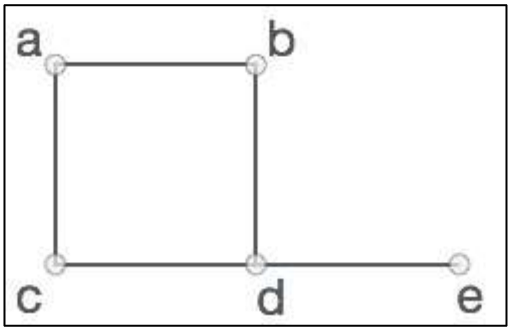
Lab 10: Implementation of Graph, DFS and BFS

Graph, Depth First Search and Breadth First Search

Graph

A Graph consists of a finite set of vertices (or nodes) and set of Edges which connect a pair of nodes. It is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as vertices, and the links that connect the vertices are called edges.

Formally, a graph is a pair of sets **(V, E)**, where **V** is the set of vertices and **E**is the set of edges, connecting the pairs of vertices. Take a look at the following graph −



In the above graph,

V = {a, b, c, d, e}

E = {ab, ac, bd, cd, de}

Graphs are used to solve many real-life problems. Graphs are used to represent networks. The networks may include paths in a city or telephone network or circuit network. Graphs are also used in social networks like LinkedIn, Facebook. For example, in Facebook, each person is represented with a vertex (or node). Each node is a structure and contains information like person id, name, gender, locale etc.

Graph Data Structure

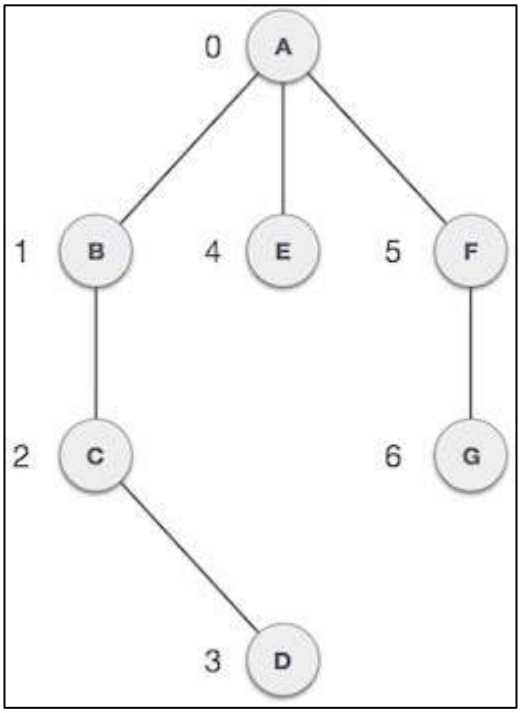
Mathematical graphs can be represented in data structure. We can represent a graph using an array of vertices and a two-dimensional array of edges. Before we proceed further, let's familiarize ourselves with some important terms −

Vertex − Each node of the graph is represented as a vertex. In the following example, the labelled circle represents vertices. Thus, A to G are vertices. We can represent them using an array as shown in the following image. Here A can be identified by index 0. B can be identified using index 1 and so on.

Edge − Edge represents a path between two vertices or a line between two vertices. In the following example, the lines from A to B, B to C, and so on represents edges. We can use a two-dimensional array to represent an array as shown in the following image. Here AB can be represented as 1 at row 0, column 1, BC as 1 at row 1, column 2 and so on, keeping other combinations as 0.

Adjacency − Two node or vertices are adjacent if they are connected to each other through an edge. In the following example, B is adjacent to A, C is adjacent to B, and so on.

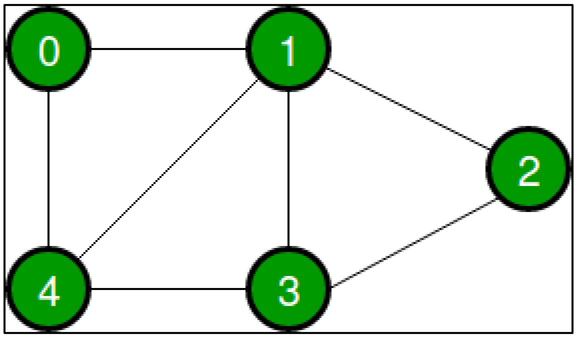
Path − Path represents a sequence of edges between the two vertices. In the following example, ABCD represents a path from A to D.



