Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
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An MCM Paper Made by Team 2417377 Based on

Summary

As the...//Considering...(background)

For problem 1, we

For problem 2, we

For problem 3, we

Eventually,

Keywords: LotkaVolterra, mathematics, LaTeX.

Contents

1 Introduction

1.1 Problem Background

A population that is necessarily dependent on reproduction for its development and continuation. Based on sexual reproduction, the growth rate of a population is greatly influenced by the sex ratio. Therefore, although many species exhibit a 1:1 sex ratio at birth, species tend to deviate from an even sex ratio in order to adapt to their environment and continue their populations. This is known as adaptive sex ratio variation.[1] The lamprey is one such typical species.

However, the role of the lamprey is complex. the seven-gill eel is complex. For some lake habitats, it is a parasite that is harmful to the ecosystem, and we want to reduce its reproduction; but at the same time, the seven-gill eel is a source of food in some parts of the world, and we want to promote its reproduction.[2] Therefore, we address these questions by studying the pros and cons of the ability to alter their sex ratio based on resource availability, modeling their relationship with growth rates, and studying their impact on other species in the ecosystem

Figure 1: a fish with lamprey

1.2 Restatement of the Problems

- Develop and examine a model to provide insights into the impact on the larger ecological system when the population of lampreys can alter its sex ratio
- Evaluate the advantages and disadvantages of the ability to change sex ratios for the population itself and for the external ecosystem under the same resource availability conditions, taking into account the modeling of Question 1
- Study the impact of sex ratio on ecosystem stability, specifically speaking,to evaluate the impact on ecosystems by considering the influence of changing population sex ratios on other populations on the basis of previous models.
- Developing a model to describe the relationship between lampreys and its parasites, study whether ecosystems with changing population sex ratios can provide advantages to other species in the ecosystem such as parasites

1.3 Overview of Our work

In order to find the relationship between sex ratio and environmental availability, we modeled the dynamics of the population and gradually added predator and parasite interactions to the model, constituting a model of the effect of variable sex ratio on the ecosystem, and finally, based on the model, we proposed the role of sex ratio control for human beings in the ecosystem.

- Assumptions are made to reduce complex ecosystems and the interactions within them (e.g., food chain relationships) to plausible resource-pressure relationships, and typical predation and parasitism relationships are included for objective modeling of population changes.
- Modeling of populations. Based on the Logitics model and the population dynamics equation, a sex ratio-environmental availability model was developed to study its effects on the population itself, as well as on the ecosystem.
- Modeling ecosystem interactions. Based on the Lotka-Volterra equation, predators and parasitoids were introduced respectively, and the model was improved so that it could describe the interactions of various groups in the ecosystem to study the role of sex ratios, and the TOPSIS evaluation model improved by the entropy weight method was used to study the effects on ecosystem stability
- Analyze the strengths and weaknesses of the model and make the conclusions

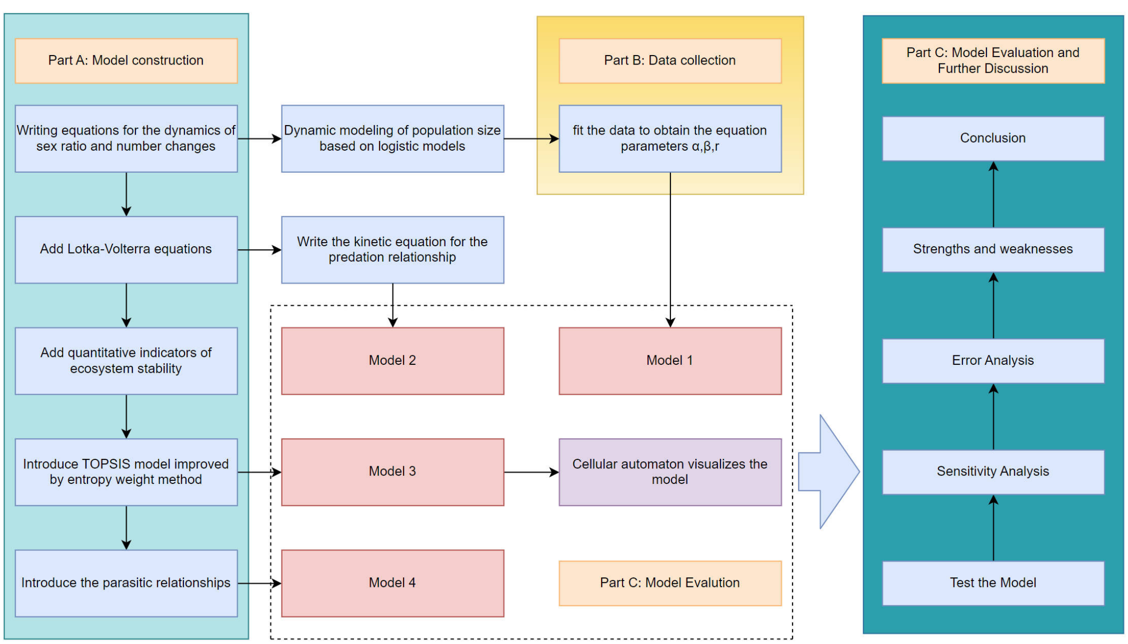


Figure 2: our work

2 Assumptions and Justification

While ensuring the correctness,to simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1:**
  ↪Justification:...[1]
- **Assumption 2:**
  ↪Justification:...[2]
- **Assumption 3:**
  ↪Justification:...[3]

3 Notations

The primary notations used in this paper are listed in Table.

