Contents

0.1 How can I print off and use this document?

Frankly, in just about any way thats useful to you. I am going to try something here, where I will try to make more or less the entirety of the notes associated with the Winter 2019 semester of BIOMEDE 211, Circuits, Systems, and Signals in Biomedical Engineering, to you, dear reader.

Please dont plagiarize this. If you were raised right, you ought to know what that is. If youd like my judgment on any sort of action, my opinions can be laid bare.

The first assignment I am giving you (worth 4% of your grade and which must be completed by the end of the semester) is to figure out where this document is located online, download it, print it off, sign your name to it, and get it to me. If you know who I am, I would expect a competent engineer to find that without much to-do about it. Start with Google, go from there. Further, for those in the class, BIOMEDE 211, Winter 2019, you must join Github and make at least four substantive contributions to this repository. The term all you engineers (and lawyers) cant wait to parse is substantive to which I will always enter a judgment which I deem final in this class, and I am ever in favor of beneficence over stricture. So, just help out the class in a way you think is helpful and watch those around you do the same. Failure to contribute to this living document by the end of the semester for those in this class will result in a loss of up to 4% of one's total grade outright.

0.2 How to contribute to GitHub

Follow these general steps to propose a change to this online document:

1. Create a GitHub account

   This should be rather self-explanatory. Use your e-mail account and verify it to be able to edit. You should proceed with the following steps while logged onto your account.

2. Find Dr. Belmont's GitHub page and go to the biomede-211-w19 repository ("repo"). Then click on the biomede-211-w19.tex file.

3. Edit the file

   You will find a small pencil icon on the right side of the page. Click on this to create your own branch ("forking"), and edit the file as you wish.

4. Propose file change

   After making your changes, you should scroll to the bottom of the page, find the message box that says, 'Propose file change', and fill it out. The first line should say what you have updated and can be explained in the description.

5. Create pull request

   After finishing your file, you will be brought to a page that displays what you have modified on the original document. Press the green 'Create pull request' button to let Dr. Belmont know that you want to create a change. Once he has approved via his own GitHub account, your changes should now be in the updated master branch!

0.3 Who comprises this class and how can they be reached?

0.3.1 The Captain at the helm

Barry Belmont
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0.3.2 The A-Team

Annabelle St. Pierre
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astpierr@umich.edu

Alice Tracey
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0.3.3 You, yourselves

In this class, we will be learning a lot from each other. You are encouraged to learn from one another. You are encouraged to talk to one another. You are are encouraged to share ideas and at times data. You are not encouraged and are hereby expressly forbidden to submit the work of another as your own. If you get help from others, you will put their name on it somewhere. Too much of this and you are committing plagiarism, not enough and you are committing fraud. Please be honest and let's all learn together.

0.4 The policies of this class

Part I

Circuits

Chapter 1

I. Potential, current, energy, conservation

01/10/2019

Contents

1.1 What is electricity?

1.
2.
3.

1.2 Charge

1. Charge is the property of matter that causes it to experience a force when placed in an electromagnetic field; measured in coulombs (C)
2.
3. **How many electrons are needed to form one coulomb?** (What is the weight of all those electrons?)
4. One byte is eight bits. Bits are essentially a single electron stored in a transistor. **If we were to take all the electrons from one terabyte of well distributed information (equal number of ones and zeros), how many coulombs would we have?**

1.3 Current

1. The time rate of charge (charged particles) in motion; measured in amperes (A); defined mathematically as

$$i := dq/dt \tag{1.1}$$

   where i is current, q is charge, and t is time

2. Conversely, the total charge transferred over time can be expressed as

$$Q := \int_{t_0}^{t} idt \tag{1.2}$$

3. 1 ampere is equal to 1 coulomb/second
4. Direct current, "DC"
5. Alternative current, "AC"

1.3.1 The directionality of current

Ultimately, the direction in which we say ”current” flows is largely arbitrary. As arbitrary as choosing one type of charge and calling it “positive” and another “negative”. The reason it doesn’t matter is that the only consequence of having chosen a “wrong direction” for the current in a given analysis is that we have to switch the sign of the value. Thus, 3 amps in one direction is *the exact same thing* as -3 in the opposite direction.