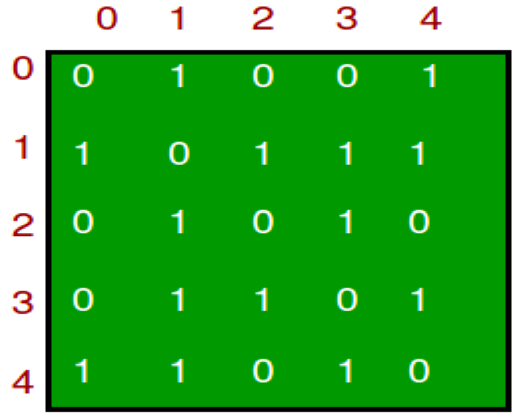
Graph Representation

Following two are the most commonly used representations of a graph.
1. Adjacency Matrix
2. Adjacency List

Adjacency Matrix:
Adjacency Matrix is a 2D array of size V x V where V is the number of vertices in a graph. Let the 2D array be adj[][], a slot adj[i][j] = 1 indicates that there is an edge from vertex i to vertex j. Adjacency matrix for undirected graph is always symmetric. Adjacency Matrix is also used to represent weighted graphs. If adj[i][j] = w, then there is an edge from vertex i to vertex j with weight w.



The adjacency matrix for the above example graph is:



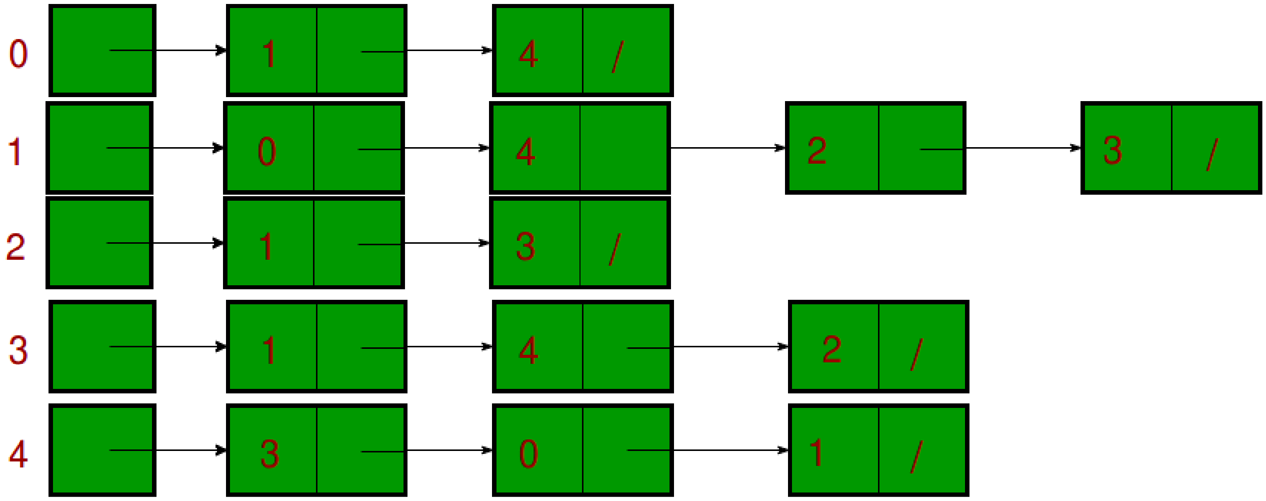
	0	1	2	3	4
0	0	1	0	0	1
1	1	0	1	1	1
2	0	1	0	1	0
3	0	1	1	0	1
4	1	1	0	1	0

Pros: Representation is easier to implement and follow. Removing an edge takes O(1) time. Queries like whether there is an edge from vertex ‘u’ to vertex ‘v’ are efficient and can be done O(1).

Cons: Consumes more space O(V^2). Even if the graph is sparse (contains less number of edges), it consumes the same space. Adding a vertex is O(V^2) time.