Table 1: Notations

Symbol	Definition
N_F	the number of female lamprey
N_M	the number of male lamprey
TNF	total number of lamprey
RA	resource availability
c	Population fertility coefficient
r	the growth rate relative to sex ratio
α	the proportion of male in growth lamprey
β	the decrease rate in relation to number
U	the number of hosts and predators
h	the growth rates of hosts and predators
ϵ	the predation conversion factor
a	predatory capacity of lamprey

Other notations instructions will be given in the text.

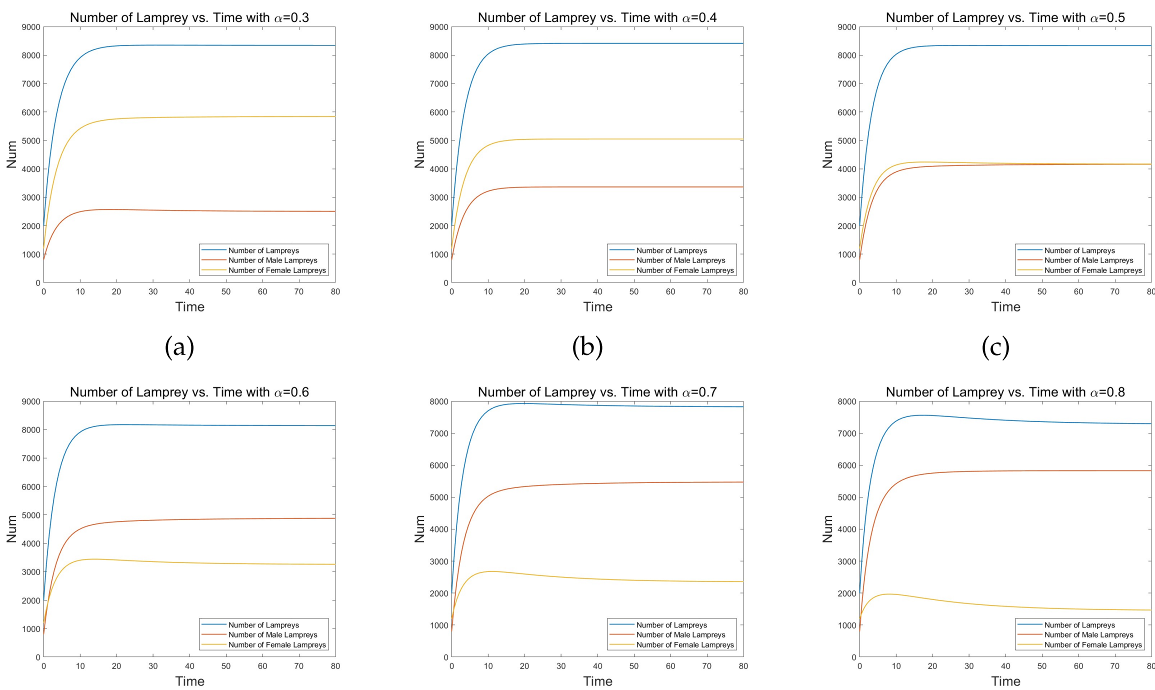
4 The Models and The solution

4.1 Model 1 and Solution

4.1.1 Details about Model 1

We define the max number of RA as RA_{max}

$$\begin{cases} \frac{\mathrm{d}N_M}{\mathrm{d}t} = r \cdot \alpha RA - \beta N_M - \frac{r \cdot RA}{\lambda} \\ \\ \frac{\mathrm{d}N_F}{\mathrm{d}t} = r \cdot (1-\alpha) RA - \beta N_F - \frac{r \cdot RA}{\lambda} \end{cases} \tag{4-1}$$



(a) (b) (c)

