Behaves like a voltage-controlled voltage source
2. They can amplify, sum, subtract, multiply, differentiate, integrate
3. They are active circuit elements
4. Though they have somewhat more complicated internal workings, we typically represent them in electrical circuits as a triangle with three (sometimes five) very important terminals:

   An inverting input (sign, typically represented up top for convenience, but it need not be)

   A non-inverting input (+ sign, typically on bottom)

   An output

3.2 Some rules

There are **three important features of ideal operational amplifiers** that we must understand thoroughly. These are things worth stamping in your brain.

1. **Infinite open-loop gain.** The "A" of the gain is infinitely large such that any difference in voltages V_1 and V_2 causes an enormously large output voltage. As much as is being supplied. (The real value of gain in most operational amplifiers is between 10^5 and 10^8.)

2. **Infinite input impedance.** Current cannot travel between the inverting and non-inverting terminals. (Really, the impedance is between 10^5 and 10^13 ohms and is often signal dependent.)

3. **Zero output impedance.** There is no loss transmitting a voltage difference to the output. (Really is about 10-100 ohms and is chip dependent.)

3.3 Some conveniences

1. With infinite input impedance, no current can flow into or out of the terminals and hence i_1 and i_2 are equal to 0.

2. Since no current flows across the terminals, the terminals are at equal potential. Hence "$v_1 = v_2$".

Some extra facts with operational amplifiers are that they can be combined with a capacitors to create different filters. Adding a capacitor in series with the input resistor creates a "high-pass filter" amplifier, where it passes signals with frequency higher than a specific cutoff frequency and attenutates signals lower than the cutoff. Adding a capacitor in parallel with the feedback resistor creates a "low-pass filter" amplifier, where it passes signals with frequency lower than the cutoff and attenuates signals higher than it. The corner frequency for the cutoff may be caulcated with f=1/(2piRC). Having both capacitors would create a band-pass filter which attenuates signals lower than the lower cutoff and higher than the upper cutoff frequencies.

3.4 Some examples

3.4.1 Inverting amplifier

We will apply the conservation of mass at this point to solve our equations. This is among the simplest and most effective ways to add gain to a circuit. So much so that you will use it again and again and again in life and especially in labs

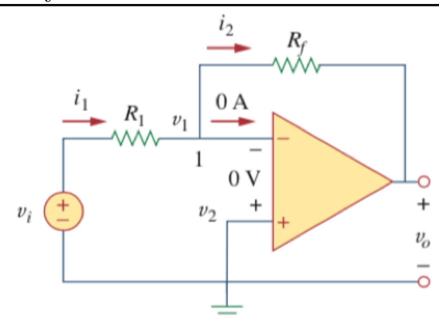


Figure 5.10
The inverting amplifier.

A key feature of the inverting amplifier is that both the input signal and the feedback are applied at the inverting terminal of the op amp.

We apply KCL at the node for v1

1. I1 i2 i3 = 0
2. I3 = 0
3. I1 i2 = 0
4. I1 = i2
5. I1 = (vi v1)/R1
6. I2 = (v1 - Vo)/Rf
7. (vi v1)/R1 = (v1 - Vo)/Rf
8. V1 = V2 = 0
9. Vi/R1 = -Vo/Rf

10. Vo = -Rf/R1 * Vi

11. R2/R1 is our gain, gain factor.

3.4.2 Non-inverting amplifier

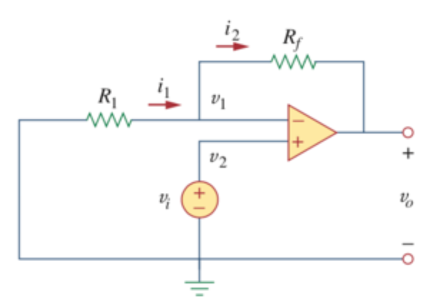


Figure 5.16
The noninverting amplifier.

Again, the name might imply what it does. It will amplify our input signal without inverting it.

We can again perform Nodal analysis.

1. I1 i2 i3 = 0

2. I3 = 0, since no current enters the non-inverting input

3. I1 = i2

4. (Vg v2)/R1 = (v2 Vo)/Rf

5. v2/R1 = (vi Vo)/Rf

6. Vo = (1 + Rf/R1) * Vi

3.4.3 Voltage follower

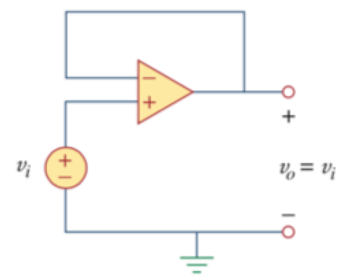


Figure 5.17
The voltage follower.

What if we didnt have any resistors? → Vi = v2 = v1 = Vo → $Vi = Vo$

3.4.4 Summing amplifier

3.4.5 Differential amplifier (as homework)

Chapter 4

Circuit analysis: I. Nodal analysis

01/22/2019

4.1 Nodes and branches

1. **A branch** is any two-terminal element. (examples: Resistor, Capacitor, Wire, etc.)

   A branch *is any two-terminal element. What are some two-terminal elements weve learned?*

2. **A node** (junction) is a point of connection between two or more branches.

   A node *is a point of connection between two or more branches. Often indicated by a dot. What else have we called a node? A junction.*

3. A loop is **independent** if at least one branch is not part of any other independent loop.

   A loop *is any closed path within a circuit. A closed path formed by starting at a node, passing through a set of nodes, returning to the starting node without passing through any node more than once.*

4.1.1 The Seven Bridges of Konigsberg

The Knigsberg bridge problem asks if the seven bridges of the city of Knigsberg (left figure; Kraitchik 1942), formerly in Germany but now known as Kaliningrad and part of Russia, over the river Preger can all be traversed in a single trip without doubling back, with the additional requirement that the trip ends in the same place it began.

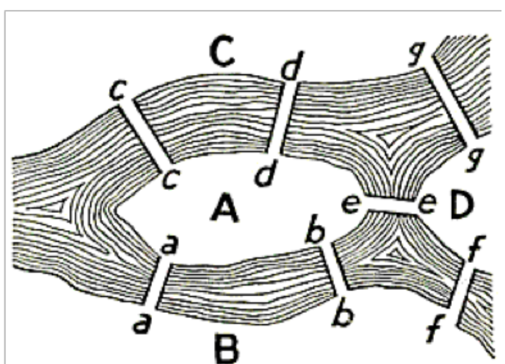


FIGURE 98. *Geographic Map: The Königsberg Bridges.*

4.1.2 Independence

A loop is independent if it contains at least one branch which is not a part of any other independent loop

1. Each of the loops in the circuit at right are independent
2. Independent loops lend themselves to sets of equations to be solved!

4.1.3 Fundamental theorem of network topology

Fundamental theorem of network topology states that the number of branches, b, must equal the sum of the independent loops, l and nodes, m minus one, that is

$$b = l + n - 1 \tag{4.1}$$

With this fundamental theorem we can also redefine/refine our definition of series and parallel.

- **Series** — two or more elements share a single node and thereby carry the same current
- **Parallel** — connected to the same two nodes and thereby have the same voltage across them (potential difference)

Practice Problem 2.4

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

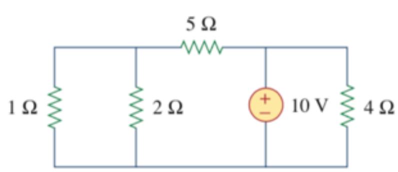


Figure 2.14
For Practice Prob. 2.4.

