UNDERSTANDING PROBABILITY INEQUALITIES

Markov and Chebyshev's Inequalities

INTRODUCTION

Probability inequalities are tools in probability theory that allow us to understand and quantify the likelihood of certain events occurring, or the behavior of random variables, even when we don't have complete information about their distributions. These inequalities provide bounds or limits on probabilities, helping us make predictions and draw conclusions in a wide range of fields, including statistics, finance, engineering, and computer science.

Two fundamental probability inequalities are Markov's Inequality and Chebyshev's Inequality.

MARKOV INEQUALITY

OVERVIEW

Markov's Inequality provides an upper bound on the probability that a non-negative random variable exceeds a certain value. It's particularly useful when we know the average or expected value of a random variable but have limited information about its distribution.

KEY POINTS:

- Markov's Inequality provides an upper bound on the probability that a non-negative random variable exceeds a certain value.
- Mathematically, for any non-negative random variable X and any positive number a, Markov's Inequality states:

$$P(X \geq a) \leq E[X] / a$$

- This inequality is particularly useful when the exact distribution of X is unknown or difficult to compute.
- Example: Suppose the average number of goals scored by a football team in a match is 2. Markov's Inequality can be used to bound the probability that they score at least 5 goals.

EXAMPLE

Imagine you have a bag of marbles, but you don't know exactly how many marbles are in it. You know that the average weight of each marble is 10 grams, but some marbles might be heavier or lighter. Now, let's say you want to know the probability that a randomly picked marble from the bag weighs at least 20 grams. Since you don't know the exact distribution of marble weights, you can't calculate this probability directly.

Markov's Inequality states:

$$P(X \geq a) \leq E[X]/a$$

Given:

- Average weight of each marble (E[X]) = 10 grams

Substituting our values:

$$P(\text{weight} \geq 20 \text{ grams}) \leq 10 \text{ grams}/20 \text{ grams} = 0.5$$

This means that the probability of picking a marble weighing at least 20 grams is less than or equal to 0.5, or 50%

CHEBYSHEV'S INEQUALITY

OVERVIEW

Chebyshev's Inequality, on the other hand, gives us an upper bound on the probability that a random variable deviates from its mean by more than a certain amount. This inequality helps us understand the spread or variability of a random variable's values around its mean.

KEY POINTS:

- Chebyshev's Inequality provides an upper bound on the probability that a random variable deviates from its mean by more than a certain amount.
- Mathematically, for any random variable X with finite mean μ and variance σ^2, and any positive number k, Chebyshev's Inequality states: $P(|X - \mu| \geq k\sigma) \leq 1 / k^2$.
- This inequality is particularly useful in analyzing the spread or dispersion of a random variable's values around its mean.
- Example: Chebyshev's Inequality can be used to estimate the probability that the waiting time for a bus exceeds twice its average waiting time.

EXAMPLE

Now let's imagine you're playing a game where you roll a fair six-sided die. You want to know how likely it is to roll a number that's far from the average roll (which is 3.5).

Chebyshev's Inequality: $P(\ |X-\mu| \geq k\sigma\ ) \leq 1k$^2

Given:

- Mean (μ) of the die roll outcomes = 3.5
- Variance (σ^2) of the die roll outcomes = (1^2+2^2+3^2+4^2+5^2+6^2)/6

=35/6 ≈5.833

- Standard deviation (σ) = 35/6 ≈ 2.415635 ≈ 2.415

Let's say we want to know the probability of rolling a number that's at least 3 standard deviations away from the mean. Using Chebyshev's Inequality:

P(|X−3.5| ≥ 3×2.415) ≤ 1/3^2

=1^9≈0.111

This means that the probability of rolling a number at least 3 standard deviations away from the mean is at most 0.111, or about 11.1%.

REAL-WORLD APPLICATIONS

- **Finance:**
  - Markov's Inequality and Chebyshev's Inequality are used in financial risk management to estimate the likelihood of extreme events, such as stock market crashes or large fluctuations in asset prices.
  - Suppose a financial analyst wants to estimate the probability that a stock's price will drop by more than 20% in a single trading day. Chebyshev's Inequality can be applied using historical price data to provide an upper bound on this probability, aiding investors in risk assessment.
- **Engineering:**
  - In structural engineering, engineers may use Markov's Inequality to estimate the probability of a bridge exceeding a certain load capacity during its lifetime. This information is crucial for ensuring the safety and reliability of infrastructure projects.
- **Computer Science:**
  - In computer science, these inequalities are employed in analyzing the performance and efficiency of algorithms and systems.
  - For instance, Chebyshev's Inequality can be used to bound the probability of a program's execution time exceeding a certain threshold, helping in optimizing computational resources.

THANK YOU

Somay Singh
B.Sc. (H) C.S.
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