Event-triggered for Rotating Consensus with Double-Integrator Multi-Agent Systems ⋆

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Abstract: This paper investigates the rotating consensus problem for a class of double-integrator multi-agent systems, where the communication networks are directed. Firstly, for convenience, we transform the original rotating consensus problem in complex field to real field by a rotation matrix. Secondly, an event-triggered mechanism with the ability to predict the system state based on known triggered state is introduced, without requiring continuous communication among agents. Thirdly, based on the rotating consensus problem and the proposed event-triggered mechanism, a distributed control protocol is developed, in which the rotating consensus will be reached with an exponential convergence rate. Then, it is shown that, with the proposed event-triggering mechanism, a strictly positive lower bound between any two consecutive triggering instants can be guaranteed, that is, Zeno-freeness can be guaranteed. Finally, the simulation example is provided to illustrate the effectiveness of the proposed control protocol.

Keywords: Event-triggered, rotating consensus, double-integrator multi-agent system, directed graph.

1. INTRODUCTION

In recent years, multi-agent dynamical systems have been studied extensively due to their wide applications in various industrial and military field, see Olfati-Saber (2006); Yu et al. (2011); Zhang and Han (2013); Yu et al. (2009); Panagou et al. (2015); Dong et al. (2016) and references therein. It is worth mentioning that many typical research directions in this field have been considered, including consensus Olfati-Saber et al. (2007); Ren (2008), leader-follower tracking Tang et al. (2015); He et al. (2016), coverage control Song et al. (2011), and competition behaviors Meng et al. (2016). With above issues, the existing control techniques always rely on the assumption that the communication networks are continuous. In other words, the continuous communication is required to reach consensus, which is not realistic due to the unavoidable delays, packet dropouts and packet disorders between communication Lin and Jia (2010a); Yang et al. (2013).

To address such a concern, other approaches have been proposed recently. With widely application background, one approach is to utilize sampled control strategy Guo et al. (2014); Guan et al. (2012). However, with sampled data control schemes, the agent receives and sends information with a regular frequency. Particularly, with a small sampled period, the requirement on equipment is too strict, which lead to great waste of energy, increase burden on communication equipment and even consequently shorten the lifespan of the system. On the other hand, if the chosen period is too large to save the energy, the considered control protocol will show pretty poor performance, unfortunately, even cause the unstability of system.

To overcome the above shortcomings, researchers put a lot of effort on the idea of triggered control Dimarogonas et al. (2011); Yue et al. (2012); Anta and Tabuada (2010). On the one hand, with self-triggered control strategies Mazo and Tabuada (2008, 2009); Gommans et al. (2014), the control input is updated with the predicted next triggered time ahead of current time, using the current measurements. On the other hand, under controlling by event-triggered controllers Dimarogonas et al. (2011); Yue et al. (2012), control input is updated by reacting to excessive deviations of the decision variable from an acceptable value, i.e., when a continuously monitored triggering condition is violated. We mainly focus on the latter approach in this paper.

All the papers mentioned above considered the event-triggered consensus control problem of multi-agent systems. The rotating consensus control is one of the most important problems in cooperative control of multi-agent systems Lin and Jia (2010b); Li et al. (2018); Huang et al. (2019), with their broad application value. In Lin and Jia (2010b), the authors got necessary and sufficient condition on rotating consensus and the sufficient condition for rotating formation problem. In Li et al. (2018), the rotating consensus is achieved with some nonuniform communication delays for an undirected graph. In Huang et al. (2019), the rotating formation problem is studied for double-integrator dynamics systems, where the

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nonuniform communication delays were considered. As far as the authors know, the rotating consensus with event-triggered control has not been studied before. Then, we consider the event-triggered rotating consensus for multi-agent systems with double-integrator dynamics under a directed graph. General event-triggered consensus control could not be used to this case directly due to that fact that the considered dynamic is in complex area. Thus, event-triggered rotating consensus control problem still remains technically challenging.

In this paper, we consider the event-triggered rotating consensus control problem for multi-agent systems. Unlike Cheng and Ugrinovskii (2016); Dimarogonas et al. (2011); Fan et al. (2013), the systems is related to complex area. Due to the difficulty on dealing with problem in complex area, we convert the original systems Lin and Jia (2010b) into real area by a rotation matrix. Furthermore, we add an exponential prediction part in event-triggered mechanism and control protocol, which reduce the consumption of energy. For directed system interconnections, we propose sufficient conditions for the design of controllers which guarantee that the rotating consensus are contained. By defining an appropriate Lyapunov candidate, it is concluded that the rotating consensus will be achieved with an exponential convergence rate, where the exponential convergence rate can be estimated with detailed calculation. Additionally, with the proposed event-triggered control protocol, the system does not exhibit Zeno behavior. These results are the main contributions of the paper.

