IMAGE COMPRESSION USING SVD

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Abstract— **This report presents an overview of image compression using Singular Value Decomposition (SVD). Image compression is essential for efficient storage and transmission of images in various applications. SVD provides a method to approximate images by reducing the amount of information required to represent them, thereby achieving significant compression ratios with minimal loss in quality. The results demonstrate the trade-off between compression ratio and image quality, highlighting SVD's potential as a viable technique for image compression.**

Keywords—singular value decomposition, image compression

I. INTRODUCTION

Image compression techniques play a critical role in reducing the storage and bandwidth requirements for images. Compression can be categorized into lossy and lossless methods, with each offering unique benefits depending on the application. Singular Value Decomposition (SVD) is a mathematical approach that decomposes an image matrix into three matrices, allowing for lower-rank approximations that retain essential visual features while discarding less significant information. This report explores the use of SVD in image compression, discussing its methodology and performance analysis.

Image Compression Techniques

Image compression techniques are generally classified into two types: lossy and lossless. Lossless compression retains all original data, making it ideal for applications requiring exact reconstruction. In contrast, lossy compression achieves higher compression ratios by sacrificing some data, which is suitable for visual media where some quality loss is acceptable. Techniques like JPEG, PNG, and GIF employ different methods to reduce image size. SVD, primarily a lossy compression technique, enables dimensionality reduction by approximating the image matrix.

Singular Value Decomposition (SVD

Singular Value Decomposition (SVD) is a mathematical technique for factorizing a matrix into three components: U, Σ, and V^T, where U and V^T are orthogonal matrices, and Σ is a diagonal matrix containing singular values. By selecting only the largest singular values, SVD enables the approximation of an image matrix with reduced rank, compressing the image while retaining significant features. This section discusses the mathematical foundations of SVD and its application in compressing images.

Power Iteration

Power iteration is simple and effective for computing the largest singular value and vector, especially for large matrices where full SVD decomposition would be computationally expensive. Repeating the process allows us to find multiple singular values in order of magnitude.

II. PROBLEM STATEMENT

Image compression using SVD, Consider a m × m image. Perform SVD on it and force the lowest k singular values to 0 and reconstruct the images back. Compute the Average signal to noise ratio for each of these images and plot this for various values of k. For various k, compute the average perceptual quality by showing the images with your friends and classmates. Plot the k versus average perceptual quality plot.

III. METHADOLOGY

The process of image compression using SVD involves the following steps:

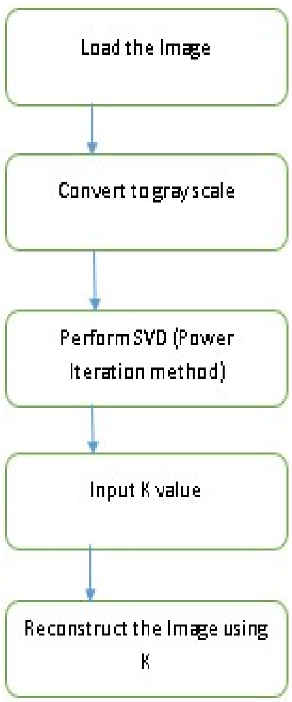

Fig.1 Flow of Exicution

Convert the image to a grayscale matrix. Apply SVD to decompose the matrix into U, S, and V^T. Select a reduced number of singular values to approximate the original image matrix. Reconstruct the compressed image using the selected values. The rank of approximation determines the balance between compression ratio and image quality.

Steps of Power Iteration

Starting with a Random Initial Vector (v):

The algorithm begins by creating a random vector v. This vector will approximate the largest right singular vector of the matrix.

v is normalized to have a unit length (by dividing it by its norm).

Iterating to Approximate Singular Vectors:

For each iteration, u and v are updated in a back-and-forth manner to converge toward the singular vectors:

Computing Left Singular Vector (u): Multiply A by v to get an approximation of the corresponding left singular vector. Normalize u.

Computing Right Singular Vector (v_new): Multiply the transpose of A by u to get a new approximation for the right singular vector. Normalize v_new.

After each update, the algorithm checks if v_new is close enough to the previous vector v by checking if the difference between v_new and v is below a small tolerance (tol). If this difference is small, it indicates that convergence has been reached, meaning v has stabilized and represents the dominant singular vector.

Calculating the Singular Value:

Once the vectors u and v have converged, the largest singular value, sigma, is calculated as the norm of the product A * v.

Storing the Singular Triplet and Deflate the Matrix:

The singular value sigma and singular vectors u and v are stored.

To find the next singular value and vectors, the matrix A_k is "deflated." This means subtracting out the effect of the current singular component, so subsequent iterations will reveal the next largest singular value.

IV. RESULTS AND ANALYSIS

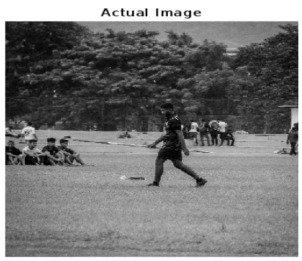

Fig.2 Actual image in gray scale

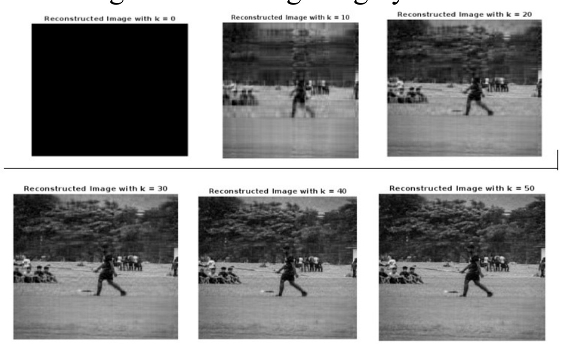

Fig.3 Reconstructed image at k=0,10,20,30,40,50

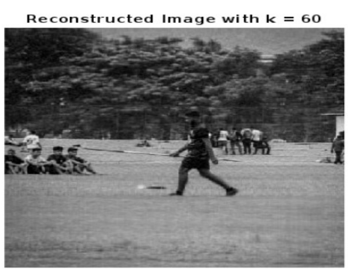

Fig.4 Reconstructed image at k=60

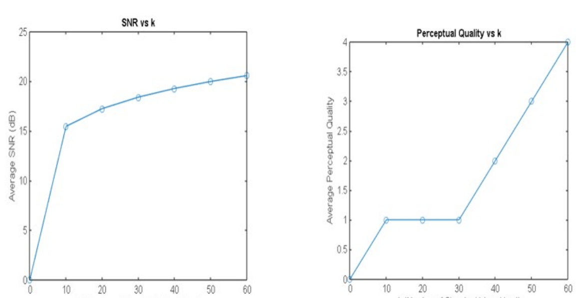

Fig.5 Ploting SNR and Perceptual Quality with K

The results show that by varying the rank of approximation, one can control the trade-off between the compression ratio and image quality. Higher ranks retain more details but offer lower compression ratios, while lower ranks result in higher compression ratios with increased quality loss. The analysis demonstrates that SVD-based image compression is effective in achieving high compression with acceptable quality loss for practical applications.

V. CONCLUSION

This report demonstrates the efficacy of Singular Value Decomposition (SVD) using Power Iterations method in image compression. SVD provides a method to approximate images by reducing the rank of the matrix representation, thus achieving compression. The results indicate that SVD offers a balance between compression ratio and quality, making it a valuable tool for applications where storage and bandwidth constraints are critical. Future work may explore optimizations to improve processing time and quality retention further.

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