Supplementary Material - Efficient Rigid-Body Dynamics and Derivatives for Nonlinear Model Predictive Control of Robot Manipulators

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TABLE S1: Number of atomic operations in the functions for rigid-body dynamics and their Jacobians computed both with algorithmic differentiation ($\mathbf{J}_{FD}^c, \mathbf{J}_{ID}^c$) and analytical derivatives ($\mathbf{J}_{FD}^a, \mathbf{J}_{ID}^a$).

| | Function | Number of atomic operations (FLOPs) per type | | | | | | | | | |
|------------------------|---|--|--------|------|------|-----|------|-----|-----|-----|--------|
| Robot | | ADD/SUB | MUL | DIV | NEG | SQ | SQRT | ŚĬN | COS | INV | TOTAL |
| Double | FD | 31 | 47 | 3 | - | - | - | 2 | 2 | - | 85 |
| | \mathbf{J}_{FD}^{a} | 132 | 179 | 9 | 2 | - | - | 2 | 2 | 1 | 327 |
| Pendulum | $egin{array}{c} \mathbf{J}_{FD}^a \ \mathbf{J}_{FD}^c \ ID \end{array}$ | 104 | 167 | 12 | - | - | - | 4 | 4 | 1 | 292 |
| $(n_b = 2)$ | ID | 24 | 37 | - | - | - | - | 2 | 2 | - | 65 |
| | \mathbf{J}_{ID}^{c} | 104 | 130 | - | 1 | 2 | - | 2 | 2 | - | 241 |
| | $egin{array}{c} \mathbf{J}_{ID}^c \ \mathbf{J}_{ID}^c \end{array}$ | 64 | 90 | - | - | - | - | 4 | 4 | - | 162 |
| KUKA iiwa7 $(n_b = 7)$ | FD | 1207 | 1424 | 32 | - | - | - | 6 | 6 | - | 2675 |
| | $\begin{array}{c c} \mathbf{J}^a_{FD} \\ \mathbf{J}^c_{FD} \\ ID \end{array}$ | 5479 | 6102 | 101 | 106 | 66 | - | 7 | 7 | 6 | 11874 |
| | \mathbf{J}_{FD}^{c} | 15151 | 17706 | 272 | 1 | - | - | 12 | 12 | 6 | 33160 |
| | ID | 509 | 629 | - | - | - | - | 6 | 6 | - | 1150 |
| | \mathbf{J}_{ID}^{c} | 3643 | 4014 | - | - | 68 | - | 7 | 7 | - | 7739 |
| | $egin{array}{c} \mathbf{J}_{ID}^c \ \mathbf{J}_{ID}^c \end{array}$ | 4323 | 5125 | - | - | - | - | 12 | 12 | - | 9472 |
| | FD | 1543 | 1833 | 34 | - | - | - | 7 | 7 | - | 3424 |
| Kinova | \mathbf{J}_{FD}^{a} | 6734 | 7505 | 114 | 114 | 78 | - | 7 | 7 | 6 | 14565 |
| Gen3 | $egin{array}{c} \mathbf{J}_{FD}^a \ \mathbf{J}_{FD}^c \ ID \end{array}$ | 18205 | 21521 | 291 | - | - | - | 14 | 14 | 6 | 40051 |
| $(n_b = 7)$ | ID | 657 | 821 | - | - | - | - | 7 | 7 | - | 1492 |
| | \mathbf{J}_{ID}^{c} | 4444 | 4890 | - | - | 78 | - | 7 | 7 | - | 9426 |
| | $egin{array}{c} \mathbf{J}_{ID}^c \ \mathbf{J}_{ID}^c \end{array}$ | 5391 | 6536 | - | - | - | - | 14 | 14 | - | 11955 |
| | FD | 2788 | 3314 | 78 | - | - | - | 14 | 14 | - | 6208 |
| ABB | $egin{aligned} \mathbf{J}^a_{FD} \ \mathbf{J}^c_{FD} \end{aligned}$ | 16846 | 18744 | 270 | 295 | 204 | - | 14 | 14 | 14 | 36401 |
| Yumi | \mathbf{J}_{FD}^{c} | 45868 | 53104 | 814 | 34 | - | - | 28 | 28 | 14 | 99890 |
| $(n_b = 14)$ | $\mid ID \mid$ | 1068 | 1316 | - | 4 | - | - | 14 | 14 | - | 2416 |
| | $egin{array}{c} \mathbf{J}_{ID}^c \ \mathbf{J}_{ID}^c \end{array}$ | 10398 | 11422 | - | 12 | 204 | - | 14 | 14 | - | 22064 |
| | $ \mathbf{J}_{ID}^c $ | 11922 | 14240 | - | 46 | - | - | 28 | 28 | - | 26264 |
| Atlas $(n_b = 30)$ | FD | 5073 | 6171 | 218 | - | 36 | 6 | 30 | 30 | - | 11564 |
| | $egin{array}{c} \mathbf{J}_{FD}^a \ \mathbf{J}_{FD}^c \ ID \end{array}$ | 77499 | 84232 | 512 | 2239 | 402 | 6 | 30 | 30 | 25 | 164975 |
| | \mathbf{J}_{FD}^c | 317576 | 377703 | 7628 | - | 36 | 6 | 60 | 60 | 25 | 703094 |
| | ID | 2035 | 2532 | - | - | - | - | 30 | 30 | - | 4627 |
| | $egin{array}{c} \mathbf{J}_{ID}^c \ \mathbf{J}_{ID}^c \end{array}$ | 24680 | 27498 | - | 366 | - | - | 30 | 30 | - | 52604 |
| | $ \mathbf{J}_{ID}^c $ | 33168 | 41321 | - | 19 | - | - | 60 | 60 | - | 74628 |

The types of operations included in this table are additions and substractions (ADD/SUB), multiplications (MUL), divisions (DIV), negations (NEG), square (SQ), square root (SQRT), sine (SIN), cosine (COS), and inversion (INV). Assignment of inputs, outputs and constant values is not included in the count of atomic operations.