The rest of this paper is organized as follows: section 2 includes some preliminaries; the problem formulation is introduced in section 3; the main results are given in section 4, where the convergence rate is given and the Zeno behavior is excluded; in section 5, a numerical example is provided to illustrate the proposed algorithm; the conclusions are given in section 6 finally.

2. PRELIMINARIES

Let $R^{n\times m}$ and R^n be, respectively, the set of $n \times n$ real matrices and $n \times 1$ real column vector. Let 1_n denotes the $n \times 1$ real column vector of all ones. I_m represents the m dimensional identity matrix. 0 represents zero vectors or zero matrices with appropriate dimension. x^T denotes the transposition of x. $\mathrm{diag}(d_1, \cdots, d_n)$ is a diagonal matrix whose diagonal entries are $d_1, \cdots,$ and d_n and all off-diagonal entries are zero. $\otimes$ denotes the Kronecker product. We let $\|x\|$ represent the Euclidean norm of vector x, and $\|M\| = sup_{x\neq 0}(\|Mx\|/\|x\|)$, where M is a matrix. It is said that $M > 0$ if matrix M is a positive definite matrix and $M < 0$ if matrix M is a negative definite matrix.

Define $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ as a directed graph with N agents, where $\mathcal{V} = \{1, 2, ..., N\}$ is the nodes set, $\mathcal{E} \subset \{(i, k) : i, k \in \mathcal{V}\}$ is the edges set and $\mathcal{A} = [a_{ik}] \in R^{N\times N}$ is the adjacency matrix of graph $\mathcal{G}$. Node i can obtain information from node k if $a_{ik} > 0$. The neighbors set of agent i is defined as $\mathcal{N}_i = \{k \in \mathcal{V} : (i, k) \in \mathcal{E}\}$. The Laplacian matrix L of a graph $\mathcal{G}$ is defined as $L = D - \mathcal{A}$, where $D = \mathrm{diag}\{d_1, d_2, ..., d_N\}$, $d_i = \sum_{k\in\mathcal{N}_i} a_{ik} = \sum_{k=1}^{N} a_{ik}$.

Then, we provide some Assumptions and Lemmas which will be used later.

Assumption 2.1. The directed graph $\mathcal{G}$ is strongly connected.

Lemma 2.1. (Lu and Chen (2006)). Under Assumption 2.1, there exists a vector $s = [s_1, s_2, \cdots, s_N]^T$, with $s_i > 0$, $i = 1, \cdots, N$, $s^T L = 0$. Furthermore, let $S = \mathrm{diag}(s_1, s_2, \cdots, s_N)$. Then $\hat{L} = \frac{SL+L^TS}{2}$ is a symmetric matrix.

Lemma 2.2. (Yu et al. (2009)). Under Assumption 2.1, the following inequality holds:

$$\gamma_L = \min_{x^T s=0, x\neq 0} \frac{x^T \hat{L} x}{x^T S x} > 0,$$

where $\hat{L} = \frac{SL+L^TS}{2}$, $s = [s_1, s_2, \cdots, s_N]^T$, and $S = \mathrm{diag}(s_1, s_2, \cdots, s_N)$, with $s_i > 0$, $i = 1, \cdots, N$, $s^T L = 0$, and $\sum_{i=1}^{N} s_i = 1$.

3. PROBLEM FORMULATION

3.1 Rotating Consensus

Consider a group of N agents, and the ith agent has the following double-integrator dynamics model:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= \omega R v_i(t) + u_i(t), \end{aligned} \tag{1}$$

where $R = R(\frac{\pi}{2}) = \begin{bmatrix} \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is the rotation matrix, ω is the angular velocity of agent i, $x_i(t), v_i(t), u_i(t) \in R^2$ represent the position, velocity and the control input of the ith agent at time t, respectively, with the initial conditions $x_i(s) = x_i(0)$, $s \in (-\infty, 0]$, $v_i(s) = v_i(0)$, $s \in (-\infty, 0]$.