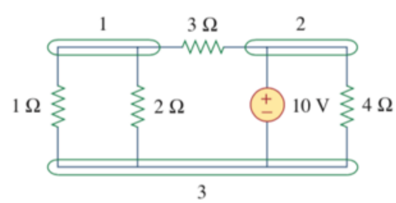


Figure 2.15
Answer for Practice Prob. 2.4.

4.2 Kirchhoff's Laws

I am not personally a fan named laws of nature, especially ones which are mere recapitulations of already perfectly good laws. Thus do we introduce Kirchhoffs laws, known as

- Kirchhoffs current law ("KCL")
- Kirchhoffs voltage law ("KVL")

These are, as far as Im concerned, mere restatements of *the conservation of mass* and *the conservation of energy*, respectively.

4.2.1 Kirchhoff's Current Law

Kirchhoff's current law states that any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

- Put differently, the algebraic sum of currents entering a node (or any closed boundary) is zero
- For those mathematically inclined among us, that is: $\sum_{x=1}^{n} i_x = 0$
- Another way this often gets stated is by saying that the algebraic sum of charges within a system cannot change and is thus sometimes referred to as "the conservation of charge".
- Well since all of our charge carriers are merely particles (electrons, protons, ions, etc.), this is just another layer over the top of the underlying law which is that mass cannot be created or destroyed.
- However, Kirchhoff's formulation of this law (conserving charge, mass) is useful in circuits as it gives us a great tool, being able to say that current going in is equal to current going out
- For the figure $i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \rightarrow i_1 + i_3 + i_4 = i_2 + i_5$
- KCL forms the basis of a technique well spend the next couple of lectures on known as nodal analysis because we evaluate the current going into and coming out of nodes

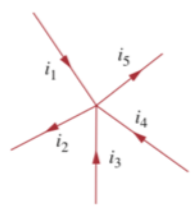


Figure 2.16
Currents at a node illustrating KCL.

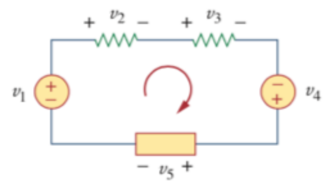


Figure 2.19
A single-loop circuit illustrating KVL.

4.2.2 Kirchhoff's Voltage Law

Kirchhoff's voltage law states that the sum of electrical potential differences (voltage) around any closed network is zero

- That is, for any closed path (loop), the sum of voltages is zero.
- Mathematically: $\sum_{m=1}^{M} v_m = 0$, where M is the number of voltage drops (caused by circuit elements) in the loop and vm is the mth voltage drop
- The sum of voltage rises = the sum of voltage drops

  $v_1 + (-v_2) + (-v_3) + v_4 + (-v_5) = 0$

  $v_1 + v_4 = v_2 + v_3 + v_5$

4.2.3 A few examples

1. Simple 1 Vs 1 R circuit; Vs = 10, R = 1 kohm, I = 0.01 A / 10 mA
2. Simple 1 Vs 2 R in series; Vs = 10, R = 1 kohm, I = 0.005 A / 5 mA

   What does KCL tell us? (Current is the same through resistors).
3. Simple 10 mA source, 2 R (1 kohm) in parallel; What is the voltage?

   $i_1 + (-i_2) + (-i_3) = 0] \rightarrow i_1 = i_2 + i_3 \rightarrow i_1 = V/R_1 + V/R_2$

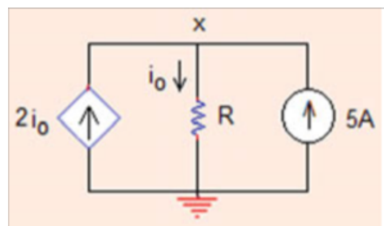
   $0.01 = V/1000 + V/1000 \rightarrow 0.01 = 2V/1000 \rightarrow 0.010/2 * 1000 = 5$ V

   Nodes are at the same voltage!

4. 10 mA up, 5 mA down, 2 R (1 kohm), Rl (500 ohm), all in parallel. What is the current through load Rl? [have someone come to the board and solve]

   When current sources are in parallel they add together

5. For the circuit shown below, use KCL to find the remaining branch currents



2.13 For the circuit in Fig. 2.77, use KCL to find the branch currents I_1 to I_4.

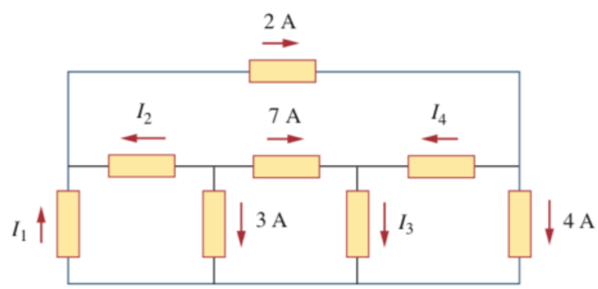


Figure 2.77
For Prob. 2.13.

For Figure 2.13, you know to use KCL because the circuit contains two current sources. Ideal current sources will deliver whatever potential is necessary to obtain the desired current so we don't know how potential behaves through it. If we look at node x and arbitrarily set i_1 as entering the node from the left, i_2 as exiting the node down the branch containing the resistor, and i_3 as entering the node from the right, then:

$i_1 - i_2 + i_3 = 0$

$2i_0 - i_0 + 5 = 0$

$i_0 = -5A$ The negative sign just means that the arbitrary direction we chose for our analysis is opposite of the actual direction of current flow

For Figure 2.77, start at a node with only one unknown current such as the upper right node.

$2 - i_4 - 4 = 0$

$i_4 = -2A$ Repeat the process for remaining nodes

$7 + i_4 - i_3 = 0$

$i_3 = 7 + (-2) = 5A$

$-i_2 - 3 - 7 = 0$

$i_2 = -10A$

$i_1 + i_2 - 2 = 0$

$i_1 = 2 - (-10) = 12A$

We could check our work by performing nodal analysis at the last node. However, it is not a necessary step as we have already found the desired unknowns.

4.3 Nodal analysis

Nodal analysis — a general circuit analysis technique in which we try to determine the potential difference between nodes by applying KCL and KVL (in my experience, usually focusing a bit more on KCL)

Your textbook offers the following description of the technique which seems pretty good to me:

1. Select a node as a reference. Assign voltages $(v_1, v_2, ..., v_{n-1})$ for the remaining n-1 nodes, all of which will be referenced with respect to the reference node.
2. Apply KCL to each of the n-1 nonreference nodes. Use Ohms law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations (system of equations) for each unknown node voltage.
4. It's as easy as that! But, well, actually, it can get a little hairy once you start to apply it in earnest.

4.3.1 The procedure

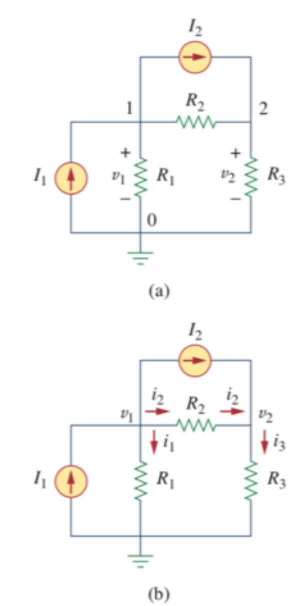


Figure 3.2
Typical circuit for nodal analysis.

1. Begin by putting a reference, usually a "ground"

   This ground can be one of two sorts: (1) **Earth ground** in which ultimately the whole earth is used as the reference point; or **Chassis ground** in which the case of the device in which the circuit is in will act as a reference as it will presumably be sufficiently large as to serve fine [this is also partly the reason why you can "feel" a MacBook charge up its charger does not utilize a traditional "earth ground"

   Either will suffice for our purposes here

2. Next we label all the nodes.

   How many branches, nodes, and loops?

