Notes on BIOMEDE 211, or: Circuits, Systems, & Signals in Biomedical Engineering

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January 22, 2019

Contents

0.1 How can I print off and use this document?

Frankly, in just about any way thats useful to you. I am going to try something here, where I will try to make more or less the entirety of the notes associated with the Winter 2019 semester of BIOMEDE 211, Circuits, Systems, and Signals in Biomedical Engineering, to you, dear reader.
Please dont plagiarize this. If you were raised right, you ought to know what that is. If youd like my judgment on any sort of action, my opinions can be laid bare.
The first assignment I am giving you (worth 4% of your grade and which must be completed by the end of the semester) is to figure out where this document is located online, download it, print it off, sign your name to it, and get it to me. If you know who I am, I would expect a competent engineer to find that without much to-do about it. Start with Google, go from there. Further, for those in the class, BIOMEDE 211, Winter 2019, you must join Github and make at least four substantive contributions to this repository. The term all you engineers (and lawyers) cant wait to parse is substantive to which I will always enter a judgment which I deem final in this class, and I am ever in favor of beneficence over stricture. So, just help out the class in a way you think is helpful and watch those around you do the same. Failure to contribute to this living document by the end of the semester for those in this class will result in a loss of up to 4% of one's total grade outright.

0.2 How to contribute to GitHub

Follow these general steps to propose a change to this online document:

1. Create a GitHub account

   This should be rather self-explanatory. Use your e-mail account and verify it to be able to edit. You should proceed with the following steps while logged onto your account.

2. Find Dr. Belmont's GitHub page and go to the biomede-211-w19 repository ("repo"). Then click on the biomede-211-w19.tex file.

3. Edit the file

   You will find a small pencil icon on the right side of the page. Click on this to create your own branch ("forking"), and edit the file as you wish.

4. Propose file change

   After making your changes, you should scroll to the bottom of the page, find the message box that says, 'Propose file change', and fill it out. The first line should say what you have updated and can be explained in the description.

5. Create pull request

   After finishing your file, you will be brought to a page that displays what you have modified on the original document. Press the green 'Create pull request' button to let Dr. Belmont know that you want to create a change. Once he has approved via his own GitHub account, your changes should now be in the updated master branch!

0.3 Who comprises this class and how can they be reached?

0.3.1 The Captain at the helm

Barry Belmont
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0.3.2 The A-Team

Annabelle St. Pierre
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0.3.3 You, yourselves

In this class, we will be learning a lot from each other. You are encouraged to learn from one another. You are encouraged to talk to one another. You are are encouraged to share ideas and at times data. You are not encouraged and are hereby expressly forbidden to submit the work of another as your own. If you get help from others, you will put their name on it somewhere. Too much of this and you are committing plagiarism, not enough and you are committing fraud. Please be honest and let's all learn together.

0.4 The policies of this class

Part I

Circuits

Chapter 1

I. Potential, current, energy, conservation

01/10/2019

Contents

1.1 What is electricity?

1. A form of energy resulting from the existence of charged particles
2. The physical phenomena arising from the existence, presence, and motion of charged particles
3. Rather ill-defined in common vernacular we will generally avoid its use

1.2 Charge

1. Charge is the property of matter that causes it to experience a force when placed in an electromagnetic field; measured in coulombs (C)
2. Charges are found in nature in discrete, integral multiples of electronic charge: e = -1.602 x 10^{-19} C (the charge of one electron)
3. **How many electrons are needed to form one coulomb?** (What is the weight of all those electrons?)
4. One byte is eight bits. Bits are essentially a single electron stored in a transistor. **If we were to take all the electrons from one terabyte of well distributed information (equal number of ones and zeros), how many coulombs would we have?**

1.3 Current

1. The time rate of change of charge charges (charged particles) in motion; measured in amperes; defined mathematically as

$$i := dq/dt \tag{1.1}$$

   where i is current, q is charge, and t is time

2. Conversely, the total charge transferred over time can be expressed as

$$Q := \int_{t_0}^{t} idt \tag{1.2}$$

3. 1 ampere is equal to 1 coulomb/second
4. Direct current, “DC”, is current that remains constant with time
5. Alternating current, “AC”, is current that varies sinusoidally with time

1.3.1 The directionality of current

Ultimately, the direction in which we say "current" flows is largely arbitrary. As arbitrary as choosing one type of charge and calling it "positive" and another "negative". The reason it doesn't matter is that the only consequence of having chosen a "wrong direction" for the current in a given analysis is that we have to switch the sign of the value. Thus, 3 amps in one direction is *the exact same thing* as -3 in the opposite direction.

1. Thanks to Benjamin Franklin we say that current is

   i. **Positive in the direction in which positively charged particles flow** and

   ii. **Negative in the direction in which negatively charged particles**

   iii. We also now know that current results primarily from the movement of negatively charged particles (electrons) and therefore our convention is wrong in one sense, though convenient and entrenched enough that were not liable to change it in our life time (besides, the math comes out the same, and the actual flow of electrons will only matter to us in a few special circumstances, diodes)

1.3.2 The at times deadly serious nature of current

Much of the point of learning this material here is its eventual application by our hands or by the hands of those we work with. Before we put any of this stuff in our hands, we should probably know what is and is not safe.