The following definition is given to illustrate the concept of rotating consensus.

Definition 3.1. Consider the multi-agent system (1) with the directed graph $\mathcal{G}$. Develop a distributed event-triggered control protocol, such that, for any finite $x_i(0)$, $v_i(0)$, $i = 1, \cdots, n$, if the following conditions are satisfied

$$\begin{aligned} &\lim_{t\to\infty} [x_i(t) - x_k(t)] = 0 \\ &\lim_{t\to\infty} [v_i(t) - v_k(t)] = 0 \\ &\lim_{t\to\infty} [\dot{v}_i(t) - \omega R(\frac{\pi}{2}) v_i(t)] = 0, \end{aligned}$$

it is said that the rotating consensus problem is solved.

Remark 3.1. To simplify the analysis below, we assume $\omega = 1$ and the general case can be discussed in a similar way.

3.2 Event-triggered control protocol

Let $z_i(t) = [x_i^T(t), v_i^T(t)]^T$, we can rewrite (1) into the following compact form:

$$\dot{z}_i(t) = A z_i(t) + B u_i(t), \tag{2}$$

where $A = \begin{bmatrix} 0 & I_2 \\ 0 & R \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}$.

Consider the following distributed event-triggered control protocol for agent i as:

$$u_i(t) = K\sum_{j=1}^{N}[e^{A(t-t^j_{k_j})}z_j(t^j_{k_j}) - e^{A(t-t^i_{k_i})}z_i(t^i_{k_i})], \quad (3)$$

where $K = \mu B^T P$, μ is a positive real, $P \in R^{4\times 4}$ is a positive definite matrix, which will be designed later, $t^i_{k_i}$ is the k_ith triggered time instance for agent i.

We define the following measurement error:

$$e_i(t) = e^{A(t-t^i_{k_i})}z_i(t^i_{k_i}) - z_i(t), \quad (4)$$

where $e^{A(t-t^i_{k_i})}z_i(t^i_{k_i})$ is a prediction based on the $z_i(t^i_{k_i})$.

If the current triggering time instant is denoted by t^i_k , the next triggering time t^i_{k+1} can be determined by the following triggering mechanism:

$$t^i_{k+1} = \inf\{t > t^i_k | \|e_i(t)\|^2 - \alpha_i e^{-\theta_i t} > 0\}, \quad (5)$$

where $\alpha_i > 0$, $\theta_i > 0$.

By (4), we transform (3) into

$$\begin{aligned} u_i(t) &= K\sum_{j=1}^{N}[(e_j(t) + z_j(t)) - (e_i(t) + z_i(t))] \\ &= K\sum_{j=1}^{N}[(e_j(t) - e_i(t)) + (z_j(t) - z_i(t))]. \end{aligned} \quad (6)$$

Let $z(t) = [z^T_1(t), \cdots, z^T_N(t)]^T$, $e(t) = [e^T_1(t), \cdots, e^T_N(t)]^T$, $u(t) = [u^T_1(t), \cdots, u^T_N(t)]^T$. Then, with (2) and (6), one has

$$\begin{aligned} \dot{z}(t) &= (I_N \otimes A)z(t) + (I_N \otimes B)(-L \otimes K)[e(t) + z(t)] \\ &= (I_N \otimes A - L \otimes BK)z(t) - (L \otimes BK)e(t). \end{aligned} \quad (7)$$

We define the disagreement vector as

$$\delta(t) = z(t) - (1_N s^T \otimes I_N)z(t). \quad (8)$$

Then, the derivative of $\delta(t)$ is

$$\dot{\delta}(t) = (I_N \otimes A - L \otimes BK)\delta(t) + (-L \otimes BK)e(t). \quad (9)$$

4. MAIN RESULTS

In this section, we will give the main result of this paper.

Theorem 4.1. Consider the multi-agent system (1). Under Assumption 2.1 and distributed event-triggered control protocol (3), the event-triggered rotating consensus is reached with triggered time instance $t^i_{k_i}$ determined by

$$t^i_{k_i+1} = \inf\left\{t > t^i_{k_i}\middle| \|e_i(t)\|^2 - \alpha_i e^{-\theta_i t} \geq 0\right\}, \quad (10)$$

where $K = \mu B^T P$, $0 < \mu < \frac{2\gamma_L}{s_M\|L\|^2} = \mu^*$ and $P > 0$ is the solution of $PA + A^T P - \beta_1 \hat{B} + I_N = 0$, $\beta_1 = 2\gamma_L\mu - s_M\mu^2\|L\|^2 > 0$, $s_M = \max[s_1, \cdots, s_N]$, γ_L, is computed as in Lemma 2.2, $\hat{B} = PBB^T P$. Furthermore, the Zeno behavior is excluded for each agent.