   5 branches, 3 nodes, 3 loops. Satisfies our network condition.

   You can give them any label you want, but I find working your way up from the ground in a clockwise manner and numbering them sequentially is a good habit to get into.

   Keep in mind that we typically set our reference node to have a voltage of 0. We can actually set it to be anything wed like, but the math is often easier if we just make it 0.

3. Then we apply KCL to each nonreference node in the circuit.

   At node 1, $IA - i1 - i2 - IB \rightarrow IA = IB + i1 + i2$

   At node 2, $IB + i2 - i3IB + i2 = i3$

   Once weve got that, now its a matter of applying Ohms law. Thought typically written as V = iR, it is perhaps more helpful to write its full extension here and note that (Va Vb) = iR

   $I = (Va - Vb)/R$

   Thus we can state

   $I1 = (v1 - v0)/R1 \rightarrow I1 = G1(v1 - v0)$

   $I2 = (v1 - v2)/R2 \rightarrow I2 = G2(v1 - v2)$

4.4 Solving simultaneous equations

4.4.1 Cramer's Rule

When given a system of linear equations, Cramer's Rule allows us to solve directly for the specific variable whose value we are looking for.

To use Cramer's Rule, the system of equations must satisfy two conditions:

1. There must be the same number of equations as variables (the coefficient matrix must be a square).

2. The determinant of the coefficient must be non-zero.

Use the following steps to apply Cramer's Rule:

1. Write the coefficient matrix of the system (call this matrix A). Make sure this is a square matrix; otherwise Cramer's Rule is not applicable.

2. Compute the determinant of matrix A. Make sure that this value is non-zero; otherwise Cramer's Rule is not applicable here.

3. Suppose the first variable of the system is x. Write the matrix Ax by placing the column of numbers to the right of the equals sign as the first column of Ax and using the non-x coefficients of matrix A as the remaining columns.

4. The value of x is the determinant of Ax divided by the determinant of A.

5. Repeat steps 3 and 4 to solve for any other variable as needed.

Example implementing Cramer's Rule: We will use the following system of equations to demonstrate how to use Cramer's Rule to solve for the value of x:

x + y + z = 4 -x + 2y = 1 -y + z = 1

1. Write the coefficient matrix A and check that it is a square matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

2. Solve for the determinant of A. (—A— = 4). The determinant does not equal zero.

3. Write the matrix Ax:

$$Ax = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

4. Solve for the determinant of Ax. (—Ax— = 4).

5. Solve for the value of x:

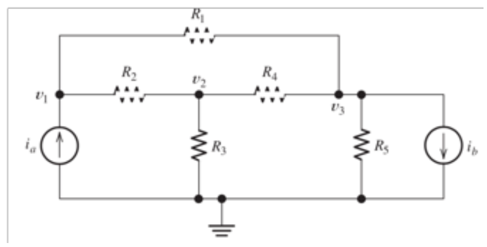
$$\text{x} = \frac{|Ax|}{|A|} \text{ x} = 1$$

Reference: Explanation and example inspired by "Solving System of Linear Equations: (lesson 4 of 5), Cramers Rule", MathPortal: https://www.mathportal.org/algebra/solving-system-of-linear-equations/cramers-rule.php. [Accessed 23 January, 2019].

4.5 Worksheet

4.5.1 Problem 1, KCL at a few nodes

Use KCL to write equations at each node.



Solution

4.5.2 Problem 2, matrix notation

Write the matrix form of the equations written above.

Solution

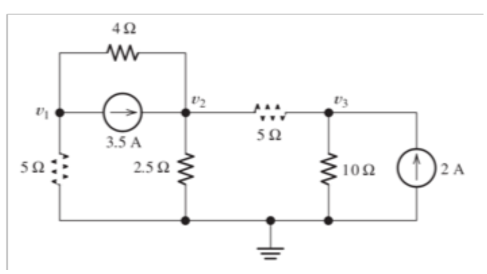
4.5.3 Problem 3, Cramer's rule

Using Cramers rule on the matrix equations above, what are the results?

Solution

4.5.4 Problem 4, Straight to the matrix

Write the node-voltage equations to the circuit at right in the matrix form.



Solution

Homework I

Assigned January 22, 2019. Due January 31, 2019 as *both* a hard copy in class and a pdf on Canvas.

1.1 Problem 1

(15 points) A typical automated external defibrillator (AED) delivers 200-1000 V in less than 10 ms.

1. If the AED in front us delivers a pulse of 600 V, how much current is needed to deliver 120, 240, and 360 Joules?

2. Assuming two human hands have a mass of approximately 1 kg, how many chest compressions would be needed to deliver an equivalent amount of energy if each compression had a depth of 5 cm and was delivered at a constant speed of 0.5 m/s every 2 seconds?

3. If you had the option of having your heart jump-started would you choose an AED or the bare hands of a stranger? Justify your answer.

1.2 Problem 2

(5 points) A 3.3 V battery you are considering for a wearable you are designing has a total charge of 300 mAh. How many joules is this battery capable of delivering?

1.3 Problem 3

(5 points) A fellow engineer bought a 12 V battery rated for 60 Ah. The experiment you both have in mind will draw 2 A over a 1 kΩ load. How long can your experiment last?

1.4 Problem 4

(5 points) A piece of medical equipment supplies 135 W at 220 V. How much electrical charge flows through the device in the 10 hours a nurse is on-call using it? And how many electrons does this charge correspond to?

1.5 Problem 5

(10 points) During an in vitro (petri dish) experiment, the peak electric power that a group of stem cells can tolerate without some serious functional consequence is known to have a threshold of about 1 mW. If the power delivered to this group of cells is defined as $p(t) = 2e^{-t} \sin 5t$ [mW], will the cell be harmed and if so how long can power be delivered before the cells are harmed?

1.6 Problem 6

(10 points) Find the equivalent resistances. (Show your work.)

- 1 square of resistors (R = 10 kΩ) measured across corners.
- 2 resistors (R1 = 10 kΩ, R2 = 4 kΩ) in series.
- 3 resistors (R1 = R2 = R3 = 10 kΩ) in parallel.
- 4 resistors (R1 = 1 kΩ, R1 = 2 kΩ, R1 = 3 kΩ, R1 = 4 kΩ) in parallel.
- 5 resistors (R = 10 kΩ) in a pentagon, measured across each resistor.

1.7 Problem 7

(10 points) Hospitals often employ what is known as a dynamic uninterruptible power system (D-UPS) comprising a diesel generator, a synchronous machine, and a kinetic energy unit. By way of example, if the energy to a particular hospital fails, the kinetic energy unit continues to feed 600 kW to the hospital for 20 seconds, allowing the generator and synchronous machine to take over and feed the load.

1. What is the total energy capacity of this kinetic energy unit?
2. If the hospital were operating at 240 kW (instead of the aforementioned 600 kW), how much longer can the unit feed the load?

1.8 Problem 8

(10 points) Current passing through an electrical element in an AC situation can generally be defined as $i(t) = A \sin \omega t$.

1. Determine the energy on this element if the voltage across it is $v(t) = B \cos \omega t$.
2. Plot current, voltage, and energy as functions of time using MATLAB (or an equivalent software package). You may use any values of A, B, and ω that youd like, but unit values (=1) might make your life/work easier. (Present the code you used to arrive at your plot.)