1. Thanks to Benjamin Franklin we say that current is

   i. **Positive in the direction in which positively charged particles flow** and

   ii. **Negative in the direction in which negatively charged particles**

   iii. We also now know that current results primarily from the movement of negatively charged particles (electrons) and therefore our convention is wrong in one sense, though convenient and entrenched enough that were not liable to change it in our life time (besides, the math comes out the same, and the actual flow of electrons will only matter to us in a few special circumstances, diodes)

1.3.2 The at times deadly serious nature of current

Much of the point of learning this material here is its eventual application by our hands or by the hands of those we work with. Before we put any of this stuff in our hands, we should probably know what is and is not safe.

1. 1 mA
2. 10 mA
3. 100 mA
4. 1000 mA

1.3.3 The “speed” of current

A possible misconception is that the electrons inside a wire travels at the speed of light. The speed of current is actually relatively slow. If one were to imagine an electron starting at the wire next to a light switch in an average classroom, it would take a very long period of time for it to travel to the light itself. The light’s immediate reaction to a switch is due to a ”hose effect"; the electrons inside the wire push other electrons in the direction opposite to the [conventional] current. This cascade of electrons is what happens close to the speed of light, not the electron movement itself.

1.4 Potential (difference)

1. The amount of work needed to move a unit of (positive) charge from a reference point to another point [without producing an acceleration]).

2. Potential is measured in “volts” and is often called “voltage”. In this class we will endeavor to avoid such a term as it can be very confusing to talk about potential as if there were such a *thing* as voltage.

3. Defined as
$$v := \frac{dw}{dq} \tag{1.3}$$

4. Potential describes the *potential* to do something. Increasing the potential is akin to increasing the height of a cliff. The height does not *do* anything other than increase what can be done on the drop. If potential is the cliff’s height, charge would be pebbles you’d drop off the side, and current describes how fast those pebble fall.

5. In this class, and for the vast vast majority of electrical engineering work, we care about the *difference* in potential. One element held at 100 billion volts and another held at 100 billion + 1 volts has a *potential difference* of 1 V, which is less than a single AA battery.

6. Some typical voltages to be aware of

   Consumer level batteries (AA, AAA):

   Car batteries:

   The “mains” (levels provided by power companies to consumers):

   Power transmission lines:

1.5 Power

1. The time rate of expending or absorbing energy.

2. Quantifies the rate of energy transfer.

3. Mathematically:

4. Measured in watts: 1 W =

5. **Passive sign convention**:

1.6 Energy

1.
2.
3.
4.

1.7 Conservation

Here, as elsewhere, things will be conserved. In electrical circuits there are two laws of conservation that will matter most for us:

1. **The Conservation of Mass.**
2. **The Conservation of Energy.**

In evaluating circuits, the main focus of the first third of this class, it will be the application of these two conservative laws that will enable us to "solve" them. That is, by understanding (1) how energy is generated and used and (2) how charges move around in closed loops ("circuits") we will be able to predict the behavior of the myriad electrical systems which may cross our paths.

1.8 Worksheet

1.8.1 Problem 1, constant charge through a cross-section

How much charge passes through a cross-section of a conductor in 60 seconds if a DC current value is measured at 0.1 mA? **Solution**

1.8.2 Problem 2, arbitrary charge through a cross-section

Determine the total charge entering a terminal between $t = 0$ seconds and $t = 10$ seconds if the current (in amps) passing through is

$$i(t) = \frac{1}{\sqrt{5t+2}}. \tag{1.4}$$

Solution

1.8.3 Problem 3, a "tera"ble puzzle

Approximately how much current is necessary to transmit one terabyte of information in an hour? **Solution**

1.8.4 Problem 4, power necessary to run a pacemaker

A cardiac pacemaker will provide approximately 5,000 J of energy over 5 years. Determine the capacity of a 5 V lithium battery necessary to drive this pacing such that only 40% of its energy is spent over that time. **Solution**

1.8.5 Problem 5, energy needed to excite a neuron

A colleague of yours has been in their lab ginning up new neurons. You, as their resident electrical expert, are tasked with determining the energy consumed by the cell. If the current and voltage variations are found to be functions of time ($t \geq 0$)

$$i(t) = 3t \tag{1.5}$$
$$v(t) = 10e^{6t} \tag{1.6}$$

determine the energy consumed between 0 and 2 ms. **Solution**

1.8.6 Problem 6, a thump to the chest

(a) A typical defibrillator delivers 200-1000 V in less than 10 ms. How much current is needed to deliver 120, 240, and 360 Joules?
(b) A human heart ways about 300 grams. From approximately how high of a cliff would one have to drop a heart such that the impact was equivalent to the energy delivered to someone's chest from a defibrillator? **Solution**

Chapter 2

II. Circuit elements

01/15/2019

2.1 Active v. passive

Active elements are electrical components that can generate their own energy. Examples of active elements would include generators, batteries, and transistors. Due to the fact that they produce energy, active elements are often known as sources. In contrast to this, passive circuit elements do not produce their own energy. Examples of passive elements would be resistors, capacitors, and inductors.