Proof. We consider the following Lyapunov function candidate:

$$V(t) = \delta^T(t)(S \otimes P)\delta(t). \quad (11)$$

Then, the derivative of (11) along the trajectory (9) is:

$$\begin{aligned} \dot{V}(t) &= \dot{\delta}^T(t)(S \otimes P)\delta(t) + \delta^T(t)(S \otimes P)\dot{\delta}(t) \\ &= 2\delta^T(t)(S \otimes \hat{A} - \hat{L} \otimes \mu\hat{B})\delta(t) \\ &\quad -2\delta^T(t)(SL \otimes \mu\hat{B})e(t), \end{aligned} \quad (12)$$

where $\hat{A} = \frac{1}{2}(PA + A^T P)$, $\hat{B} = PBB^T P$, $\hat{L} = \frac{SL+L^T S}{2}$.

By using inequality $\gamma a^T Y a + \frac{1}{\gamma} b^T Y b \geq \pm 2a^T Y b$, where $a \in R^q$, $b \in R^q$, $Y \in R^{q\times q}$, $\gamma > 0$, one has

$$\begin{aligned} &-2\delta^T(t)(SL \otimes \mu\hat{B})e(t) \\ &\leq \delta^T(t)(SLL^T S \otimes \mu^2\hat{B})\delta(t) + e^T(t)(I_N \otimes \hat{B})e(t). \end{aligned} \quad (13)$$

Then, substituting (13) into (12), we get

$$\begin{aligned} \dot{V}(t) &\leq \delta^T(t)(S \otimes 2\hat{A} - 2\hat{L} \otimes \mu\hat{B} + SLL^T S \otimes \mu^2\hat{B})\delta(t) \\ &\quad + e^T(t)(I_N \otimes \hat{B})e(t). \end{aligned} \quad (14)$$

By using Lemma 2.2, one has $x^T\hat{L}x > \gamma_L x^T S x$. Then, we can further get that

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^{N}\delta^T_i(t)s_i[2\hat{A} - (2\gamma_L\mu - s_i\mu^2\|L\|^2)\hat{B}]\delta_i(t) \\ &\quad + \|\hat{B}\|\sum_{i=1}^{N}\|e_i(t)\|^2. \end{aligned} \quad (15)$$

Let $\beta_1 = 2\gamma_L\mu - s_M\mu^2\|L\|^2$, where $s_M = \max[s_1, \cdots, s_N] > 0$. We can choose $0 < \mu < \frac{2\gamma_L}{s_M\|L\|^2} = \mu^*$ to let $\beta_1 > 0$. Then, by solving algebraic Riccati equation $2\hat{A} - \beta_1\hat{B} = -I_N$, we get a solution $P > 0$. With $s_M \geq s_i$, $2\gamma_L\mu - s_i\mu^2\|L\|^2 \geq 2\gamma_L\mu - s_M\mu^2\|L\|^2$, we can get $2\hat{A} - (2\gamma_L - s_i\|L\|^2)\hat{B} \leq 2\hat{A} - (2\gamma_L - s_M\|L\|^2)\hat{B} = -I_N$.

Then, we have

$$\dot{V}(t) \leq -\sum_{i=1}^{N}s_i\|\delta_i(t)\|^2 + \|\hat{B}\|\sum_{i=1}^{N}\|e_i(t)\|^2. \quad (16)$$

By (5), we can know $\|e_i(t)\|^2 \leq \alpha_i e^{-\theta_i t}$.