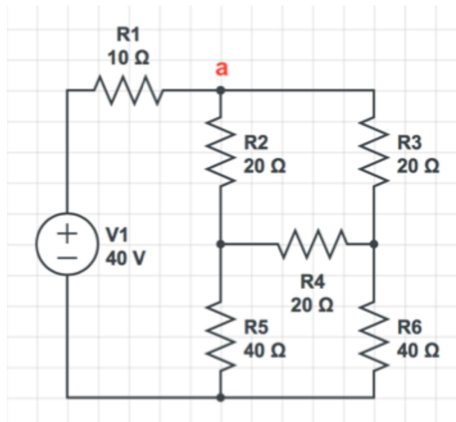
1.9 Problem 9

(10 points) The materiality of the human body ensures that it puts up some resistance to the flow of current. Find some values of impedance for the following human body parts. Be sure to cite your source(s) and explain why you trust it. (Given that there is a frequency dependence to these values, you may present any value of impedance found between 1 — 100 kHz.)

1. The whole body, from head to toe.
2. A single limb (such as an arm or a leg).
3. Blood.
4. Muscle.
5. Fat.

1.10 Problem 10

(20 points) For the circuit shown below.



1. Use the delta-wye transformation rule to determine the power dissipated by R1.
2. Find the voltage at node a (just after R1).
3. If R4 were changed to 100 Ω, would the results to (10.1) and/or (10.2) change? How do you know?
4. How many nodes, branches, and loops are there?
5. What is the voltage at each node, the current through each branch?

Chapter 5

Circuit analysis: II. Mesh analysis

01/24/2019

5.1 Clarifications on the homework

5.2 Mesh analysis

- A mesh is a loop which does not contain any other loops within it.
- ABEF and BCDE are meshes. ABCDEF is not.
- We apply KVL to find the mesh currents within a given circuit (this can be particularly helpful / convenient when dealing with parallel circuits).

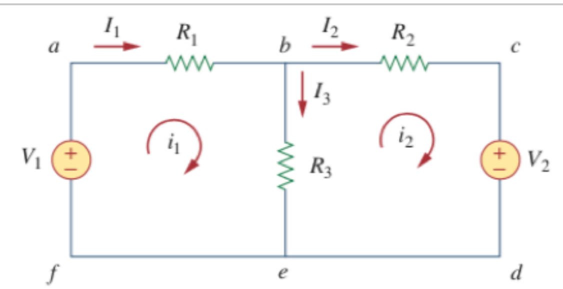


Figure 3.17
A circuit with two meshes.

5.3 Steps of mesh analysis

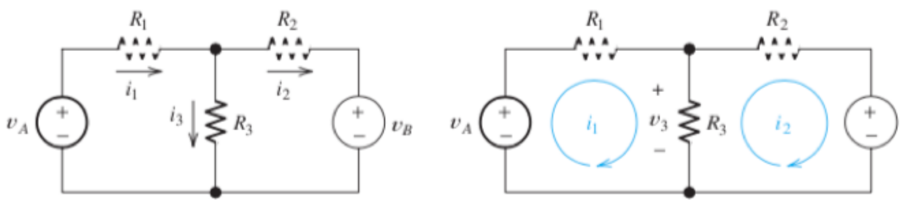
1. **For each of your n meshes assign a mesh current (i_1, i_2, ..., i_n).** You may draw them in any direction you want, but generally a nice clockwise direction will help with consistency.
2. **Apply KVL to each of the n meshes.** Use Ohms law to express the voltages in terms of the mesh currents
3. **Solve the resulting n simultaneous equations to get the mesh currents.**

5.3.1 Important condition for use

Unlike nodal analysis, mesh analysis only works for "planar networks" — that is one which can be written in two-dimensions without any branches.[1]

[1] Nonplanar networks can still be solved using nodal analysis.

5.4 An example



(a) Circuit with branch currents (b) Circuit with mesh currents

For the example seen above, we might at this point in our lives be tempted to solve the circuit with branch currents (the current that actually flows into and out of every branch. That is certainly a valid way by which to look at the problem. But now we're going to try another. We're going to try to understand "mesh currents".

Those branch currents can be formed into a pair of coupled equations

$$V_A - R_1 i_1 - R_3 i_3 = 0 \tag{5.1}$$
$$V_B - R_2 i_2 - R_3 i_3 = 0 \tag{5.2}$$

We can also apply KCL at the node between the three resistors and recognize a third bounding equation: $i_1+i_2-i_3 = 0$. We can then solve these three equations through any number of means (substitution, elimination, etc.).

But mesh analysis allows us to model the situation more simply and often more efficiently.

We begin by defining two arbitrary mesh currents, i_1 and i_2. Then we apply KVL as we do a "walk" around the loop defined by the mesh current.

$$V_A - R_1 i_1 - R_3(i_1 - i_2) = 0 \tag{5.3}$$
$$-R_3(-i_1 + i_2) - R_2 i_2 - V_B = 0 \tag{5.4}$$

This given us two equations with two unknowns (the arbitrary mesh currents, i_1 and i_2, we defined).

5.5 Another example

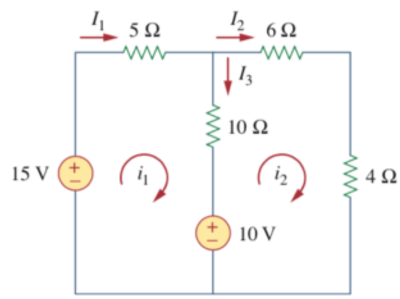


Figure 3.18
For Example 3.5.

For the circuit above, let's try to find the branch currents by performing mesh analysis (i.e., by. first finding the mesh currents).

Solution.

Obtain mesh current using KVL. For mesh 1:

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \tag{5.5}$$

which simplifies to

$$3i_1 - 2i_2 = 1 \tag{5.6}$$

For mesh 2:

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0 \tag{5.7}$$

simplifying to

$$i_1 = 2i_2 - 1 \tag{5.8}$$

5.5.1 Solving it one way

Perhaps the technique we'd most likely reach for if we knew nothing else about a situation is substitution. In this case, let's substitute equation **??** into equation **??**:

$$6i_2 - 3 - 2i_2 = 1 \rightarrow i_2 = 1 \text{ A} \tag{5.9}$$

And plugging that result back in

$$i_1 = 2i_2 - 1 = 2 - 1 = 1 \text{ A} \tag{5.10}$$

And thus I_3 being equal to the difference in current between i_1 and i_2 is equal t zero.

5.5.2 Solving it another

Another way of doing this is by recognizing some of the techniques afforded to us by linear algebra. If we squint at the equations with a little bit of good brain wrinkling, we can see that

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{5.11}$$

is just as valid of way of putting it as any other.

In this form we can use a few techniques to solve for the unknowns. One technique to be aware of given its deployability in those times in which you might need to write out your work is *Cramer's rule*.

Cramer's rule says that

$$i_1 = \frac{\Delta_1}{\Delta} \tag{5.12}$$

$$i_2 = \frac{\Delta_2}{\Delta} \tag{5.13}$$

where

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = (3)(2) - (-1)(-2) = 4 \tag{5.14}$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = (1)(2) - (1)(-2) = 4 \tag{5.15}$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = (3)(1) - (-1)(1) = 4 \tag{5.16}$$

$$\tag{5.17}$$

Thus

$$i_1 = \frac{4}{4} = 1 \tag{5.18}$$

$$i_2 = \frac{4}{4} = 1 \tag{5.19}$$

which agrees with the results we obtained before.