Adjacency List:

An adjacency list represents a graph as an array of linked list. Size of the array is equal to the number of vertices. The index of the array represents a vertex and each element in its linked list represents the other vertices that form an edge with the vertex. Let the array be array[]. An entry array[i] represents the list of vertices adjacent to the ***i***th vertex. This representation can also be used to represent a weighted graph. The weights of edges can be represented as lists of pairs. Following is adjacency list representation of the above graph.



Basic Operations

Following are basic primary operations of a Graph −

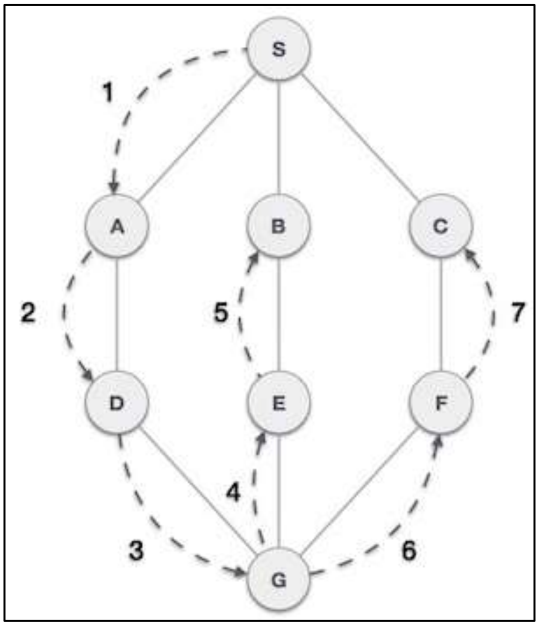
Add Vertex − Adds a vertex to the graph.

Add Edge − Adds an edge between the two vertices of the graph.

Display Vertex − Displays a vertex of the graph.

Depth First Search

Depth First Search (DFS) algorithm traverses a graph in a depth ward motion and uses a stack to remember to get the next vertex to start a search, when a dead end occurs in any iteration.

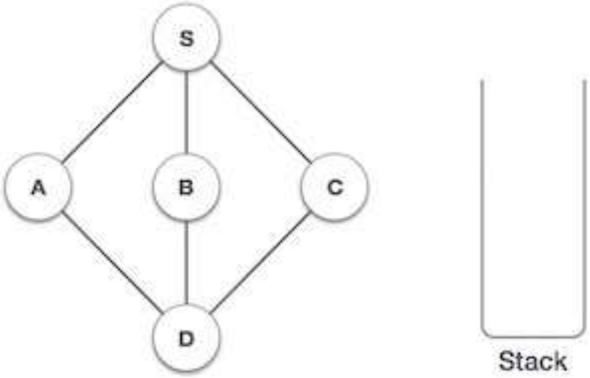
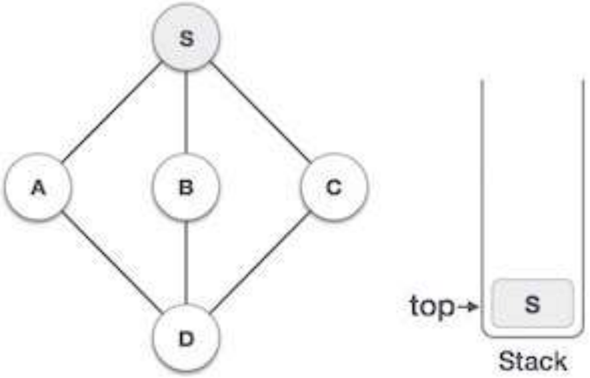
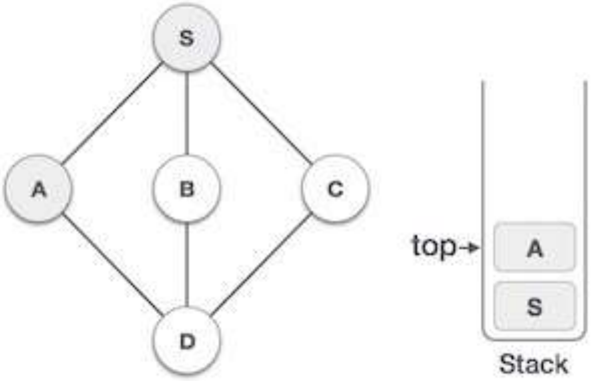