Substituting this into (16), one has

$$\begin{aligned} \dot{V}(t) &\leq -\sum_{i=1}^{N}s_i\|\delta_i(t)\|^2 + \|\hat{B}\|\sum_{i=1}^{N}\alpha_i e^{-\theta_i t} \\ &\leq -\sum_{i=1}^{N}\frac{s_i}{\lambda_M(P)}\delta^T_i(t)P\delta_i(t) + \|\hat{B}\|\sum_{i=1}^{N}\alpha_i e^{-\theta_i t} \\ &= -\frac{1}{\lambda_M(P)}V(t) + \|\hat{B}\|\sum_{i=1}^{N}\alpha_i e^{-\theta_i t} \\ &= -\beta_2 V(t) + \beta_3\sum_{i=1}^{N}\alpha_i e^{-\theta_i t} \\ &\leq -\beta_2 V(t) + N\beta_3\alpha_M e^{-\theta_m t}, \end{aligned} \quad (17)$$

where $\lambda_M(P)$ is the maximal eigenvalue of matrix P, $\beta_2 = \frac{1}{\lambda_M(P)} > 0$, $\beta_3 = \|\hat{B}\| > 0$, $\alpha_M = \max[\alpha_1, \cdots, \alpha_N] > 0$, $\theta_m = \min[\theta_1, \cdots, \theta_N] > 0$.

By solving $\dot{x}(t) = Fx(t) + Eu(t)$, we get $x(t) = e^{F(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{F(t-\tau)}Eu(\tau)d\tau$. It follows from (17) that

$$\begin{aligned} V(t) &\leq e^{-\beta_2 t}V(0) + N\beta_3\alpha_M\int_0^t e^{-\beta_2(t-\tau)}e^{-\theta_m\tau}d\tau \\ &= e^{-\beta_2 t}V(0) + \frac{N\beta_3\alpha_M}{\beta_2 - \theta_m}e^{-\beta_2 t}[e^{(\beta_2-\theta_m)t} - 1] \\ &= e^{-\beta_2 t}[V(0) - \frac{N\beta_3\alpha_M}{\beta_2 - \theta_m}] + e^{-\theta_m t}\frac{N\beta_3\alpha_M}{\beta_2 - \theta_m} \\ &\leq e^{-\beta_* t}V(0), \end{aligned} \quad (18)$$

where $\beta_* = \min[\beta_2, \theta_m] > 0$.

Therefore, under Assumption 2.1, the rotating consensus will be reached faster than exponential convergence rate β_*.

Next, we want to give the proof on the exclusion of the Zeno behavior, which is critical to ensure the event-triggered function (5) can be implementable in practice.

Firstly, we need to calculate the upper bound of $\|\dot{e}_i(t)\|$. The derivative of (4) is

$$\dot{e}_i(t) = Ae_i(t) - Bu_i(t). \tag{19}$$

Then the upper bound of $\|\dot{e}_i(t)\|$ is calculated as follows

$$\|\dot{e}_i(t)\| \leq \|A\| \times \|e_i(t)\| + \|Bu_i(t)\|. \tag{20}$$

Next, we calculate the upper bound of $\|e_i(t)\|$ and $\|Bu_i(t)\|$. By (5), we can get $\|e_i(t)\|^2 \leq \alpha_i e^{-\theta_i t} \leq \alpha_i \leq \alpha_M$ and $\|e(t)\|^2 \leq N\alpha_M$.

Then, $\|Bu_i(t)\|$ is upper bounded by $\|(I_N \otimes B)u(t)\|$ as follows

$$(I_N \otimes B)u(t) = (-L \otimes BK)[e(t) + \delta(t)]. \tag{21}$$

Then, $\|(I_N \otimes B)u(t)\|$ is limited as

$$\|(I_N \otimes B)u(t)\| \leq \|(-L \otimes BK)\|(\|e(t)\| + \|\delta(t)\|). \tag{22}$$

Next, we need to calculate the upper bound of $\|\delta(t)\|$. By (18), we can know that $V(t) \leq V(0)$. Then,

$$\begin{aligned}
\|\delta(t)\|^2 &\leq \sum_{i=1}^{N} \delta_i^T(t)\delta_i(t) \\
&\leq \frac{1}{s_m \lambda_m(P)} \sum_{i=1}^{N} s_i \delta_i^T(t) P \delta_i(t) \\
&= \frac{1}{s_m \lambda_m(P)} \delta^T(t)(S \otimes P)\delta(t) \\
&\leq \frac{1}{s_m \lambda_m(P)} \delta^T(0)(S \otimes P)\delta(0) \\
&= \beta_4^2,
\end{aligned} \tag{23}$$

where $s_m = \min[s_1, \cdots, s_N] > 0$, $\beta_4 > 0$, $\lambda_m(P) > 0$ is the minimual eigenvalue of matrix P. Then, we have $\|\delta(t)\| \leq \beta_4$.