5.6 Yet another example

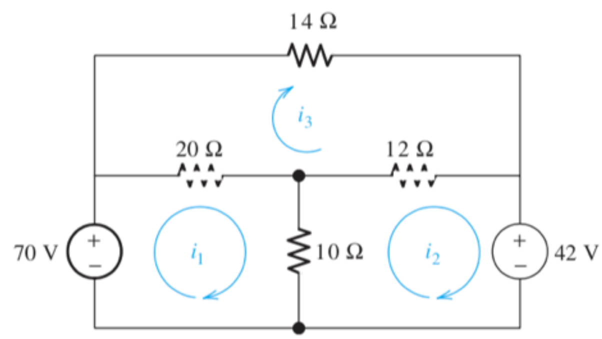


Figure 2.34 Circuit of Example 2.13.

Let's begin by defining the mesh currents (flowing clockwise around each mesh)

Mesh 1

$$20(i_1 - i_3) + 10(i_1 - i_2) - 70 = 0 \tag{5.20}$$

Mesh 2

$$10(i_2 - i1) + 12(i_2 - i_3) + 42 = 0 \tag{5.21}$$

Mesh 3

$$20(i_3 - i_1) + 14i_3 + 12(i_3 - i_2 = 0 \tag{5.22}$$

Putting the equations into standard form:

$$30i_1 - 10i_2 - 20i_3 = 70 \tag{5.23}$$
$$-10i_2 + 22i_2 - 12i_3 = -42 \tag{5.24}$$
$$-20i_1 - 12i_2 + 46i_3 = 0 \tag{5.25}$$

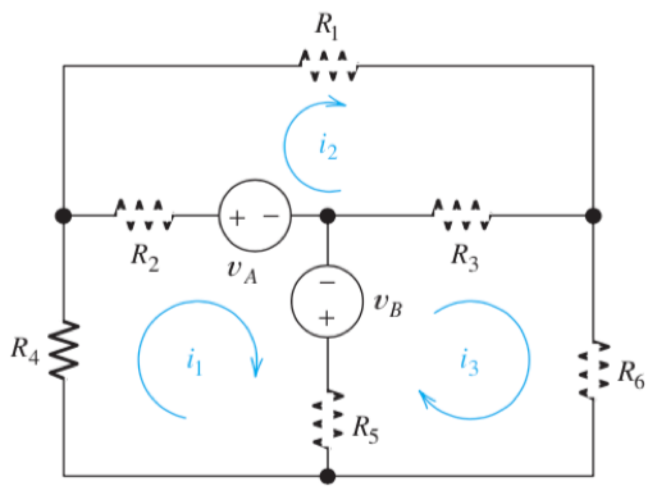
In matrix notation that becomes:

$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 22 & -12 \\ -20 & -12 & 46 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 70 \\ -42 \\ 0 \end{bmatrix} \tag{5.26}$$

While this can be solved any number of ways, in MATLAB one can simply write `I = R/V`, which should tell you something about what is "going on" mathematically in these operations.

5.7 Writing mesh equations directly in matrix form

Let's take a look at yet one more example.



We might, by now, begin to appreciate that one of our matrices is simply the resistances (impedances) of a mesh bundled up in one matrix, the currents bundled up in another, and the potentials bundled up in another, shown below.

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \tag{5.27}$$

To properly assemble these matrices there's just a few simple rules.

First, let's take a loop around each mesh and determine the equivalent impedance (of that mesh!)

- **Mesh 1.** $R_2 + R_5 + R_6$
- **Mesh 2.** $R_2 + R_1 + R_3$
- **Mesh 3.** $R_3 + R_6 + R_5$

These will comprise values along the diagonal of the resistance (impedance) matrix (r_{11}. r_{22}, and r_{33}).

Next, let's look for elements that are shared among meshes

- **Mesh 1 & 2** share R_2, so r_{21} and r_{12} both become $-R_2$
- **Mesh 2 & 3** share R_3, so r_{23} and r_{32} both become $-R_3$
- **Mesh 3 & 1** share R_5, so r_{13} and r_{31} both become $-R_5$

Thus, our entire resistance matrix becomes

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} (R_2 + R_5 + R_6) & -R_2 & -R_5 \\ -R_2 & (R_2 + R_1 + R_3) & -R_3 \\ -R_5 & -R_3 & (R_3 + R_6 + R_5) \end{bmatrix} \tag{5.28}$$

The matrix it buts up against is easy enough to construct, it is simply the mesh current of each mesh

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \tag{5.29}$$

And the potential matrix is the sum of sources in the direction of the mesh current

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -v_A + v_B \\ v_A \\ -v_B \end{bmatrix} \tag{5.30}$$

Giving rise to a final form which can be solved efficiently by any computation engine we put before us.

$$\begin{bmatrix} (R_2 + R_5 + R_6) & -R_2 & -R_5 \\ -R_2 & (R_2 + R_1 + R_3) & -R_3 \\ -R_5 & -R_3 & (R_3 + R_6 + R_5) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -v_A + v_B \\ v_A \\ -v_B \end{bmatrix} \tag{5.31}$$

Chapter 6

Circuit analysis: III. Supernodes and supermeshes

01/29/2019 Lecture 6.

6.1 A review of nodal and mesh analysis

6.1.1 Nodal analysis

1. Select a node as a reference. Assign voltages (v_1, v_2, ..., v_{n-1}) for the remaining n-1 nodes, all of which will be referenced with respect to the reference node.

2. Apply KCL to each of the n-1 nonreference nodes. Use Ohms law to express the branch currents in terms of node voltages.

3. Solve the resulting simultaneous equations (system of equations) for each unknown node voltage.

4. (Its as easy as that! But, well, actually, it can get a little hairy once you start to apply it in earnest.)

6.2 Nodal analysis with an independent current source

Let's begin by analyzing the following circuit.

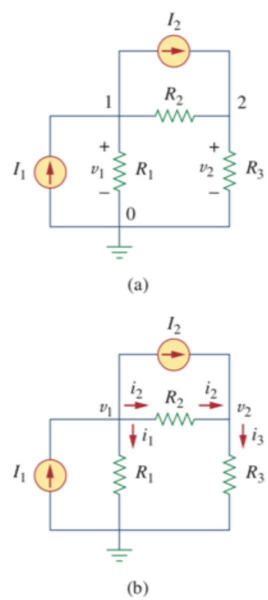


Figure 3.2
Typical circuit for nodal analysis.

1. Begin by putting a reference, usually a "ground".
2. Next we label all the nodes.

   How many branches, nodes, and loops? (5 branches, 3 nodes, 3 loops. Satisfies our network condition.)

   You can give them any label you want, but I find working your way up from the ground in a clockwise manner and numbering them sequentially is a good habit to get into.

   Keep in mind that we typically set our reference node to have a voltage of 0. We can actually set it to be anything wed like, but the math is often easier if we just make it 0.