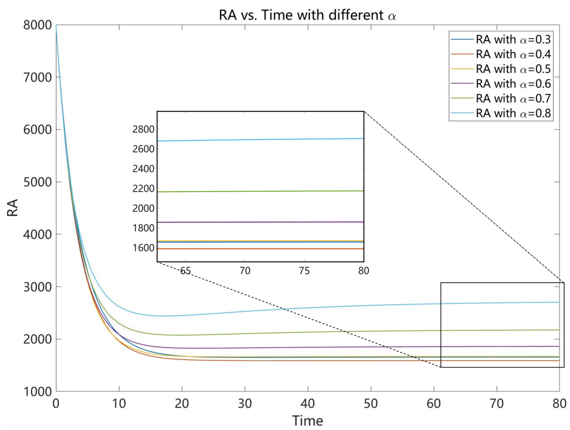
(d) (e) (f)

Figure 3: 111

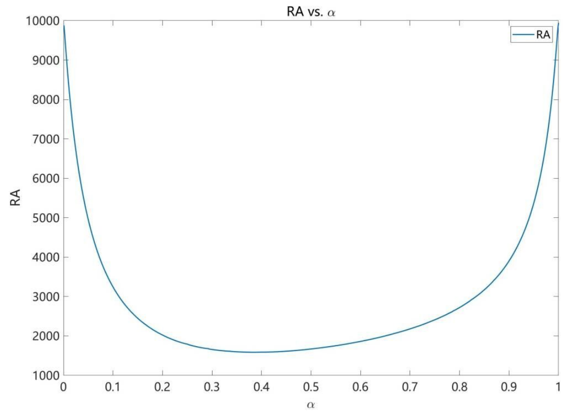
Table 2: The data of sea lamprey

Year	% Males	n	Males	Females
2007	53%	3,124	1,666	1,460
2008	57%	2,228	1,264	964
2009	54%	2,725	1,485	1,240
2010	58%	8,841	5,146	3,695
2011	60%	10,912	6,555	4,357
2012	60%	14,047	8,442	5,605
2013	61%	8,947	5,495	3,452
2014	59%	8,696	5,131	3,565

1. Sex ratio of sea lamprey in tributaries to Lakes Michigan and Huron[3]
2. the sum number of the Males is 59,522, for females the number is 59,522



(a) Image on the left



(b) Image on the right

Figure 4: Two images

4.2 Model 2 and Solution

4.2.1 Details about Model 2

4.2.2 Conclusion of Model 2

The results are shown in Figure , where t denotes the time in seconds, and c refers to the concentration of water in the boiler.

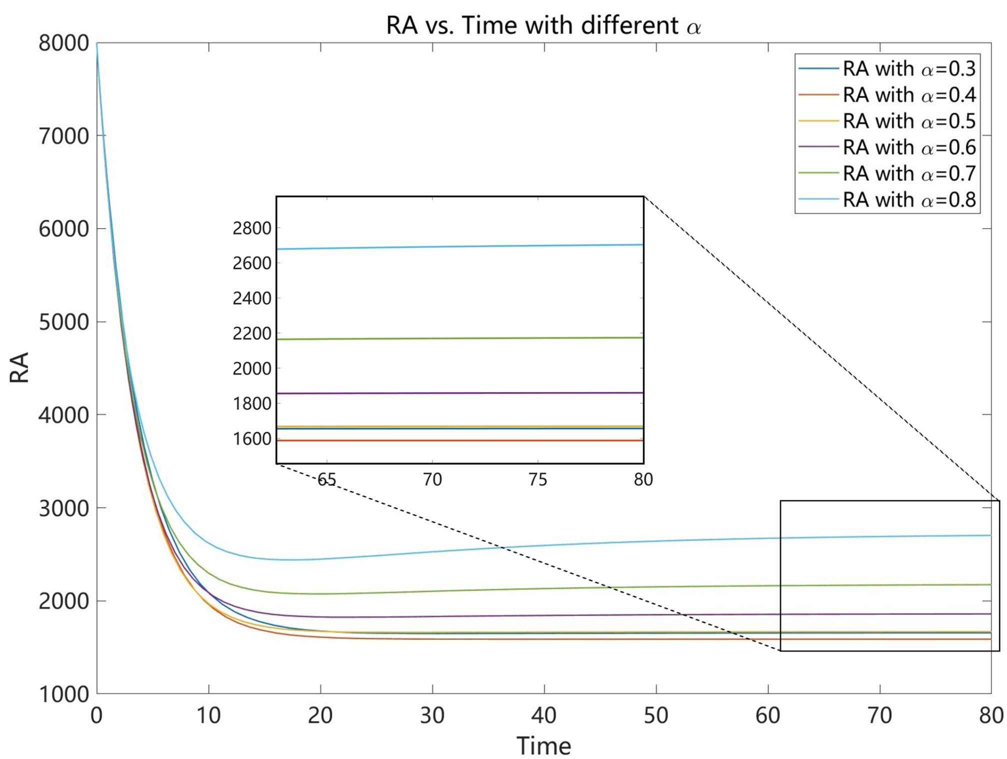


Figure 5: The result of Model 2

4.2.3 Commetary on Model 2

The instance of long and wide tables are shown in Table 3.