1. 1 mA, you will feel
2. 10 mA, you will really feel
3. 100 mA, you will likely die
4. 1000 mA, you will definitely die

1.3.3 The "speed" of current

A possible misconception is that the electrons inside a wire travels at the speed of light. The speed of current is actually relatively slow. If one were to imagine an electron starting at the wire next to a light switch in an average classroom, it would take a very long period of time for it to travel to the light itself. The light's immediate reaction to a switch is due to a "hose effect"; the electrons inside the wire push other electrons in the direction opposite to the [conventional] current. This cascade of electrons is what happens close to the speed of light, not the electron movement itself.

1. **The *signal* of electrical current (that is electromagnetic radiation) travels anywhere between about 50-99% the speed of light** (dependent on a number of conditions) depending upon the material through which it travels (based on a dielectric behavior known as permittivity)

2. **The drift velocity of electrons** within a copper wire is 25 μm/s, so how does anything ever turn on?

3. **The hose effect** - The electrons at the light switch will almost certainly never pass through a light bulb, but they will move around and bump into their neighbors which bump into their neighbors which bump into their neighbors, etc., until it causes the electrons nearest the light to pass through. This is how water at a spigot is able to push water at the end of a hose.

1.4 Potential (difference)

1. The amount of work needed to move a unit of (positive) charge from a reference point to another point [without producing an acceleration]).

2. Potential is measured in "volts" and is often called "voltage". In this class we will endeavor to avoid such a term as it can be very confusing to talk about potential as if there were such a *thing* as voltage.

3. Defined as
$$v := \frac{dw}{dq} \tag{1.3}$$

4. Potential describes the *potential* to do something. Increasing the potential is akin to increasing the height of a cliff. The height does not *do* anything other than increase what can be done on the drop. If potential is the cliff's height, charge would be pebbles you'd drop off the side, and current describes how fast those pebble fall.

5. In this class, and for the vast vast majority of electrical engineering work, we care about the *difference* in potential. One element held at 100 billion volts and another held at 100 billion + 1 volts has a *potential difference* of 1 V, which is less than a single AA battery.

6. Some typical voltages to be aware of

   Consumer level batteries (AA, AAA): 1.5 V (DC); 9 V (DC)

   Car batteries: 12 V (DC)

   The "mains" (levels provided by power companies to consumers): 110-120 V (AC) and 220-240 V (AC) in America

   Power transmission lines: 110-1200 kV (AC), transformers are used to step up and down the potential before used by consumers

1.5 Power

1. The time rate of expending or absorbing energy.

2. Quantifies the rate of energy transfer.

3. Mathematically:
$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i \tag{1.4}$$

4. Measured in watts: 1 W = 1 $\frac{\mathrm{J}}{\mathrm{s}}$ = 1 $\frac{\mathrm{N \cdot m}}{\mathrm{s}}$ = 1 $\frac{\mathrm{kg \cdot m^2}}{\mathrm{s^3}}$ = 1 V · 1 A

5. **Passive sign convention**: If current enters through the positive terminal of an element, $p = +vi$; if current enters through the negative terminal of an element, $p = -vi$.

1.6 Energy

1. The capacity to do work.

2. Measured in joules.

3. $E = \int \frac{dw}{dt} dt \to$ power x time

4. $\mathrm{J} = \frac{\mathrm{kg \cdot m^2}}{\mathrm{s^2}} = \mathrm{N \cdot m} = \mathrm{Pa \cdot m^3} = \mathrm{W \cdot s} = \mathrm{C \cdot V}$

1.7 Conservation

Here, as elsewhere, things will be conserved. In electrical circuits there are two laws of conservation that will matter most for us:

1. **The Conservation of Mass.** The conservation of mass means that no mass can be added to or removed from a circuit without being accounted for. Put differently, in a closed system (the type we will concern ourselves with here) no mass is added or removed.

   In electrical circuits, the mass we care the most about are the charges whipping around. Thus, for us, *the amount of charge within a circuit must remain constant.*

2. **The Conservation of Energy.** The conservation of energy means that no energy can be added to or removed from a circuit without being accounted for. Put differently, in a closed system (the type we will concern ourselves with here) no energy is added or removed.

   In electrical circuits, the energy we care the most about is the potential provided by sources and depleted by other elements in the circuits. Thus, for us, *the sum of potentials within a circuit must equal zero.*

In evaluating circuits, the main focus of the first third of this class, it will be the application of these two conservative laws that will enable us to "solve" them. That is, by understanding (1) how energy is generated and used and (2) how charges move around in closed loops ("circuits") we will be able to predict the behavior of the myriad electrical systems which may cross our paths.

1.8 Worksheet

1.8.1 Problem 1, constant charge through a cross-section

How much charge passes through a cross-section of a conductor in 60 seconds if a DC current value is measured at 0.1 mA? **Solution**

1.8.2 Problem 2, arbitrary charge through a cross-section

Determine the total charge entering a terminal between $t = 0$ seconds and $t = 10$ seconds if the current (in amps) passing through is

$$i(t) = \frac{1}{\sqrt{5t+2}}. \tag{1.5}$$

Solution

1.8.3 Problem 3, a "tera"ble puzzle

Approximately how much current is necessary to transmit one terabyte of information in an hour? **Solution**

1.8.4 Problem 4, power necessary to run a pacemaker

A cardiac pacemaker will provide approximately 5,000 J of energy over 5 years. Determine the capacity of a 5 V lithium battery necessary to drive this pacing such that only 40% of its energy is spent over that time. **Solution**