3. Then we apply KCL to each nonreference node in the circuit.

   At node 1, $IA - i1 - i2 - IB \to IA = IB + i1 + i2$

At node 2, $IB + i2 - i3 - IB + i2 = i3$

Once weve got that, now its a matter of applying Ohms law. Thought typically written as $V = iR$, it is perhaps more helpful to write its full extension here and note that $(VaVb) = iR$

4. $I = (Va - Vb)/R$, Thus we can state

   - $I1 = (v1 - v0)/R1 \rightarrow I1 = G1(v1 - v0)$
   - $I2 = (v1 - v2)/R2 \rightarrow I2 = G2(v1 - v2)$
   - $I3 = (v2 - v0)/R3 \rightarrow I3 = G3(v2 - v0)$

5. Now we can substitute these relationships into our previous equations

6.3 A brief review of Cramer's rule

system of equations	coefficient matrix's determinant	answer column	D_x: coefficient determinant with answer-column values in x-column
$2x + 1y + 1z = 3$ $1x - 1y - 1z = 0$ $1x + 2y + 1z = 0$	$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$	$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$	$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}$

Similarly, D_y and D_z would then be:

$$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} \qquad D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

Evaluating each determinant, we get:

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (-2)+(-1)+(2) -(-1)-(-4)-(1) = 3$$

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = (-3)+(0)+(0) -(0)-(-6)-(0) = -3+6 = 3$$

$$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} = (0)+(-3)+(0) -(0)-(0)-(3) = -3-3 = -6$$

$$D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = (0)+(0)+(6) -(-3)-(0)-(0) = 6+3 = 9$$

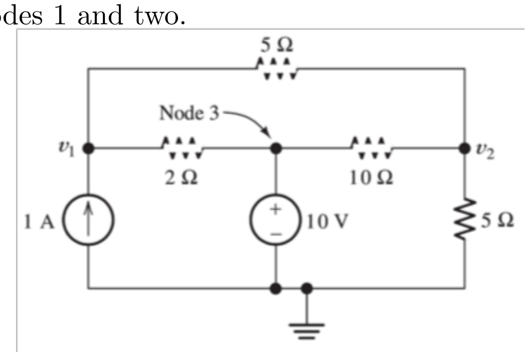
Cramer's Rule says that $x = D_x \div D$, $y = D_y \div D$, and $z = D_z \div D$. That is:

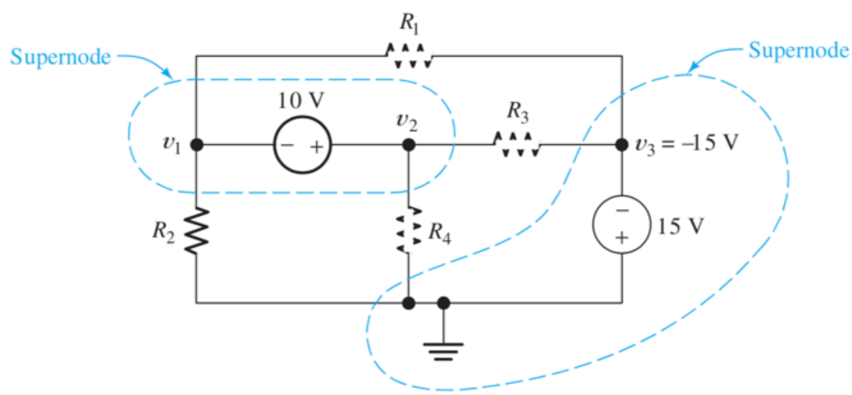
$x = {}^3/_3 = 1$, $y = {}^{-6}/_3 = -2$, and $z = {}^9/_3 = 3$

6.4 Nodal analysis with voltage sources, Supernodes

We often pick the reference node at one end of the source (and then we have one less unknown node voltage to solve.

Typical examples can be typically solved. Write current equations at nodes 1 and two.





If we try to write a current equation at node 1, we must include a term for current through the 10V source. We could assign an unknown, or...

Or we can create a "supernode" which we do by drawing a dashed line around several nodes, including the elements between them, and apply KCL more broadly

- Recall that KCL says that the net current flowing through any closed surface must be equal to zero. Thus, we apply KCL to the supernode.

$$\frac{v_1}{R_2} + \frac{}{v_1 - (-15)} R_! + \frac{v_2}{R_4} + \frac{v_2 = (-15)}{R_3} = 0 \qquad (6.1)$$

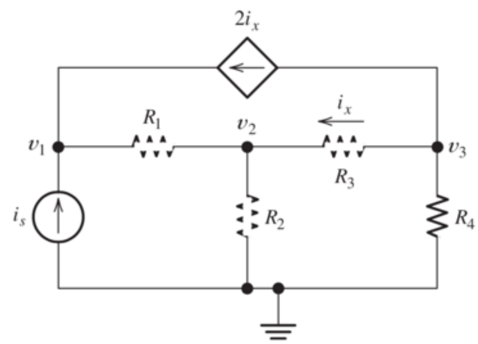
Now you might be tempeted to write another current equation for the other super node, however, youd find that the equation is equivalent to the one we just found. If we tried to solve by substitution, all the terms would drop out and wed see that the matrix was singular (the determinant is 0, it won't tell you anything new.

Instead we apply KVL, noting that $-v_1 - 10 + v_2 = 0$

Next, we find an expression for the controlling variable ix in terms of the node voltages. Notice that since in this case ix is the current flowing away from node 3 through R3, we can say ix = (v3 v2)/R3.

6.5 Nodal analysis with controlled sources

We approach it the same way, but we're mindful of the dependence



Write KCL equations at each node, including the current of the controlled source, just as if it were an ordinary current source

$$\frac{v_1 - v_2}{R_1} = i_s + 2i_x$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$
$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$

Next, we find an expression for the controlling variable ix in terms of the node voltages. Notice that since in this case ix is the current flowing away from node 3 through R3, we can say $ix = (v3 - v2)/R3$.

$$\frac{v_1 - v_2}{R_1} = i_s + 2\frac{v_3 - v_2}{R_3}$$
$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$
$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2\frac{v_3 - v_2}{R_3} = 0$$

So long as only three of those variable are unknown (say the voltages), then it can be solved by whatever method youd like.

6.6 Mesh analysis with current sources

6.7 Mesh analysis with controlled sources, Supermeshes

Chapter 7

Circuit analysis: IV. Circuit theorems

02/05/2019 Lecture 7.

7.1 Circuit theorems

Circuit theorems are ways of quickly describing, summarizing, analyzing circuit, based on some mathematical tricks, two of which well review today: Thevenin and Norton equivalent circuits.

- **Thevenin** — taking a complex circuit seen between two terminals and representing it as a resistor in series with a voltage source
- **Norton** — taking a complex bit of circuitry between two terminals and representing it as a current source in parallel with a current source

1. These only apply to "linear circuits" that is, circuits made of (ideal) resistors, capacitors, inductors, op-amps, filters
2. Non-linear elements include things like diodes, transistors, and some of the digital logic stuff well get into later in the semester.
3. It might pay us dividends for us to consider what we mean by linearity at this point in the semester as it will become very relevant in our upcoming analyses

7.2 Linearity

Comprises two separate yet equally important properties

1. Homogeneity — If the input (the excitation) is multiplied by a constant, the output (the response) is multiplied by the same constant

   $v = iR \to kv = kiR$

2. Additivity — The response to a sum of excitations is equal to the responses to each individual excitation

   $v_1 = i_1 R; v_2 = i_2 R; \to v = (i_1 + i_2)R = i_1 R_1 + i_2 R = v_1 + v_2$

Much of what we will do in this class is linear and much in life, given sufficient approximating, can be considered linear. Such relationships are useful, if not always strictly true (recall that even wires have resistivities and resistors have frequency ranges)

Note that since power is $i^2 R = v^2/R$, the relationship is a quadratic and thus our necessary assumptions of linearity are not applicable.

7.3 Superposition

- Until now, weve been considering everything in a circuit always “on”, but one of the things the linearity of the system allows us to do is to selectively turn on and off sources while we solve, so long as we sum them up in the end.

- This principle — **superposition** — states that the voltage (or current) through an element (in a linear circuit) is the algebraic sum of the voltages across (or current through) that element due to each independent source acting alone.

- To apply the superposition principle

  1. Turn off all independent source but one and find the output (voltage or current) due to that source [using any of the techniques youve previously learned]
  2. Do this for each independent source
  3. Find the total contribution [for any given element, or indeed for all of them] by adding all the contributions due to the independent sources
  4. (Leave in all dependent sources since they are controlled by circuit variables)

- To turn off a source:

  Voltage sources get replaced by a 0V source / a short circuit — **Turn voltage sources into wires**

  Current sources get replaced by a 0A source / an open circuit — **Cut current sources out**

Example 4.5

For the circuit in Fig. 4.12, use the superposition theorem to find i.

