This submission did not make use of a machine that is running Linux. Rather, this submission was done on a Windows 11 machine using an MSYS2 MinGW 64-bit Shell in order to simulate multithreading via OpenMP.

Additionally, I used time, which is shell-provided, in order to get the machine's execution time of each task.

These commands were used to compile the C files for each task:

```
Shell

gcc -02 task1.c -o task1

gcc -02 -mavx2 task2.c -o task2

gcc -02 task3.c -o task3 -fopenmp

gcc -02 -mavx2 task4.c -o task4 -fopenmp
```

I made use of the shell-provided time instruction in order to get execution time for running a task. I then made a shell script execution_time.sh which runs a task 10 times, and then gets the average run time of the runs. For example, for running task 1:

```
Shell
./execution_time.sh ./task1
```

Execution Times

Task 1 - Basic Implementation

```
Shell
gcc -02 task1.c -o task1
./execution_time.sh ./task1
```

```
Shiro@LAPTOP-U2KIJPRU MINGW64 /c/Users/Shiro/cs21lab09
$ ./execution_time.sh ./task1
Run 1:
Appr: 3.1415926636
Time: 0.670 seconds

Run 2:
Appr: 3.1415926636
Time: 0.265 seconds

Run 3:
Appr: 3.1415926636
Time: 0.239 seconds

Run 4:
Appr: 3.1415926636
Time: 0.247 seconds

Run 5:
Appr: 3.1415926636
Time: 0.213 seconds

Run 6:
Appr: 3.1415926636
Time: 0.239 seconds

Run 7:
Appr: 3.1415926636
Time: 0.239 seconds

Run 7:
Appr: 3.1415926636
Time: 0.223 seconds

Run 8:
Appr: 3.1415926636
Time: 0.249 seconds

Run 9:
Appr: 3.1415926636
Time: 0.283 seconds

Run 10:
Appr: 3.1415926636
Time: 0.269 seconds

Average runtime over 10 runs: 0.2897 seconds
```

Task 2- Vectorized Implementation

```
Shell
gcc -02 -mavx2 task2.c -o task2
./execution_time.sh ./task2
```

```
$ ./execution_time.sh ./task2
Run 1:
Appr: 3.1415926436
Time: 0.291 seconds
Run 2:
Appr: 3.1415926436
Time: 0.148 seconds
Run 3:
Appr: 3.1415926436
Time: 0.138 seconds
Run 4:
Appr: 3.1415926436
Time: 0.147 seconds
Run 5:
Appr: 3.1415926436
Time: 0.165 seconds
Run 6:
Appr: 3.1415926436
Time: 0.155 seconds
Run 7:
Appr: 3.1415926436
Time: 0.155 seconds
Run 7:
Appr: 3.1415926436
Time: 0.154 seconds
Run 8:
Appr: 3.1415926436
Time: 0.154 seconds
Run 9:
Appr: 3.1415926436
Time: 0.154 seconds
Run 9:
Appr: 3.1415926436
Time: 0.155 seconds
Run 9:
Appr: 3.1415926436
Time: 0.151 seconds
Run 10:
Appr: 3.1415926436
Time: 0.151 seconds
Average runtime over 10 runs: 0.1636 seconds
```

Task 3 - Multicore Implementation

```
Shell
gcc -02 task3.c -o task3 -fopenmp
./execution_time.sh ./task3
```

```
$ ./execution_time.sh ./task3
Run 1:
Appr: 3.1415926636
Time: 0.391 seconds
Run 2:
Appr: 3.1415926636
Time: 0.101 seconds
Run 3:
Appr: 3.1415926636
Time: 0.091 seconds
Run 4:
Appr: 3.1415926636
Time: 0.096 seconds
Run 5:
Appr: 3.1415926636
Time: 0.087 seconds
Run 6:
Appr: 3.1415926636
Time: 0.106 seconds
Run 7:
Appr: 3.1415926636
Time: 0.106 seconds
Run 7:
Appr: 3.1415926636
Time: 0.107 seconds
Run 8:
Appr: 3.1415926636
Time: 0.087 seconds
Run 9:
Appr: 3.1415926636
Time: 0.107 seconds
Run 9:
Appr: 3.1415926636
Time: 0.091 seconds
Run 9:
Appr: 3.1415926636
Time: 0.091 seconds
Run 10:
Appr: 3.1415926636
Time: 0.089 seconds
Average runtime over 10 runs: 0.1246 seconds
```

Task 4 - Hybrid Implementation

```
Shell
gcc -02 -mavx2 task4.c -o task4 -fopenmp
./execution_time.sh ./task4
```

```
Shiro@LAPTOP-UZKIJPRU MINGW64 /c/Users/Shiro/cs211ab09
$ ./execution_time.sh ./task4
Run 1:
Appr: 3.1415926436
Time: 0.259 seconds
Run 2:
Appr: 3.1415926436
Time: 0.093 seconds
Run 3:
Appr: 3.1415926436
Time: 0.099 seconds
Run 4:
Appr: 3.1415926436
Time: 0.097 seconds
Run 5:
Appr: 3.1415926436
Time: 0.098 seconds
Run 6:
Appr: 3.1415926436
Time: 0.092 seconds
Run 7:
Appr: 3.1415926436
Time: 0.094 seconds
Run 8:
Appr: 3.1415926436
Time: 0.089 seconds
Run 8:
Appr: 3.1415926436
Time: 0.089 seconds
Run 9:
Appr: 3.1415926436
Time: 0.106 seconds
Run 10:
Appr: 3.1415926436
Time: 0.104 seconds
Average runtime over 10 runs: 0.1131 seconds
```

Speedup Computations

Say you want to compute the speedup from Implementation A to Implementation B, the formula to be used for the Speedup is:

$$Speedup_{A,B} = \frac{AvgRuntime_A}{AvgRuntime_B}$$

$$Speedup_{1,2} = \frac{\frac{AvgRuntime_{1}}{AvgRuntime_{2}}}{\frac{AvgRuntime_{1}}{AvgRuntime_{2}}} = \frac{0.2897}{0.1636} = 1.7708 \ times faster$$

$$Speedup_{1,3} = \frac{\frac{AvgRuntime_{1}}{AvgRuntime_{3}}}{\frac{AvgRuntime_{1}}{AvgRuntime_{4}}} = \frac{0.2897}{0.1246} = 2.3250 \ times faster$$

$$Speedup_{1,4} = \frac{\frac{AvgRuntime_{1}}{AvgRuntime_{4}}}{\frac{AvgRuntime_{2}}{AvgRuntime_{4}}} = \frac{0.2897}{0.1131} = 2.5615 \ times faster$$

$$Speedup_{2,4} = \frac{\frac{AvgRuntime_{2}}{AvgRuntime_{4}}}{\frac{AvgRuntime_{3}}{AvgRuntime_{4}}} = \frac{0.1636}{0.1131} = 1.4465 \ times faster$$

$$Speedup_{3,4} = \frac{\frac{AvgRuntime_{3}}{AvgRuntime_{4}}}{\frac{AvgRuntime_{4}}{AvgRuntime_{4}}} = \frac{0.1246}{0.1131} = 1.1017 \ times faster$$