1.8.5 Problem 5, energy needed to excite a neuron

A colleague of yours has been in their lab ginning up new neurons. You, as their resident electrical expert, are tasked with determining the energy consumed by the cell. If the current and voltage variations are found to be functions of time ($t \geq 0$)

$$i(t) = 3t \tag{1.6}$$

$$v(t) = 10e^{6t} \tag{1.7}$$

determine the energy consumed between 0 and 2 ms. **Solution**

1.8.6 Problem 6, a thump to the chest

(a) A typical defibrillator delivers 200-1000 V in less than 10 ms. How much current is needed to deliver 120, 240, and 360 Joules?
(b) A human heart ways about 300 grams. From approximately how high of a cliff would one have to drop a heart such that the impact was equivalent to the energy delivered to someone's chest from a defibrillator? **Solution**

Chapter 2

An introduction: II. Circuit elements

Contents

01/15/2019

2.1 Active v. passive

1. Active elements are capable of generating energy while passive components cannot
2. **Active**: generators, batteries, operational amplifiers, "sources"
3. **Passive**: resistors, capacitors, inductors, i.e., most circuit elements

2.2 Ohm's Law and what it means

Ohm's Law is concerned with the relationship between voltage, or potential difference, and current across a conductor. The potential difference across a conductor is proportional to the current flowing thorugh the conductor with the proportionality constant being denoted as R, or resistance. This can be expressed as:

$$V := iR \tag{2.1}$$

This essentially states that the drop in potential across the conductor, or resistor, is equivalent to the current flowing through the conductor and its resistance. When considering impedance, the equation can be modified to state:

$$V := iZ \tag{2.2}$$

2.3 Sources

1. **An ideal independent source** is an active element that provides a specified value of potential or current, regardless of other circuit elements.

   Batteries and power supplies may be approximated as ideal potential sources.

2. **An ideal dependent (or controlled) source** is an active element in which the source quantity is controlled by another quantity (such as potential, current, temperature, measured resistance, etc.).

3. **An ideal potential source** will produce any current required to ensure that the terminal voltage stated is satisfied.

4. **An ideal current source** will produce any voltage required to ensure that the terminal current as stated is satisfied

5. Symbols

   Voltage-controlled voltage source, VCVS

   Current-controlled voltage source, CCVS

   Voltage-controlled current source, VCCS

   Current-controlled current source, CCCS

2.4 Resistors

Resistors are electrical (circuit) elements that resist the flow of electric charge (current); passive two-terminal components that implement a defined/"constant" resistance; meant to reduce current flow and change potential

2.4.1 Resistance, R

1. **Resistance** is the physical property describing an element's ability to resist current and is most often represented by R

2. Resistance is measured in "ohms", Ω, which is equivalent to 1 V/A

3. Resistance is one half of a broader physical phenomenon known as **"impedance"** - the property describing an element's ability to *impede*

current. Impedance is typically represented by Z, which we'll explore more thorough in a bit.

2.4.2 Resistivity, ρ

1. The resistance of an element (such as a resistor) depends on three things:

   Resistivity, ρ, of the material comprising the element, which is the *material*'s ability to resist the flow of charges; measured in ohm-meters

   Length, l, of the element; measured in meters

   Area, A, of the cross-section of the element; measured in m^2

   Such that $R = \rho\frac{l}{A}$

   What units are we left with?

   What are the effects of length and area?

2. **Materials with low resistivity** are generally called (and treated as) "conductors" as they are able to more effectively *conduct* the motion of electrical charges than materials with high resistivity

3. **Materials with very high resistivity** are generally used as "insulators" as they prevent the flow of current through them and thus *insulate* the current within prescribed bounds, such as with a copper wire with plastic wrapped around it.

Here is a link to a video that further explains the concepts of resistivity and resistance: `<https://www.youtube.com/watch?v=4rsswT_Rv1M>`.

2.4.3 Conductance

1. The inverse of resistance is conductance, G, which describes the ability of an element to conduct current

2. Measured in Siemens

3. Allows us express Ohm's law slightly differently, $i = Gv$, which says that the current generated through an element by a potential is directly proportional to some constant, namely conductance.

4. The material specific property **conductivity**, σ is measured in S/m

2.5 Capacitors

1. Passive two-terminal components that store energy in an electric field; introduces capacitance to a circuit.

2. Can be thought of as two conductive plates sandwiching a "dielectric" material. Essentially it is two "conductors" separated by a "non-conductive region".

3. When a capacitor is attached across a source, an electric field develops across the dielectric causing a net positive charge to collect on one conductor and a net negative charge to collect on the other.

4. We can define the capacitance of an element mathematically as

$$C = Q/V \tag{2.3}$$

   where C is capacitance in farads, Q is positive or negative charge on each conductor, and V is the potential between them

5. We can also represent capacitance by the voltage-based rate of charge accumulation: $C = dQ/dV$.

2.5.1 Its time varying behavior

Unlike resistors, capacitors have a *time-varying* element to that. That is, since $C = Q/V$, $V = Q/C$.

If we then recall Equation 1.2, we can write the time-dependent potential relationship of a capacitor

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + V(t_0) \tag{2.4}$$

We can also recall[1] Equation 1.1, and represent the time-dependent current relationship as

$$I(t) = \frac{dQ(t)}{dt} = C\frac{dV(t)}{dt} \tag{2.5}$$

[1] We could also take the derivative of the equation preceding this one and do a little rearrangement. As it turns out, these physical relationships are rather codified and thus can be gotten out by any number of means.

2.5.2 Charge accumulation

1. While charges accumulate on a capacitor, no current flows *through* the capacitor.

2. **Well, then why use them? After awhile won't the current just stop?** Yes, indeed it will – in a DC circuit!