2.2 Ohm's Law and what it means

Ohm's Law is concerned with the relationship between voltage, or potential difference, and current across a conductor. The potential difference across a conductor is proportional to the current flowing thorugh the conductor with the proportionality constant being denoted as R, or resistance. This can be expressed as:

$$V := iR \tag{2.1}$$

This essentially states that the drop in potential across the conductor, or resistor, is equivalent to the current flowing through the conductor and its resistance. When considering impedance, the equation can be modified to state:

$$V := iZ \tag{2.2}$$

2.3 Sources

2.4 Resistors

Electrical circuit elements that resist the flow of electricity and electric charge Passive two terminal element (does not generate its own energy)

2.4.1 Resistance, R

Resistance = R and is measured in Ohms 1 = 1V/1A Resistance is the subset of a broad phenomenon known as impedence (Z), which is an element's total opposition to a current when a potential is applied.

2.4.2 Resistivity, ρ

Resistivity: based on 3 parameters

1. $\rho \rightarrow$ materials ability to resist flow (Ωm)
2. $l \rightarrow$ length of the element (m)
3. $A \rightarrow$ cross sectional area of element (m^2)

$$R = \rho \frac{l}{A} \tag{2.3}$$

Resistivity and Materials Materials that exhibit a high resistivity are things that are classified as conductors. Conductors are commonly associated with metals such as silver. In contrast, materials that exhibit a low resistivity are called insulators. Some common examples of insulators would be polymers such as plastics, ceramics and glass.

Here is a link to a video that further explains the concepts of resistivity and resistance: <https://www.youtube.com/watch?v=4rsswT_Rv1M>.

2.4.3 Resistance, R

2.4.4 Conductance

2.5 Capacitors

2.5.1 Its time varying behavior

2.5.2 Charge accumulation

2.5.3 A simple example

2.6 Inductors

2.7 Impedance

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2.8 Equivalent impedance

2.8.1 Impedances in general

2.8.2 Resistors

2.8.3 Capacitors

2.8.4 Delta-Wye (Δ-Y) transformations

2.8.5 A few examples

2.9 Grounds

2.10 Conductors

2.11 Operational amplifiers

2.12 Diodes

2.13 Switches

2.14 Transistors

2.15 Transformers

2.16 Worksheet

2.16.1 Problem 1, expressing power in ohms

Utilizing Ohm's law, express units of power to include ohms.

Solution

2.16.2 Problem 2, a couple toaster based problems

A toaster draws 2 A at 120 V. What is its resistance?

Solution

How much current is drawn by a toaster with a resistance of 10 Ω at 110 V?

Solution

2.16.3 Problem 3, currently conducting power

In the circuit shown, calculate the current, i, the conductance, G, and the power, p.

Solution

2.16.4 Problem 4, conductance of a sodium channel

Conductance (G/A) of a sodium channel of a cell membrane at a specific time is 10 mS/cm^2. If the channel length as 100 nm, what is its conductivity?

Solution

2.16.5 Problem 5, resistance of a simple tissue

Determine the resistance of a homogenous and isotropic tissue with a cross-sectional area which can be described by the functions $y = 8 - x^2$ from $x = -2$ cm to $x = +2$ cm, a length of 10 cm (parallel to the z-axis), and a resistivity of 80 Ωm.

Solution

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III. Operational amplifiers

01/17/2019

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01/22/2019

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4.2.1 Kirchhoff's Current Law

4.2.2 Kirchhoff's Voltage Law

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01/24/2019

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6.2 Nodal analysis with voltage sources, Supernodes

6.3 Nodal analysis with controlled sources

6.4 Mesh analysis with current sources

6.5 Mesh analysis with controlled sources, Supermeshes

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02/12/2019

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10.7 The Laplace transform of e^{at}

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The Laplace transform of the unit step function

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Its "sifting" abilities

The Laplace transform of the unit impulse function

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The Laplace transform of the unit impulse function

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