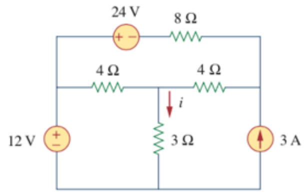


Figure 4.12
For Example 4.5.

Solution:
In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3\ \Omega$. Thus,

$$i_1 = \frac{12}{6} = 2\text{ A}$$

To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \tag{4.5.1}$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \tag{4.5.2}$$

Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$

To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \tag{4.5.3}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \tag{4.5.4}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

$$i_3 = \frac{v_1}{3} = 1\text{ A}$$

Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2\text{ A}$$

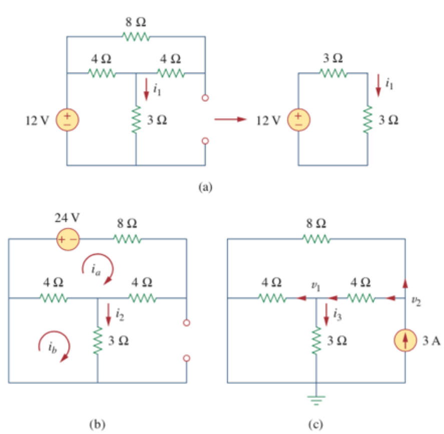


Figure 4.13
For Example 4.5.

7.4 Source transformation

Source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R or

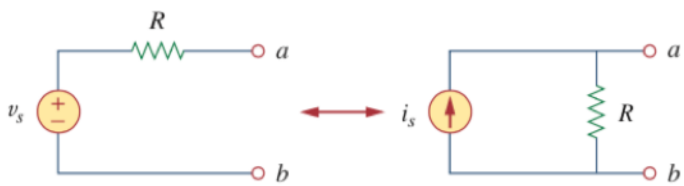


Figure 4.15
Transformation of independent sources.

vice versa.

- We can prove these two are equivalent between a-b by short circuiting the two-terminals.
- If the sources are turned off, the resistance at terminal a-b are the same (R)
- If the terminals are shorted, current flowing from a to be is $i_{ab} = Vs/R$ and $i_{ab} = i_s$. Thus $Vs/R = i_s$
- You can also replace dependent sources this way, provided youre careful, but well skip that analysis here and I wont encourage its use unless you personally feel comfortable with the material

7.5 Thevenin equivalents

- A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source, V_{th}, in series with a resistor, R_{th}

  V_{th} is equal to open-circuit voltage of original network, $V_{th} = V_{oc}$
- If we were to short the connection across terminals a and b, we can see (as we did previously), that $i_{sc} = V_{th}/R_{th}$

  Thus the short-circuit current is equal to the current for the original circuit and the Thevenin equivalent.
- These two facts allow us to say something rather interesting, namely that the **Thevenin resistance** of a circuit is equal to the ratio of its open-circuit voltage and its short-circuit current. $R_{th} = V_{oc}/I_{sc}$

- Thus, **to determine the Thevenin equivalent circuit**, we start by analyzing the original network for its open-circuit voltage and its short circuit current. [A more robust derivation of this is shown in your book in chapter 4.7]

- **To find the thevenin equivalent resistance**:

  *If there are **no** dependent sources*, turn off all independent sources in the network. R_{th} is the input resistance to the network between the two terminals of interest

  *If **there are** dependent sources*, still turn off all independent sources and apply superposition.

  Apply a voltage at the terminals and determine the resulting current. Then R_{th} is the ratio of applied voltage and elicited current v_o/i_o

  Or apply a current and determine the resulting voltage. Use which ever you feel comfortable with.

7.5.1 An example

Create thevenin equivalent circuit between a and b [R_L is a potentiometer and thus its value can change. We want to simplify our understanding of how all the complex stuff before it will act.]

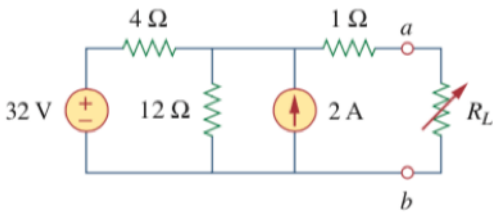


Figure 4.27
For Example 4.8.

1. Start by turning off the independent sources (32 V and 2 A), replacing them with a short- and an open-circuit respectively
2. This will yield our resistance as a 4 —— 12 + 1 = (4*12)/16 + 1 = 4 ohms
3. Next, identify what our V_{Th} would be
4. To find V_{Th} we can apply mesh analysis

$$324i1 - 12i1 + 12i2 = 0 \tag{7.1}$$
$$i2 = 2 \tag{7.2}$$
$$i1 = 0.5A \tag{7.3}$$
$$\tag{7.4}$$

   then plug it in for the voltage across the 12 ohm

$$V_{th} = 1_2(i_1 - i_1) = 30 \tag{7.5}$$

5. Or could use nodal analysis

$$(32V_{th})/4 + 2 = V_{Th}/12 \rightarrow V_{Th} = 30 \tag{7.6}$$

7.6 Norton equivalents

- A linear two-terminal circuit can be replaced by an equivalent circuit comprising a current source, I_N in parallel with a resistor R_N

  I_N is equal to the short-circuit current through the terminals

- To find the Norton equivalent resistance, we start the same way we did for R_Th — that is, short our voltage sources, open our current sources and find the resistance.

  Well then this should suggest to us that R_N is equal to R_Th, and indeed they are equal.

- The Norton current, I_N, is equal to the short-circuit current.

  So we merely find the current that would travel through the short

7.6.1 An example

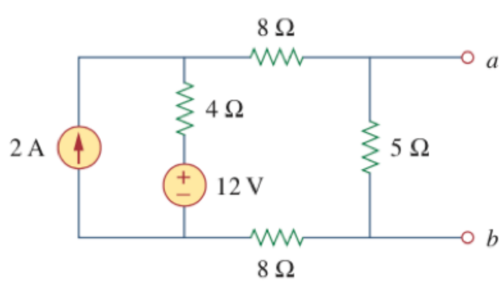


Figure 4.39
For Example 4.11.

1. We begin by zeroing all of our independent sources

   $R_N = 5||(8 + 4 + 8[+0]) = 5||20 = (5 * 20)/(5 + 20) = 4$ ohms

2. We can apply mesh analysis to find I_N

   We can ignore the 5-ohm resistor as its in parallel with a short and current will always seek the path of least resistance.