3. The capacitor will become "charged" over time, eventually reaching the same potential as that established across it, e.g., by a source. Since potential only ever travels down potential gradients, if the capacitor and the source (say, a battery) are at the same potential, no current will flow.

4. Thus, a fully charged capacitor will act as an "open" circuit, while an uncharged capacitor will act as a "short" circuit.

2.5.3 A simple example

If we consider Ohm's law for a simple RC circuit (one in which a source, a resistor, and a capacitor are in series), we can describe the system by

$$V_0 = v_R(t) + v_C(t) \tag{2.6}$$

$$V_0 = i(t)R + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau \tag{2.7}$$

Taking the derivative of both sides:

$$0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t) \tag{2.8}$$

$$0 = RC\frac{di(t)}{dt} + i(t) \tag{2.9}$$

$$i(t) = \frac{V_0}{R} \cdot e^{-t/RC} \tag{2.10}$$

$$v(t) = V_0\left(1 - e^{-t/RC}\right) \tag{2.11}$$

$$Q(t) = C \cdot V_0\left(1 - e^{-t/RC}\right) \tag{2.12}$$

2.6 Inductors

1. Passive two-terminal components that store energy in a magnetic field

2. Can be thought of as an insulated wire wound into a coil around a core (which may either be filled with a material or left open to the environment)

3. Behavior can be modeled as $L = \frac{\Phi}{I}$, where L is the inductance, Φ is the magnetic flux generated by a current, I.

4. By Faraday's law of induction, voltage induced by a change in magnetic flux through a circuit is

$$v = \frac{d\Phi}{dt} \tag{2.13}$$

which we can rewrite as

$$v = \frac{d}{dt}(Li) = L\frac{di}{dt} \tag{2.14}$$

5. In this class, at this level, and for most biomedical applications you're liable to experience in your tenure, you will not work extensively with inductors. However, you should be able to recall at least this much at a moment's notice to be able to ascertain a system's behavior.

2.7 Impedance

1. The measure of opposition a circuit element presents to a current when a potential is applied. (It is measured in ohms.)

2. It is "complex" in two sense of the term. First, the actual phenomenon itself comprises complex numbers; that is, there is both a "real" and an "imaginary" component.

   The real component is known as resistance, R

   The imaginary component is known as reactance, X

   Impedance can be represented as a combination of either

   Resistance and reactance: $\mathbf{Z} = R + \jmath X$, where $\mathbf{Z}$ is impedance, R is resistance, and X is reactance, or

   Magnitude and phase: $\mathbf{Z} = |Z|e^{\jmath\theta}$, where $|Z|$ is the magnitude of the impedance vector, $\mathbf{Z}$, and θ is the phase of said vector (i.e., the delay between current and potential). Phase, θ is equivalent to $\tan^{-1}(X/R)$

3. Impedance is also complex in the sense that it is complicated. The impedance of an object is a factor of many parameters including permittivity, geometry, quantum states, thermal stability, etc. Let us not view this sort of complexity as an impediment to our understanding of impedance.

4. The inverse of impedance is **admittance**, Y, and comprises a real component, **conductance**, G, (which is the inverse of resistance) and an imaginary component, **susceptance**, B (which is the inverse of reactance). (It is measured in Siemens.)

   $\mathbf{Y} = G + \jmath B$

2.7.1 A quick note on "imaginary" numbers

The term "imaginary" is an unfortunate name for an excellent mathematical tool. All the imaginary operator – in this class represented by $\jmath = \sqrt{-1}$ – is is a type of number "orthogonal" to our "real" numbers. Imaginary numbers are no less "real" than real numbers. Unfortunately, they aren't necessarily the most intuitive to our little mammalian brains and thus we must be trained to work with them. However, as we will see in this class, they can be quite useful.

2.8 Equivalent impedance

1. It will often be more convenient to think about the impedance which a component burdens a system with (or the conductance which it affords) rather than its resistance. **Therefore, we need to begin to think in terms of equivalent impedances as we start to evaluate circuits.**

2. Recall Ohm's law

   Resistors, $v = iR \quad \rightarrow Z_{eq,R} = R$

   Capacitors, $v = \frac{1}{C}\int idt \quad \rightarrow Z_{eq,C} = \frac{1}{\jmath\omega C}$

   Inductors, $v = L\frac{di}{dt} \quad \rightarrow \jmath\omega L$

   I want to plant a flag here for you to notice the relationship between the $\jmath\omega$ terms from the capacitor and inductor and the corresponding derivative and integral forms of current in the Ohm's law representation. This will become very important once we get into the Laplace and Fourier transforms.

3. We must also recognize that few will be the circuits comprising but a single element. As such, we should know how to find the equivalent impedance of many elements.

2.8.1 Impedances in general

Series

$$Z_{eq,series} = Z_1 + Z_2 + Z_3 + ... \tag{2.15}$$

Parallel

$$\frac{1}{Z_{eq,parallel}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + ... \tag{2.16}$$

A special case to remember. When dealing with only two elements:

$$Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \tag{2.17}$$

2.8.2 Resistors

Series

$$R_{eq,series} = R_1 + R_2 + R_3 + ... \tag{2.18}$$

Parallel

$$\frac{1}{R_{eq,parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \tag{2.19}$$

2.8.3 Capacitors

Series

$$\frac{1}{C_{eq,series}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... \tag{2.20}$$

Parallel

$$C_{eq,parallel} = C_1 + C_2 + C_3 + ... \tag{2.21}$$

2.8.4 Delta-Wye (Δ-Y) transformations

Going from Delta to Wye

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \tag{2.22}$$
$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c} \tag{2.23}$$
$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \tag{2.24}$$

Going from Wye to Delta

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1} \tag{2.25}$$
$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2} \tag{2.26}$$
$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3} \tag{2.27}$$
$$\tag{2.28}$$

2.8.5 A few examples

Example 1 Find the equivalent resistance, if a resistor $R_1 = 10$ kΩ is connected in parallel to $R_2 = 3.3$ kΩ.

