Lineare Regression

Data

x_1	x_2	x_3	...	x_n	y
x_{11}	x_{12}	x_{13}	...	x_{1n}	y_1
x_{21}	x_{22}	....		x_{2n}	y_2
$\vdots$					$\vdots$
x_{m1}	x_{m2}	....		x_{mn}	y_m

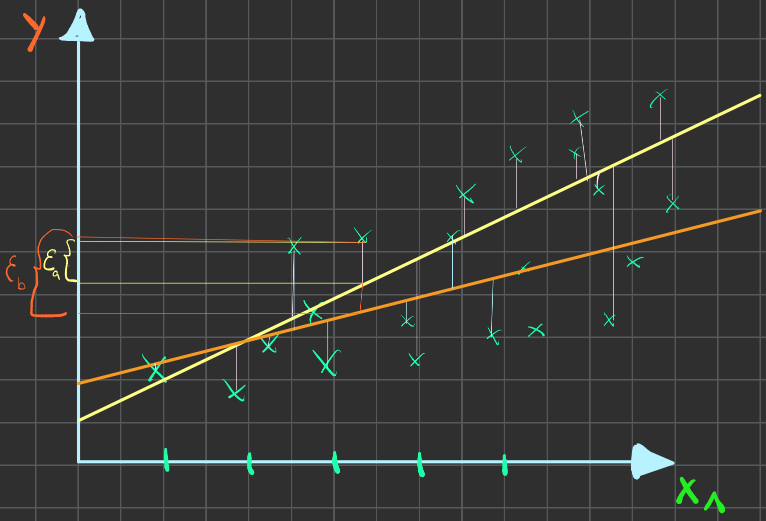
$$x_{ij} \in \mathbb{R},\ i \in \{1, \dots, m\},\ j \in \{1, \dots, n\}$$

$$y_i \in \mathbb{R},\ i \in \{1, \dots, m\}$$

$x_{ij} \Rightarrow$ Feature-Variablen

$y_i \Rightarrow$ Zielvariablen

Ziel: Finde eine Funktion, mit der sich die Zielvariable y_i aus den Feature-Variablen x_i darstellen lässt



$\hat{y}_a = mx + b \Rightarrow y = mx + b + \varepsilon_a$

$\hat{y}_b = \ell x + d \Rightarrow y = \ell x + d + \varepsilon_b$

$\Rightarrow$ Welche Vorhersage zeigt die kleinste Abweichung?

$\Rightarrow$ Minimiere die Abweichung

x_1	x_2	x_3	...	x_n	y	$\hat{y}$
x_{11}	x_{12}	x_{13}	...	x_{1n}	y_1	$\hat{y}_1$
x_{21}	x_{22}	....		x_{2n}	y_2	$\hat{y}_2$
$\vdots$					$\vdots$	$\vdots$
x_{m1}	x_{m2}	....		x_{mn}	y_m	$\hat{y}_m$

$$(y - \hat{y})^2 = (y - \hat{y})^T (y - \hat{y})$$

$$= (y_1 - \hat{y}_1,\ y_2 - \hat{y}_2, \dots, y_m - \hat{y}_m) \begin{pmatrix} y_1 - \hat{y}_1 \\ \vdots \\ y_m - \hat{y}_m \end{pmatrix}$$

$$= \sum_{i=1}^{m} (y_i - \hat{y}_i)^2$$

$$\hat{y}_1 = \omega_1 x_{11} + \omega_2 x_{12} + \dots + \omega_n x_{1n}$$

$$y_1 = \hat{y}_1 + \varepsilon$$