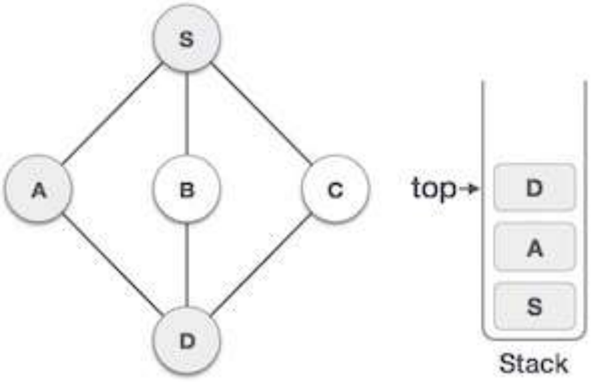
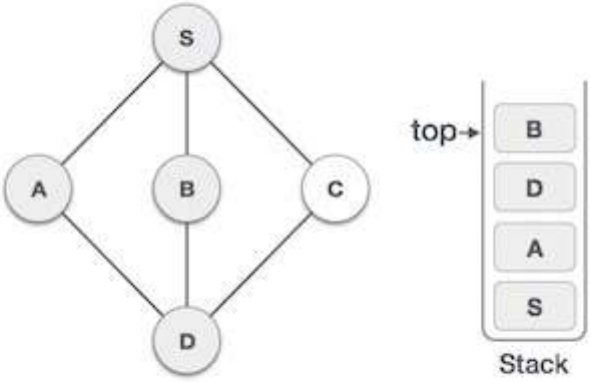
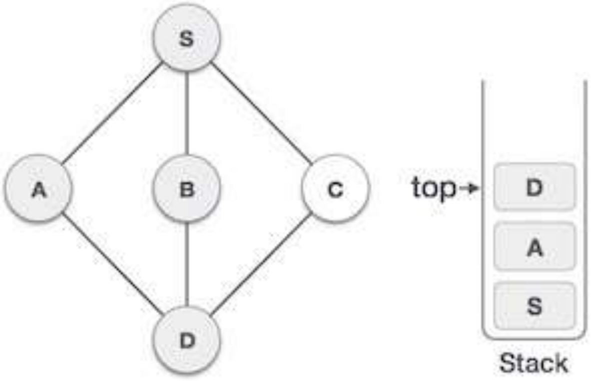
As in the example given above, DFS algorithm traverses from S to A to D to G to E to B first, then to F and lastly to C. It employs the following rules.

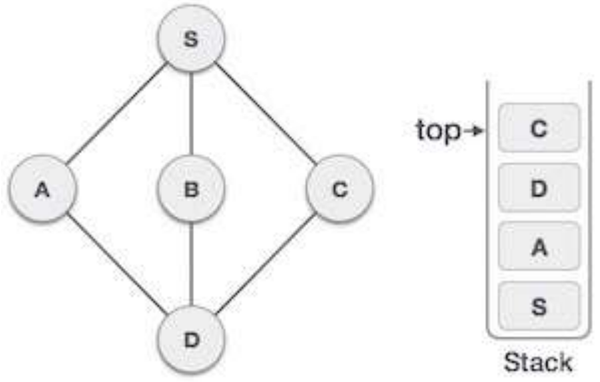
Rule 1 − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.

Rule 2 − If no adjacent vertex is found, pop up a vertex from the stack. (It will pop up all the vertices from the stack, which do not have adjacent vertices.)

Rule 3 − Repeat Rule 1 and Rule 2 until the stack is empty.

Step	Traversal	Description
1.		Initialize the stack.
2.		Mark **S** as visited and put it onto the stack. Explore any unvisited adjacent node from **S**. We have three nodes and we can pick any of them. For this example, we shall take the node in an alphabetical order.
3.		Mark **A** as visited and put it onto the stack. Explore any unvisited adjacent node from A. Both **S**and **D** are adjacent to **A** but we are concerned for unvisited nodes only.