Solution. $R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{(10)(3.3)}{10+3.3} = 2.48$ kΩ

Example 2 Find the equivalent resistance of three parallel-connected resistors of equal value. If $R = R_1 = R_2 = R_3 = 10$ kΩ, what's R_{eq}?

Solution. Recall, Equation 2.19

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow 3R_{eq} = R \rightarrow R_{eq} = \frac{R}{3} \rightarrow R_{eq} = \frac{10k}{3} = 3.33k\Omega \tag{2.29}$$

Example 3 Four resistors are connected in parallel. $R_1 = 10$ kΩ, $R_2 = 1$ kΩ, $R_3 = 5$ kΩ, and $R_4 = 3$ kΩ. Calculate their equivalent resistance.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \tag{2.30}$$
$$\frac{1}{R_{eq}} = \frac{1}{10k} + \frac{1}{1k} + \frac{1}{5k} + \frac{1}{3k} \tag{2.31}$$
$$= 612.3\ \Omega \tag{2.32}$$

2.9 Grounds

1. A reference point in an electrical circuit from which potentials are measured
2. A common return path within a circuit

2.10 Conductors

1. Allow from the transmission of electrical energy
2. Serve to connect circuit elements
3. Also known as wires and traces
4. Within circuit schematics we must be mindful of "junctions" and "jumps" in conductors

2.11 Operational amplifiers

1. Active components that deliver the amplified difference between two of its terminals
2. Will be discussed at length in the next class and along with resistors, capacitors, and sources, will be among the primary circuit components we work with

2.12 Diodes

Two-terminal circuit elements that allow current to flow only in one direction

2.13 Switches

Make/break/change circuit paths (thereby diverting current or removing potential)

1. Single pole, single throw, SPST
2. Single pole, double throw, SPDT
3. Double pole, single throw, DPST
4. Double pole, double throw, DPDT

2.14 Transistors

2.15 Transformers

1. Transfer electrical energy between circuits using induction
2. Allows for the effective transmission of power and the stepping up/down of potential
3. Crucial for the transmission, distribution, and utilization of AC

2.16 Worksheet

2.16.1 Problem 1, expressing power in ohms

Utilizing Ohm's law, express units of power to include ohms.

Solution

2.16.2 Problem 2, a couple toaster based problems

A toaster draws 2 A at 120 V. What is its resistance?

Solution

How much current is drawn by a toaster with a resistance of 10 Ω at 110 V?

Solution

2.16.3 Problem 3, currently conducting power

In the circuit shown, calculate the current, i, the conductance, G, and the power, p.

Solution

2.16.4 Problem 4, conductance of a sodium channel

Conductance (G/A) of a sodium channel of a cell membrane at a specific time is 10 mS/cm^2. If the channel length as 100 nm, what is its conductivity?

Solution

2.16.5 Problem 5, resistance of a simple tissue

Determine the resistance of a homogenous and isotropic tissue with a cross-sectional area which can be described by the functions $y = 8 - x^2$ from $x = -2$ cm to $x = +2$ cm, a length of 10 cm (parallel to the z-axis), and a resistivity of 80 Ωm.

Solution

Chapter 3

An introduction: III. Operational amplifiers

01/17/2019

Contents

3.1 Some details

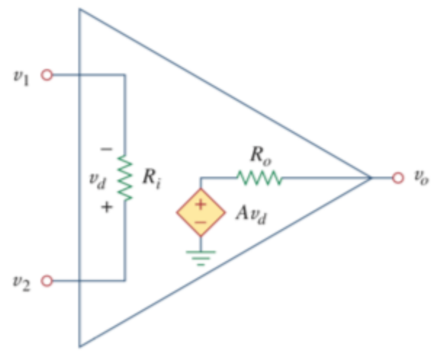


Figure 5.4
The equivalent circuit of the nonideal op amp.

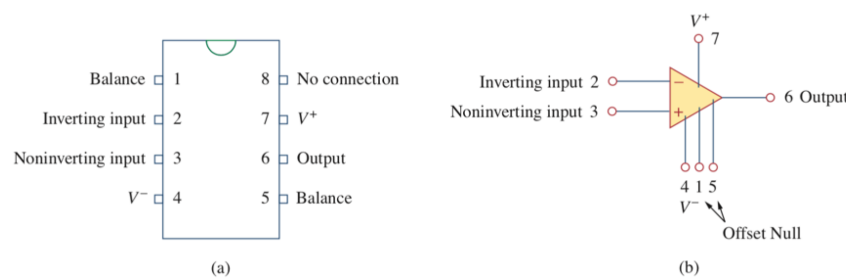


Figure 5.2
A typical op amp: (a) pin configuration, (b) circuit symbol.

1. Behaves like a voltage-controlled voltage source
2. They can amplify, sum, subtract, multiply, differentiate, integrate
3. They are active circuit elements
4. Though they have somewhat more complicated internal workings, we typically represent them in electrical circuits as a triangle with three (sometimes five) very important terminals:

   An inverting input (sign, typically represented up top for convenience, but it need not be)

   A non-inverting input (+ sign, typically on bottom)

   An output

3.2 Some rules

There are **three important features of ideal operational amplifiers** that we must understand thoroughly. These are things worth stamping in your brain.