Table 3: Basic Information about Three Main Continents (scratched from Wikipedia)

Continent	Description	Information
Africa	Africa Continent is surrounded by the Mediterranean Sea to the north, the Isthmus of Suez and the Red Sea to the northeast, the Indian Ocean to the southeast and the Atlantic Ocean to the west.	At about 30.3 million km^2 including adjacent islands, it covers 6% of Earth's total surface area and 20% of its land area. With 1.3 billion people as of 2018, it accounts for about 16% of the world's human population.
Asia	Asia is Earth's largest and most populous continent which located primarily in the Eastern and Northern Hemispheres. It shares the continental landmass of Eurasia with the continent of Europe and the continental landmass of Afro-Eurasia with both Europe and Africa.	Asia covers an area of 44,579,000 square kilometres, about 30% of Earth's total land area and 8.7% of the Earth's total surface area. Its 4.5 billion people (as of June 2019) constitute roughly 60% of the world's population.
Europe	Europe is a continent located entirely in the Northern Hemisphere and mostly in the Eastern Hemisphere. It comprises the westernmost part of Eurasia and is bordered by the Arctic Ocean to the north, the Atlantic Ocean to the west, the Mediterranean Sea to the south, and Asia to the east.	Europe covers about 10,180,000 km^2, or 2% of the Earth's surface (6.8% of land area), making it the second smallest continent. Europe had a total population of about 741 million (about 11% of the world population) as of 2018.

Figure 4 gives an example of subfigures. Figure 4a is on the left, and Figure 4b is on the right.

5 Test the Model

5.1 Sensitivity Analysis

5.2 Error Analysis

6 Model Evaluation and Further Discussion

6.1 Strengths

- First one...
- Second one ...

6.2 Weaknesses

- Only one ...

6.3 Further Discussion

6.3.1 Model Improvement

6.3.2 Model Extension

References

[1] Karlin S, Lessard S. Theoretical studies on *sex ratio evolution*[J]. 1986.

[2] Almeida P R, Arakawa H, Aronsuu K, et al. *Lamprey fisheries: History, trends and management*[J]. Journal of Great Lakes Research, 2021, 47: S159-S185.

[3] *The National Center for Biotechnology Information: sea lamprey*, from `https://www.ncbi.nlm.nih.gov/pmc/articles/PMC5378093/`

[4] Lewandoski, S. A. Brenden, T. O. (2022). Forecasting suppression of invasive sea lamprey in Lake Superior. Journal of Applied Ecology, 59, 20232035. from urlhttps://doi.org/10.1111/1365-2664.14203

Appendix: Source Code on Model

Here are the program codes we used in our research.

test.py

```
# Python code example
for i in range(10):
    print('Hello, world!')
```

test.m

```
% Model 1
clear;clc;
figure
alpha = 0.8;
Ram = 10000;
[t,y] = ode45(@(t,y) lampreyModel(t,y,alpha,Ram),[0,80],[800,1200]);
plot(t,y(:,1)+y(:,2),'DisplayName','Number of Lampreys','LineWidth',1);
hold on;
plot(t,y(:,1),'DisplayName','Number of Male Lampreys','LineWidth',1);
hold on;
plot(t,y(:,2),'DisplayName','Number of Female Lampreys','LineWidth',1);
hold on;
title(['Number of Lamprey vs. Time with \alpha=',num2str(alpha)],
'FontSize',16)
set(gcf,'Position',[50,50,600,500]);
xlabel('Time','FontSize',16);
ylabel('Num','FontSize',16);
legend('Location','southeast');
hold on;

% Model 1 RA-Time
clear;clc;
figure;
Ram = 10000;
for i = 0:5
alpha = i*(0.8-0.3)/5+0.3;
[t,y] = ode45(@(t,y) lampreyModel(t,y,alpha,Ram),[0,80],[800,1200]);
plot(t,Ram-y(:,1)-y(:,2),'DisplayName',
['RA with \alpha=',num2str(alpha)],'LineWidth',1.5);
hold on;
end
title('RA vs. \alpha')
legend;
hold off;

% Model 1 RA-Alpha
clear;clc;
figure;
```

```
Ram = 10000;
RaM = zeros(1000,1);
x = [(1:1000)*1/1000];
for i = 1:1000
alpha = i*1/1000;
[t,y] = ode45(@(t,y) lampreyModel(t,y,alpha,Ram),[0,200],[800,1200]);
RaM(i) = Ram-y(end,1)-y(end,2);
end
plot(x,RaM,'LineWidth',1.5,'DisplayName','RA');
hold on;
title('RA vs. \alpha')
legend;
```

test.cpp

```
// C++ code example
#include <iostream>
using namespace std;

int main() {
    for (int i = 0; i < 10; i++)
        cout << "hello, world" << endl;
    return 0;
}
```