   Recall our bioimpedance example where with increasing frequency caused the current to flow down a different branch, because with greater frequency the capacitor went from acting like an open circuit (at low frequencies) to a short circuit (at high frequencies), thus bypassing the other branch altogether.

$$I_1 = 2 \tag{7.7}$$
$$I_2 = 124i_2 + 4i_1 8i_2 8i_2 \rightarrow 4(2)20(i_2) = -12 \tag{7.8}$$
$$I_2 = 1 = i_{sc} = I_N \tag{7.9}$$

3. Alternatively, we could have found the Thevinin voltage [meshing it up]

$$i3 = 2 \tag{7.10}$$
$$12 - 4(i4) + 4(i1)8(i4) - 5(i4)8(i4) \rightarrow 4(2)25(i4) = -12 \tag{7.11}$$
$$I4 = 0.8A \tag{7.12}$$

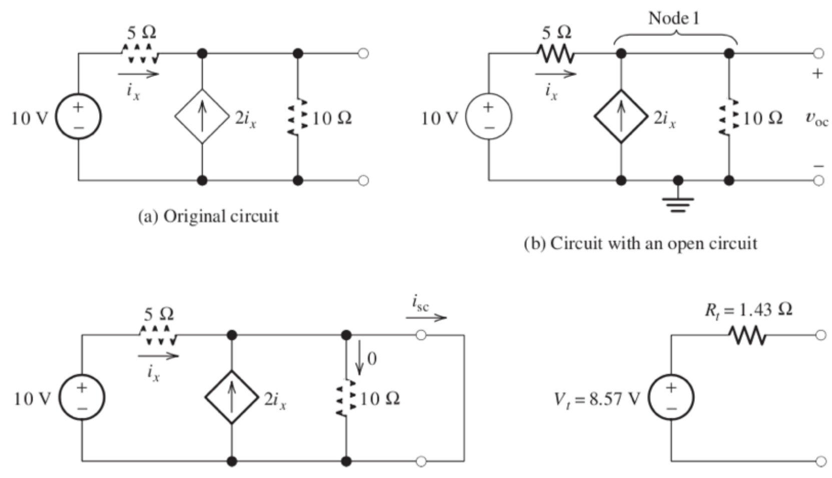
   Since we know 0.8 A goes through the 5 ohm resistor, and since we know that that 5 ohm resistor experiences the same voltage drop as the open circuit, we simple multiple 5*0.8 to give us, 4 V

4. Well since we know $R_N = R_T h$ is 4 and $V_T h$ is 4: $I_N = V_T h / R_T h = 1A$

5. Thus Norton and Thevenin circuits are two sides of the same source transformation coin

7.7 Equivalents with dependents

When weve got a dependent source, we cant just remove the sources and combine impedances. Instead, weve got to analyze the circuit to find the open-circuit voltage and the short-circuit current (and indeed such an approach will work in all cases, its just a little bit more work on our parts)



(a) Original circuit

(b) Circuit with an open circuit

(c) Circuit with a short circuit

(d) Thévenin equivalent

- Im personally a fan of finding the Thevenin open voltage because it just clicks with my brain a little better, but if you like the Norton short-circuit current, feel to take that approach
- So I apply nodal analysis

$$I_x + 2i_x V_{oc}/10 = 0 \rightarrow 3i_x = V_{oc}/10 \quad (7.13)$$

$$\text{Next we write an expression for our controlling variable, ix} \quad (7.14)$$

$$10V_{oc} = i_x(5) \rightarrow ix = (10V_{oc})/5 \quad (7.15)$$

$$\text{We can substitute this into our previous equation} \quad (7.16)$$

$$3 * (10 - V_{oc})/5 = Voc/10 \rightarrow \mathbf{V_{oc} = V_{Th} = 8.57} \quad (7.17)$$

- Now we can consider the short-circuit conditions.

$$I_x + 2i_x i_{sc} = 0 \rightarrow 3i_x = i_{sc} \quad (7.18)$$

$$I_x = 10/5 = 2 \rightarrow \mathbf{I_{sc} = 3i_x = I_N = 6} \quad (7.19)$$

- From this, we can take the ratio of the open circuit voltage and the short circuit current and find the equivalent resistance of the network via Ohm's law: 8.57 V / 6 A = 1.43 ohms

7.8 A step-by-step procedure

1. Perform two of these:

   Determine the open-circuit voltage $V_t = v_{oc}$

   Determine the short-circuit current $I_n = i_{sc}$

   Zero the independent sources and find the Thevenin resistance R_t looking back into the terminals. Do not zero dependent sources

2. Use the equations $V_t = R_t In$ to compute the remaining value

3. The Thevenin equivalent consists of a voltage source V_t in series with R_t

4. The Norton equivalent consists of a current source I_n in parallel with R_t

A Glorified Quiz I

02/07/2019

Part II

Systems

Chapter 8

The Laplace Transform: I. What it is and why it is important

02/12/2019 Lecture 10.

8.1 How do we know our world looks like this?

8.2 Euler's identity / Euler's formula

8.3 The Laplace transform

8.4 The Laplace transform of 1

8.5 The s-plane

8.6 The linearity of the Laplace transform

8.7 The Laplace transform of e^{at}

8.8 The Laplace transform of dx/dt

8.9 The Laplace transform in RLC circuits

8.9.1 Resistors

8.9.2 Inductors

8.9.3 Capacitors

8.9.4 RLC

8.10 Two important places, zeros and poles

Chapter 9

The Laplace Transform: II. How to use it

02/14/2019 Lecture 11.

9.1 The inverse Laplace transform

9.2 The Laplace transform of sin

9.3 The Laplace transform of t^n

9.4 Some applicability

Chapter 10

Circuits as ODEs: I. First-order

02/21/2019 Lecture 12.

10.1 Source-free RC circuits

10.1.1 One resistor, one capacitor

10.1.2 Two or more resistors and/or capacitors

10.2 Source-free "active" circuits

10.3 First-order systems with sources

10.4 Several singular functions

10.4.1 Unit step function, $u(t - t_0) = 1, t > t_0$

The Laplace transform of the unit step function

10.4.2 Unit impulse function, $\delta(t) = du(t)/dt$

Its "sifting" abilities

The Laplace transform of the unit impulse function

10.4.3 Unit ramp function, $r(t) = \int u(t)dt$

The Laplace transform of the unit impulse function

Chapter 11

Circuits as ODEs: II. Second-order

02/26/2019 Lecture 13.

11.1 A series RLC circuit

Chapter 12

System response: I. Convolution; Homework III

02/28/2019 Lecture 14.

12.1 An introduction to thinking in systems

Viewing everything as a "system".

12.1.1 Domains of interest, of command

12.1.2 The time-domain, or: our typical realm

12.1.3 The frequency-domain, or: our new realm

12.1.4 The s-domain, or: our magical realm

12.2 Inputs and outputs

12.3 Somewhere in the between

12.4 Convolution in the time-domain

12.5 Multiplication in the frequency- and s-domain

Chapter 13

System response: II. Stability

03/12/2019 Lecture 15.

13.1 An introduction

13.1.1 What do we mean by stability?

13.2 Undamped, $\zeta = 0$

13.3 Underdamped, $0 < \zeta < 1$

13.4 Overdamped, $\zeta > 1$

Part III

& Signals

Chapter 14

System response: III. The frequency domain

03/14/2019 Lecture 16.

Chapter 15

System response: IV. Filters

03/19/2019 Lecture 17.

Chapter 16

System response: V. Feedback; Homework IV

03/21/2019 Lecture 18.

Exam II

03/26/2019

Part IV

in Biomedical Engineering

Chapter 17

Bioelectricity: I. Passive properties

03/28/2019 Lecture 19.

17.1 Modeling biological material with a simple circuit, $R_1 + (R_2||C)$

17.2 Resistance-Reactance Plane

17.3 What can we do with this information?

Chapter 18

Bioelectricity: II. Active properties

04/02/2019 Lecture 20.

Chapter 19

Bioelectricity: III. Measurement

04/04/2019 Lecture 21.

Chapter 20

Digital circuits: I. Discretization

04/09/2019 Lecture 22.

Chapter 21

Digital circuits: II. Logic; Homework V

04/11/2019 Lecture 23.

Chapter 22

Happenstance: A few BME specific situations

04/16/2019 Lecture 24.

Chapter 23

Circumstance: A few BME specific standards

04/18/2019 Lecture 25.

Chapter 24

A philosophy of circuits, systems, and signals; Homework VI

04/23/2019 Lecture 26.

Exam III

04/26/2019