1. **Infinite open-loop gain.** The “A” of the gain is infinitely large such that any difference in voltages V_1 and V_2 causes an enormously large output voltage. As much as is being supplied. (The real value of gain in most operational amplifiers is between 10^5 and 10^8.)

2. **Infinite input impedance.** Current cannot travel between the inverting and non-inverting terminals. (Really, the impedance is between 10^5 and $10^1 3$ ohms and is often signal dependent.)

3. **Zero output impedance.** There is no loss transmitting a voltage difference to the output. (Really is about 10-100 ohms and is chip dependent.)

3.3 Some conveniences

1. With infinite input impedance, no current can flow into or out of the terminals and hence i_1 and i_2 are equal to 0.

2. Since no current flows across the terminals, the terminals are at equal potential. Hence “$v_1 = v_2$”.

3.4 Some examples

3.4.1 Inverting amplifier

We will apply the conservation of mass at this point to solve our equations. This is among the simplest and most effective ways to add gain to a circuit. So much so that you will use it again and again and again in life and especially in labs

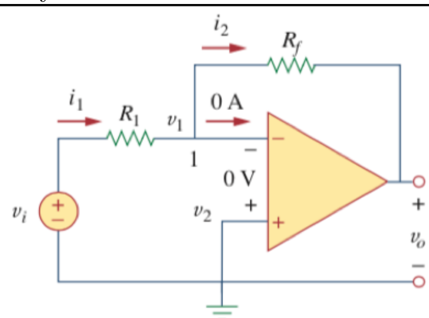


Figure 5.10
The inverting amplifier.

A key feature of the inverting amplifier is that both the input signal and the feedback are applied at the inverting terminal of the op amp.

We apply KCL at the node for v1

1. I1 i2 i3 = 0
2. I3 = 0
3. I1 i2 = 0
4. I1 = i2
5. I1 = (vi v1)/R1
6. I2 = (v1 - Vo)/Rf
7. (vi v1)/R1 = (v1 - Vo)/Rf
8. V1 = V2 = 0
9. Vi/R1 = -Vo/Rf

10. Vo = -Rf/R1 * Vi

11. R2/R1 is our gain, gain factor.

3.4.2 Non-inverting amplifier

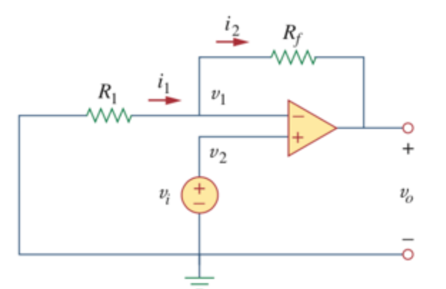


Figure 5.16
The noninverting amplifier.

Again, the name might imply what it does. It will amplify our input signal without inverting it.

We can again perform Nodal analysis.

1. I1 i2 i3 = 0

2. I3 = 0, since no current enters the non-inverting input

3. I1 = i2

4. (Vg v2)/R1 = (v2 Vo)/Rf

5. v2/R1 = (vi Vo)/Rf

6. Vo = (1 + Rf/R1) * Vi

3.4.3 Voltage follower

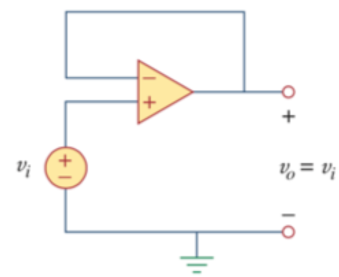


Figure 5.17
The voltage follower.

What if we didnt have any resistors? → Vi = v2 = v1 = Vo → $Vi = Vo$

3.4.4 Summing amplifier

3.4.5 Differential amplifier (as homework)

Chapter 4

Circuit analysis: I. Nodal analysis

01/22/2019

Contents

4.1 Nodes and branches

1. **A branch** is any two-terminal element. (examples: Resistor, Capacitor, Wire, etc.)

    ***A branch** is any two-terminal element. What are some two-terminal elements weve learned?*

2. **A node** (junction) is a point of connection between two or more branches.

    ***A node** is a point of connection between two or more branches. Often indicated by a dot. What else have we called a node? A junction.*

3. A loop is **independent** if at least one branch is not part of any other independent loop.

    ***A loop** is any closed path within a circuit. A closed path formed by starting at a node, passing through a set of nodes, returning to the starting node without passing through any node more than once.*

4.1.1 The Seven Bridges of Konigsberg

The Knigsberg bridge problem asks if the seven bridges of the city of Knigsberg (left figure; Kraitchik 1942), formerly in Germany but now known as Kaliningrad and part of Russia, over the river Preger can all be traversed in a single trip without doubling back, with the additional requirement that the trip ends in the same place it began.

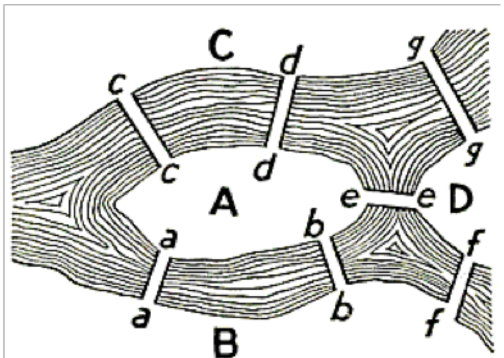


FIGURE 98. *Geographic Map: The Königsberg Bridges.*

4.1.2 Independence

A loop is independent if it contains at least one branch which is not a part of any other independent loop

1. Each of the loops in the circuit at right are independent
2. Independent loops lend themselves to sets of equations to be solved!