We can calculate $\|(I_N \otimes B)u(t)\|$ as

$$\begin{aligned}
\|(I_N \otimes B)u(t)\| &\leq \|(-L \otimes BK)\|(\|e(t)\| + \|\delta(t)\|) \\
&\leq \|(-L \otimes BK)\|(\sqrt{N\alpha_M} + \beta_4).
\end{aligned} \tag{24}$$

And the upper limit of $\|\dot{e}_i(t)\|$ is

$$\begin{aligned}
\|\dot{e}_i(t)\| &\leq \|A\| \times \|e_i(t)\| + \|Bu_i(t)\| \\
&\leq \|A\|\sqrt{\alpha_M} + \|(I_N \otimes B)u(t)\| \\
&\leq \|A\|\sqrt{\alpha_M} + \|(-L \otimes BK)\|(\sqrt{N\alpha_M} + \beta_4) \\
&= \hat{W}.
\end{aligned} \tag{25}$$

Assume that the Zeno behavior is happened for agent i at T_z. It means $\lim_{k_i \to +\infty} t^i_{k_i} = T_z$. Then, for any $\varepsilon > 0$, there exits $M(\varepsilon)$ such that $t^i_{k_i} \in (T_z - \varepsilon, T_z + \varepsilon)$ for $\forall k_i \geq M(\varepsilon)$, which implies that $t^i_{M(\varepsilon)+1} - t^i_{M(\varepsilon)} < 2\varepsilon$.

The norm of measurement error $\|e_i(t)\|$ is reset to 0 when (5) is triggered. At the previous moment $t^i_{k_i-}$ of trigger time $t^i_{k_i}$, we have $\sqrt{\alpha_i} e^{-\frac{\theta_i}{4} t^i_{k_i-}} \leq \|e_i(t^i_{k_i-})\| \leq \sqrt{\alpha_i} e^{-\frac{\theta_i}{2} t^i_{k_i-}}$. It follows that

$$[t^i_{M(\varepsilon)+1} - t^i_{M(\varepsilon)}]\hat{W} \geq \sqrt{\alpha_i} e^{-\frac{\theta_i}{4} t^i_{k_i-}}. \tag{26}$$

Let $\varepsilon > 0$ be a solution of the following equation:

$$\frac{1}{\hat{W}} \sqrt{\alpha_i} e^{-\frac{\theta_i}{4} T_z} = 2\varepsilon e^{\frac{\theta_i}{4}\varepsilon}, \tag{27}$$

namely,

$$\frac{1}{\hat{W}} \sqrt{\alpha_i} e^{-\frac{\theta_i}{4}(T_z + \varepsilon)} = 2\varepsilon. \tag{28}$$

Due to the fact that $t^i_{M(\varepsilon)+1} \leq T_z + \varepsilon$, one has

$$e^{-\frac{\theta_i}{4}(t^i_{M(\varepsilon)+1})} \geq e^{-\frac{\theta_i}{4}(T_z + \varepsilon)}. \tag{29}$$

So we have

$$\begin{aligned}
t^i_{M(\varepsilon)+1} - t^i_{M(\varepsilon)} &\geq \frac{1}{\hat{W}} \sqrt{\alpha_i} e^{-\frac{\theta_i}{4} t^i_{k_i-}} \\
&= \frac{1}{\hat{W}} \sqrt{\alpha_i} e^{-\frac{\theta_i}{4} t^i_{M(\varepsilon)+1}} \\
&\geq \frac{1}{\hat{W}} \sqrt{\alpha_i} e^{-\frac{\theta_i}{4}(T_z + \varepsilon)} \\
&= 2\varepsilon,
\end{aligned} \tag{30}$$

which is contrary to the assumption that $t^i_{M(\varepsilon)+1} - t^i_{M(\varepsilon)} < 2\varepsilon$. Thus, the Zeno behavior is excluded.

5. SIMULATION

In this section, we present a simulation example to illustrate the correctness of the obtained theoretical results. As is shown in Fig. 1, the topology is directed strongly connected with 4 nodes, in which the adjacency matrix and Laplacian matrix are

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix},$$

respectively.

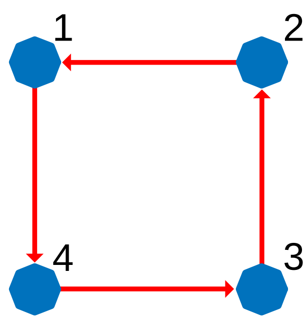

Fig. 1. Directed graph $\mathcal{G}$.

