Supplementary Material - Mixed Use of Analytical Derivatives and Algorithmic Differentiation for NMPC of Robot Manipulators

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TABLE S1: Number of atomic operations in the functions for rigid-body dynamics and their Jacobians computed both with algorithmic differentiation ($\mathbf{J}^c_{FD}, \mathbf{J}^c_{ID}$) and analytical derivatives ($\mathbf{J}^a_{FD}, \mathbf{J}^a_{ID}$).

Robot	Function	Number of atomic operations (FLOPs) per type									
		ADD/SUB	MUL	DIV	NEG	SQ	SQRT	SIN	COS	INV	TOTAL
Double Pendulum ($n_b = 2$)	FD	31	47	3	-	-	-	2	2	-	**85**
	$\mathbf{J}^a_{FD}$	132	179	9	2	-	-	2	2	1	**327**
	$\mathbf{J}^c_{FD}$	104	167	12	-	-	-	4	4	1	**292**
	ID	24	37	-	-	-	-	2	2	-	**65**
	$\mathbf{J}^c_{ID}$	104	130	-	1	2	-	2	2	-	**241**
	$\mathbf{J}^c_{ID}$	64	90	-	-	-	-	4	4	-	**162**
KUKA iiwa7 ($n_b = 7$)	FD	1207	1424	32	-	-	-	6	6	-	**2675**
	$\mathbf{J}^a_{FD}$	5479	6102	101	106	66	-	7	7	6	**11874**
	$\mathbf{J}^c_{FD}$	15151	17706	272	1	-	-	12	12	6	**33160**
	ID	509	629	-	-	-	-	6	6	-	**1150**
	$\mathbf{J}^c_{ID}$	3643	4014	-	-	68	-	7	7	-	**7739**
	$\mathbf{J}^c_{ID}$	4323	5125	-	-	-	-	12	12	-	**9472**
Kinova Gen3 ($n_b = 7$)	FD	1543	1833	34	-	-	-	7	7	-	**3424**
	$\mathbf{J}^a_{FD}$	6734	7505	114	114	78	-	7	7	6	**14565**
	$\mathbf{J}^c_{FD}$	18205	21521	291	-	-	-	14	14	6	**40051**
	ID	657	821	-	-	-	-	7	7	-	**1492**
	$\mathbf{J}^c_{ID}$	4444	4890	-	-	78	-	7	7	-	**9426**
	$\mathbf{J}^c_{ID}$	5391	6536	-	-	-	-	14	14	-	**11955**
ABB Yumi ($n_b = 14$)	FD	2788	3314	78	-	-	-	14	14	-	**6208**
	$\mathbf{J}^a_{FD}$	16846	18744	270	295	204	-	14	14	14	**36401**
	$\mathbf{J}^c_{FD}$	45868	53104	814	34	-	-	28	28	14	**99890**
	ID	1068	1316	-	4	-	-	14	14	-	**2416**
	$\mathbf{J}^c_{ID}$	10398	11422	-	12	204	-	14	14	-	**22064**
	$\mathbf{J}^c_{ID}$	11922	14240	-	46	-	-	28	28	-	**26264**
Atlas ($n_b = 30$)	FD	5073	6171	218	-	36	6	30	30	-	**11564**
	$\mathbf{J}^a_{FD}$	77499	84232	512	2239	402	6	30	30	25	**164975**
	$\mathbf{J}^c_{FD}$	317576	377703	7628	-	36	6	60	60	25	**703094**
	ID	2035	2532	-	-	-	-	30	30	-	**4627**
	$\mathbf{J}^c_{ID}$	24680	27498	-	366	-	-	30	30	-	**52604**
	$\mathbf{J}^c_{ID}$	33168	41321	-	19	-	-	60	60	-	**74628**

The types of operations included in this table are additions and substractions (ADD/SUB), multiplications (MUL), divisions (DIV), negations (NEG), square (SQ), square root (SQRT), sine (SIN), cosine (COS), and inversion (INV). Assignment of inputs, outputs and constant values is not included in the count of atomic operations.