4.1.3 Fundamental theorem of network topology

Fundamental theorem of network topology states that the number of branches, b, must equal the sum of the independent loops, l and nodes, m minus one, that is

$$b = l + n - 1 \tag{4.1}$$

With this fundamental theorem we can also redefine/refine our definition of series and parallel.

- **Series** — two or more elements share a single node and thereby carry the same current
- **Parallel** — connected to the same two nodes and thereby have the same voltage across them (potential difference)

Practice Problem 2.4

How many branches and nodes does the circuit in Fig. 2.14 have? Identify the elements that are in series and in parallel.

Answer: Five branches and three nodes are identified in Fig. 2.15. The 1-Ω and 2-Ω resistors are in parallel. The 4-Ω resistor and 10-V source are also in parallel.

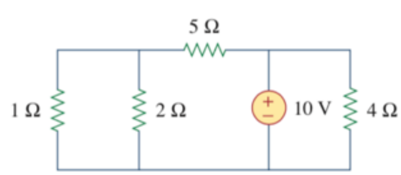


Figure 2.14
For Practice Prob. 2.4.

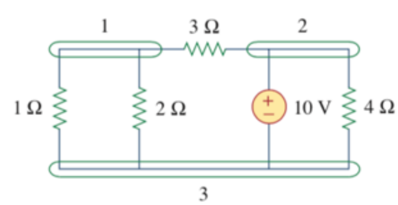


Figure 2.15
Answer for Practice Prob. 2.4.

4.2 Kirchhoff's Laws

I am not personally a fan named laws of nature, especially ones which are mere recapitulations of already perfectly good laws. Thus do we introduce Kirchhoffs laws, known as

- Kirchhoffs current law ("KCL")
- Kirchhoffs voltage law ("KVL")

These are, as far as Im concerned, mere restatements of *the conservation of mass* and *the conservation of energy*, respectively.

4.2.1 Kirchhoff's Current Law

Kirchhoff's current law states that any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

- Put differently, the algebraic sum of currents entering a node (or any closed boundary) is zero
- For those mathematically inclined among us, that is: $\sum_{x=1}^{n} i_x = 0$
- Another way this often gets stated is by saying that the algebraic sum of charges within a system cannot change and is thus sometimes referred to as "the conservation of charge".
- Well since all of our charge carriers are merely particles (electrons, protons, ions, etc.), this is just another layer over the top of the underlying law which is that mass cannot be created or destroyed.
- However, Kirchhoff's formulation of this law (conserving charge, mass) is useful in circuits as it gives us a great tool, being able to say that current going in is equal to current going out
- For the figure $i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0 \rightarrow i_1 + i_3 + i_4 = i_2 + i_5$
- KCL forms the basis of a technique well spend the next couple of lectures on known as nodal analysis because we evaluate the current going into and coming out of nodes

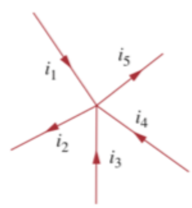


Figure 2.16
Currents at a node illustrating KCL.

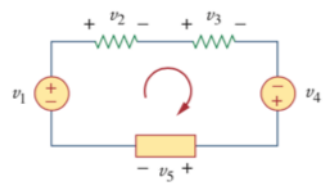


Figure 2.19
A single-loop circuit illustrating KVL.

4.2.2 Kirchhoff's Voltage Law

Kirchhoff's voltage law states that the sum of electrical potential differences (voltage) around any closed network is zero

- That is, for any closed path (loop), the sum of voltages is zero.
- Mathematically: $\sum_{m=1}^{M} v_m = 0$, where M is the number of voltage drops (caused by circuit elements) in the loop and vm is the mth voltage drop
- v1 + (-v2) + (-v3) + v4 v5 = 0 v1 + v4 = v2 +v3 + v5 iv. The sum of voltage rises = the sum of voltage drops

4.2.3 A few examples

1. Simple 1 Vs 1 R circuit; Vs = 10, R = 1 kohm, I = 0.01 A / 10 mA
2. Simple 1 Vs 2 R in series; Vs = 10, R = 1 kohm, I = 0.005 A / 5 mA

   What does KCL tell us? (Current is the same through resistors).
3. Simple 10 mA source, 2 R (1 kohm) in parallel; What is the voltage?

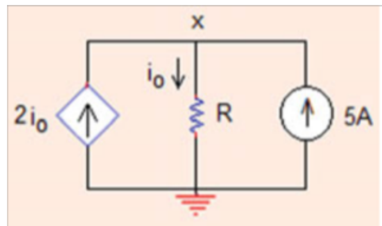
   $i_1 + (-i_2) + (-i_3) = 0] \rightarrow i_1 = i_2 + i_3 \rightarrow i_1 = V/R_1 + V/R_2$

   $0.01 = V/1000 + V/1000 \rightarrow 0.01 = 2V/1000 \rightarrow 0.010/2 * 1000 = 5$ V

   Nodes are at the same voltage!
4. 10 mA up, 5 mA down, 2 R (1 kohm), Rl (500 ohm), all in parallel. What is the current through load Rl? [have someone come to the board and solve]

   When current sources are in parallel they add together

5. For the circuit shown below, use KCL to find the remaining branch currents



2.13 For the circuit in Fig. 2.77, use KCL to find the branch currents I_1 to I_4.

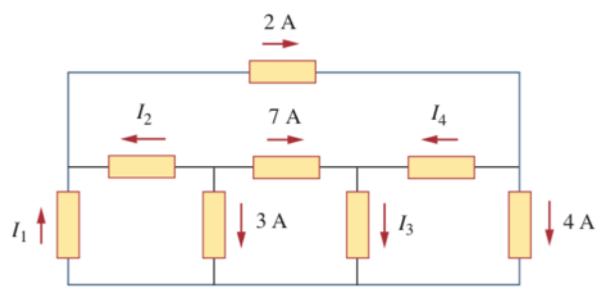


Figure 2.77
For Prob. 2.13.