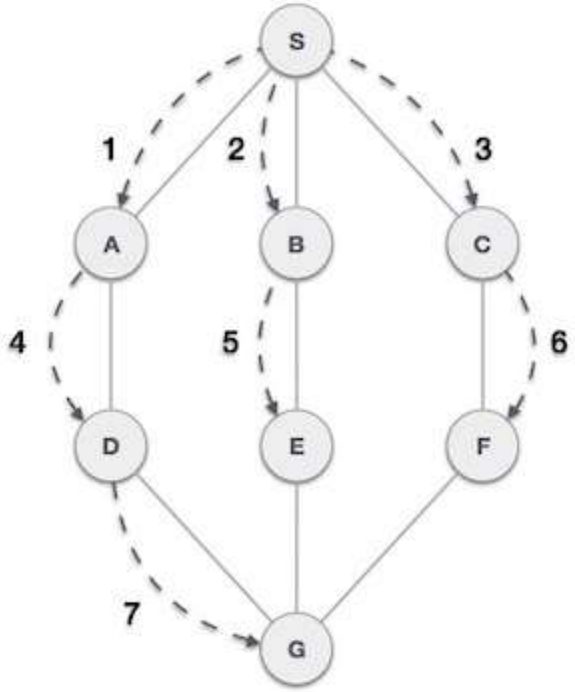
4.		Visit **D** and mark it as visited and put onto the stack. Here, we have **B** and **C** nodes, which are adjacent to **D** and both are unvisited. However, we shall again choose in an alphabetical order.
5.		We choose **B**, mark it as visited and put onto the stack. Here **B**does not have any unvisited adjacent node. So, we pop **B**from the stack.
6.		We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find **D** to be on the top of the stack.

7.		Only unvisited adjacent node is from **D** is **C** now. So we visit **C**, mark it as visited and put it onto the stack.

As **C** does not have any unvisited adjacent node so we keep popping the stack until we find a node that has an unvisited adjacent node. In this case, there's none and we keep popping until the stack is empty.

Breadth First Search

Breadth First Search (BFS) algorithm traverses a graph in a breadth ward motion and uses a queue to remember to get the next vertex to start a search, when a dead end occurs in any iteration.

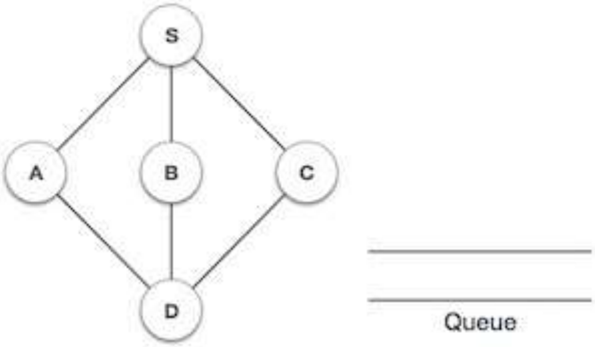
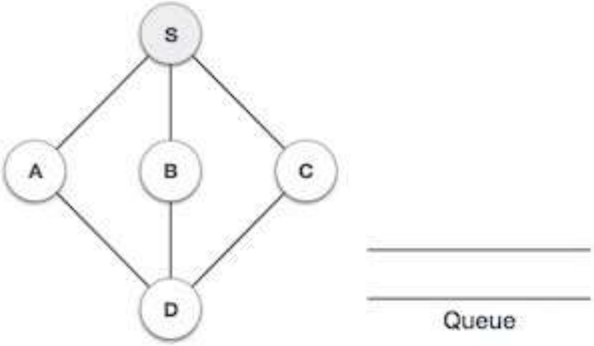