With simple calculations, we get $s = [0.25, 0.25, 0.25, 0.25]^T$, $s_M = s_m = 0.25$, $\|L\|^2 = 4$, $\gamma_L = 1.0$, $\mu^* = 2.0$, we choose $\mu = 1.2$, $\beta_1 = 0.96$, and the P and K matrices are listed as follows:

$$P = \begin{bmatrix} 1.95 & -0.00 & 0.87 & -0.54 \\ -0.0 & 1.95 & 0.54 & 0.87 \\ 0.87 & 0.54 & 1.69 & 0.00 \\ -0.54 & 0.87 & -0.00 & 1.69 \end{bmatrix},$$

$$K = \begin{bmatrix} 1.05 & 0.65 & 2.04 & 0.00 \\ -0.65 & 1.05 & -0.00 & 2.04 \end{bmatrix}.$$

The simulation results are shown in Figs. 2, 3 and 4. It is clear that the rotating consensus is reached even the communication is discrete based on the distributed event-triggered control protocol, which is consistent with Theorem 4.1.

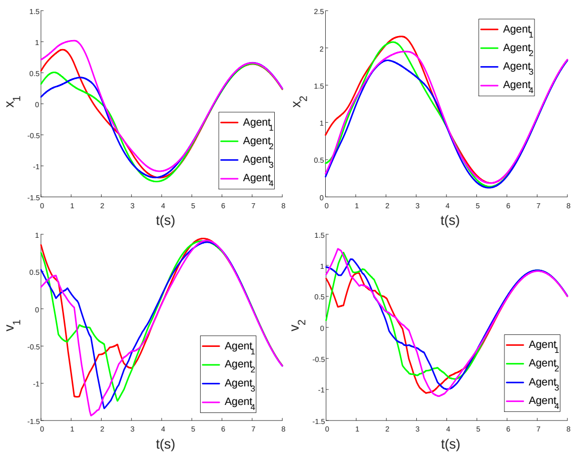

Fig. 2. All agents' positions first dimension (upper left), second dimension (upper right) and velocities first dimension (lower left), second dimension (lower right).

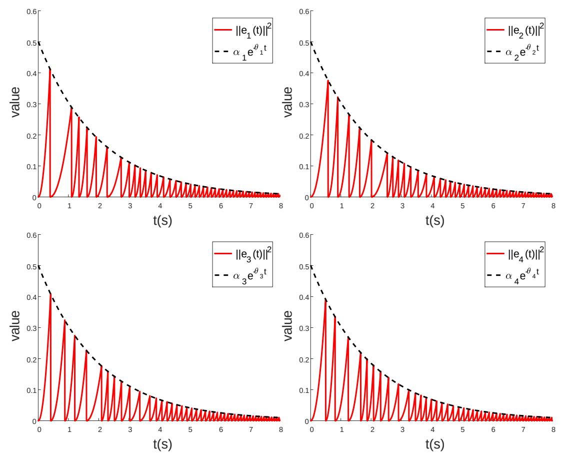

Fig. 3. The errors $\|e_i(t)\|^2$ and the thresholds $\alpha_i e^{-\theta_i t}$ of each agent .

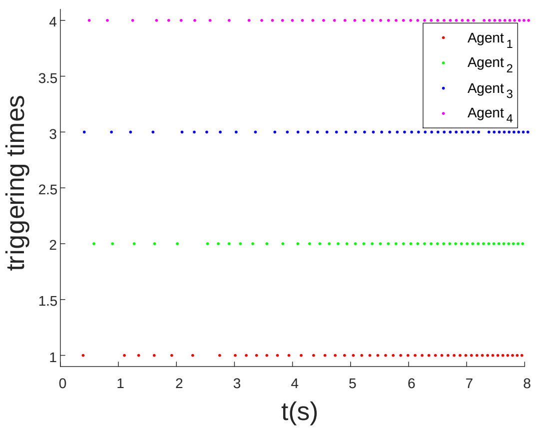

Fig. 4. All agents' triggering times.

6. CONCLUSIONS

This paper has proposed an event-triggered mechanism and a suitable control protocol to solve the rotating consensus problem, where the topology is directed. Under such conditions, all agents reach rotating consensus with an exponential convergence rate and the Zeno behavior is ruled out. In the future, we will focus on the event-triggered control of multi-agent systems taken the time-varying communication delay into consideration.

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