4.3 Nodal analysis

Nodal analysis — a general circuit analysis technique in which we try to determine the potential difference between nodes by applying KCL and KVL (in my experience, usually focusing a bit more on KCL)

Your textbook offers the following description of the technique which seems pretty good to me:

1. Select a node as a reference. Assign voltages $(v_1, v_2, ..., v_{n-1})$ for the remaining n-1 nodes, all of which will be referenced with respect to the reference node.
2. Apply KCL to each of the n-1 nonreference nodes. Use Ohms law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations (system of equations) for each unknown node voltage.
4. It's as easy as that! But, well, actually, it can get a little hairy once you start to apply it in earnest.

4.3.1 The procedure

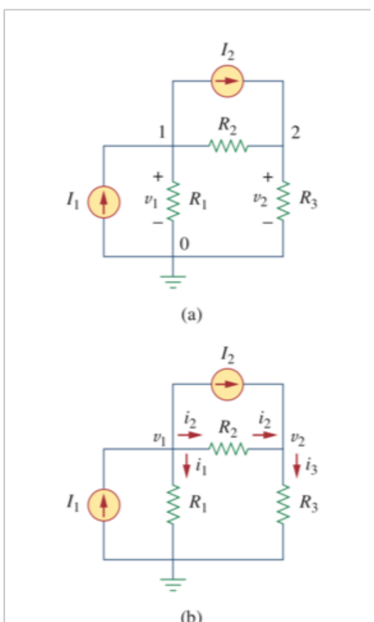


Figure 3.2
Typical circuit for nodal analysis.

1. Begin by putting a reference, usually a "ground"

   This ground can be one of two sorts: (1) **Earth ground** in which ultimately the whole earth is used as the reference point; or **Chassis ground** in which the case of the device in which the circuit is in will act as a reference as it will presumably be sufficiently large as to serve fine [this is also partly the reason why you can "feel" a MacBook charge up its charger does not utilize a traditional "earth ground"

   Either will suffice for our purposes here

2. Next we label all the nodes.

   How many branches, nodes, and loops?

   5 branches, 3 nodes, 3 loops. Satisfies our network condition.

   You can give them any label you want, but I find working your way up from the ground in a clockwise manner and numbering them sequentially is a good habit to get into.

   Keep in mind that we typically set our reference node to have a voltage of 0. We can actually set it to be anything wed like, but the math is often easier if we just make it 0.

3. Then we apply KCL to each nonreference node in the circuit.

   At node 1, $IA - i1 - i2 - IB \rightarrow IA = IB + i1 + i2$

   At node 2, $IB + i2 - i3IB + i2 = i3$

   Once weve got that, now its a matter of applying Ohms law. Thought typically written as V = iR, it is perhaps more helpful to write its full extension here and note that (Va Vb) = iR

   $I = (Va - Vb)/R$

   Thus we can state

   $I1 = (v1 - v0)/R1 \rightarrow I1 = G1(v1 - v0)$

   $I2 = (v1 - v2)/R2 \rightarrow I2 = G2(v1 - v2)$

4.4 Solving simultaneous equations

4.4.1 Cramer's Rule

Chapter 5

Circuit analysis: II. Mesh analysis; Homework I

01/24/2019

5.1 Mesh analysis

5.2 Steps of mesh analysis

5.3 Writing mesh equations directly in matrix form

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Circuit analysis: III. Supernodes and supermeshes

01/29/2019 Lecture 6.

6.1 Nodal analysis with an independent current source

6.2 Nodal analysis with voltage sources, Supernodes

6.3 Nodal analysis with controlled sources

6.4 Mesh analysis with current sources

6.5 Mesh analysis with controlled sources, Supermeshes

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01/31/2019 Lecture 7.

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02/12/2019

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10.3 The Laplace transform

10.4 The Laplace transform of 1

10.5 The s-plane

10.6 The linearity of the Laplace transform

10.7 The Laplace transform of e^{at}

10.8 The Laplace transform of dx/dt

10.9 The Laplace transform in RLC circuits

10.9.1 Resistors

10.9.2 Inductors

10.9.3 Capacitors

10.9.4 RLC

10.10 Two important places, zeros and poles

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12.2 Source-free "active" circuits

12.3 First-order systems with sources

12.4 Several singular functions

12.4.1 Unit step function, $u(t - t_0) = 1, t > t_0$

The Laplace transform of the unit step function

12.4.2 Unit impulse function, $\delta(t) = du(t)/dt$

Its "sifting" abilities

The Laplace transform of the unit impulse function

12.4.3 Unit ramp function, $r(t) = \int u(t)dt$

The Laplace transform of the unit impulse function

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15.2 Undamped, $\zeta = 0$

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