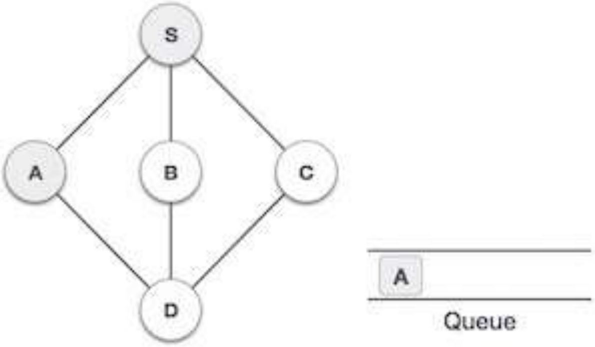
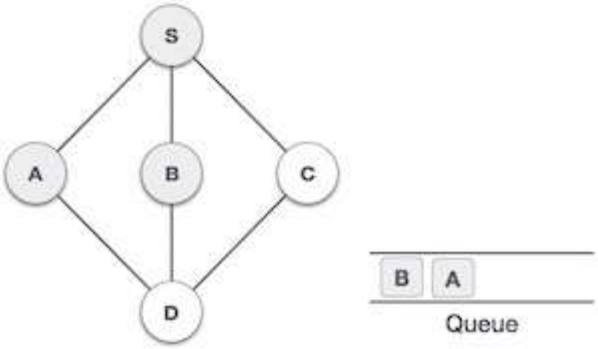
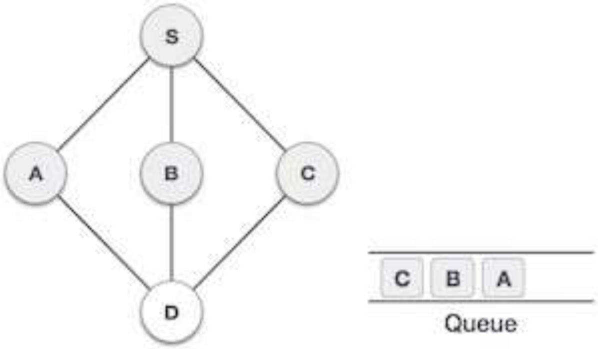
As in the example given above, BFS algorithm traverses from A to B to E to F first then to C and G lastly to D. It employs the following rules.

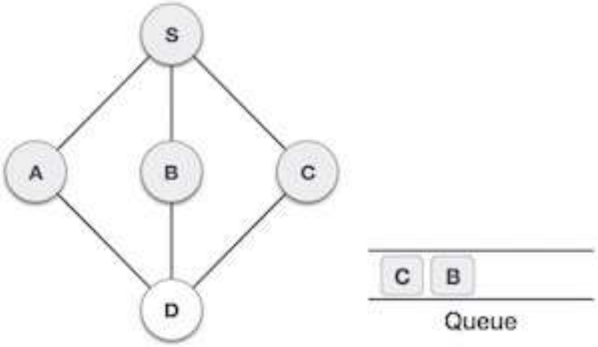
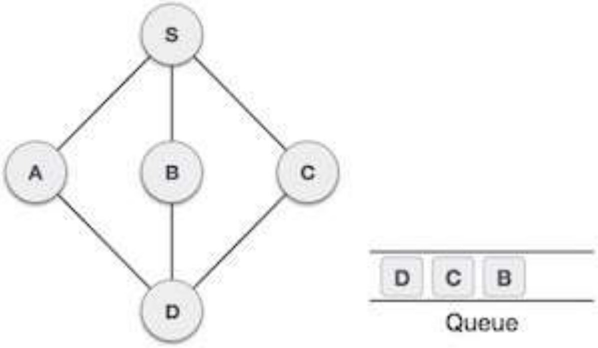
Rule 1 − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Insert it in a queue.

Rule 2 − If no adjacent vertex is found, remove the first vertex from the queue.

Rule 3 − Repeat Rule 1 and Rule 2 until the queue is empty.

Step	**Traversal**	**Description**
1.		Initialize the queue.
2.		We start from visiting **S**(starting node), and mark it as visited.

3.		We then see an unvisited adjacent node from **S**. In this example, we have three nodes but alphabetically we choose **A**, mark it as visited and enqueue it.
4.		Next, the unvisited adjacent node from **S** is **B**. We mark it as visited and enqueue it.
5.		Next, the unvisited adjacent node from **S** is **C**. We mark it as visited and enqueue it.

6.		Now, **S** is left with no unvisited adjacent nodes. So, we dequeue and find **A**.
7.		From **A** we have **D** as unvisited adjacent node. We mark it as visited and enqueue it.

At this stage, we are left with no unmarked (unvisited) nodes. But as per the algorithm we keep on dequeuing in order to get all unvisited nodes. When the queue gets emptied, the program is over.