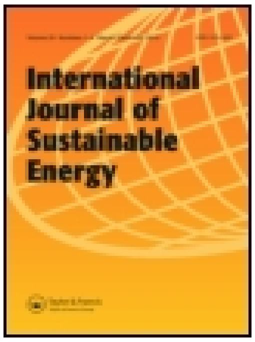

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A COMPARISON OF THREE MODELS FOR ESTIMATION OF GLOBAL SOLAR IRRADIATION ON TILTED AND ORIENTED SURFACES IN ATHENS

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Solar radiation data from one location in Greece (Athens), are used to test the applicability of three models for computing global irradiance on tilted surfaces. The models selected for discussion, applicability, and comparisons are the model suggested by Liu and Jordan dated 1962[1], the model of Klein dated 1977[3], and the model of Klein and Theilacker dated 1981[4]. All these three models are applied, tested and compared for their reliability. The comparative study of the models shows that all these models are rather accurate for locations in Greece. But it seems that the Klein and Theilacker's analytical method for estimating $\bar{R}$ leads to better results. $\bar{R}$ values calculated in this manner and through the other methods are compared. Therefore, the above three models are used to estimate global solar irradiation for one location on surfaces of various tilt angles and azimuths, and the results are presented and discussed.

KEY WORDS: Global solar irradiation, Tilted surfaces, Solar energy, Solar constant, Reflectance, Inclination angle, Angle of incidence, Diffuse irradiance, Extraterrestrial irradiance.

1 INTRODUCTION

There is a necessity for accurate estimates of available solar radiation on inclined surfaces since solar energy applications increase and it is important for the design of conversion systems, and other applications. Monthly averages of the daily global irradiation incident upon a horizontal surface are available for many locations. However, radiation data on tilted surfaces are generally not available. As a result, theoretical estimates of the available solar energy have to be utilized. Thus, various algorithms have been developed that produce available global solar irradiation from other existing data.

An estimation of the average global irradiation for each month for tilted surfaces facing the equator was proposed by Liu and Jordan[1]. In Klein's review paper[3] the above work has been summarized. Klein[3] gives an additional relationship for $\bar{R}_b$, the ratio of the average beam radiation on the tilted surface to that on the horizontal surface for each month, for tilted surfaces facing angle α east or west of the due south.

Several mathematical methods have been developed by a number of investigators to calculate global irradiance on tilted surfaces from measurements on a horizontal surface[3-4]. In addition, there are some other methods to compute global irradiance on tilted surfaces from measured daily global and diffuse irradiation on a horizontal surface[5,6,7]. A significant difference among the various models appears in the assessment of the sky-diffuse component. If the global irradiation on a horizontal surface is known, a different way to calculate the diffuse component has been proposed by Liu and Jordan[18] and improved by Collares-Pereira and Rabl[15], Erbs *et al.*[19] and others[30]. This is based on the correlation of the average daily sum of diffuse irradiation, $\bar{D}_m$, and with the average daily sum of global irradiation on a horizontal surface, $\bar{G}_m$, the correlation parameter is the "clearness index", $\bar{K}_T$, which is equal to the ratio of $\bar{G}_m$ to the daily sum of the extraterrestrial global solar irradiation, $\bar{G}_{om}$. Similar correlations have also been putforth for mean hourly values on the basis of the knowledge of global solar irradiation.

In particular, for Greece, Kambezidis *et al.*[2] and Pissimanis *et al.*[12] developed instead algorithms that utilize only routine meteorological data as input and require very limited computational resources. Lalas *et al.*[20] also effectuated correlations for calculations of the diffuse component of the daily and monthly average global irradiation in terms of the "clearness index" $\bar{K}_T$ for three Greek locations (Athens included). They found simple linear or, at most, quadratic regression equations. Additionally, Lalas *et al.*[29] developed a computer program for estimating global irradiance components with quite realistic results.

Liu and Jordan's and Page's methods are widely used because of their relative simplicity as they require only mean daily global irradiation on a horizontal surface for each month. Here, the Liu-Jordan[1], Page[21] and Lalas *et al.*[20] relationships are used for the $\bar{D}_m$ calculation. Refering now to the average daily irradiance on a tilted surface, $\bar{G}\ (\beta,\alpha)$ calculation, Simonson[5] attempted to improve Klein's formulae by calculating one of its factors, $\bar{R}_b$, in a more accurate way. In addition, the models of Liu and Jordan[1], Klein[3], and Klein and Theilacker[4] are chosen for application, comparison, and discussion in the present study. These models are supposed to be applicable to locations anywhere in the world. Results obtained by the above models were compared with each other in order to examine their applicability. Thus, the purpose of this study is

(i) to compare statistically three widely used models for estimating global irradiance on inclined surfaces for their applicability, and to recommend the most accurate one for estimates for climatic locations similar to Greek region, and
(ii) to compute global irradiance on surfaces at various tilt angles and azimuths for a location in Greece.

Solar radiation data over a period of 37 years (1953-1989) at the solar radiation station of Athens (ϕ = 37°58′N, λ = 23°45′E, z = 107m above m.s.l.) are utilized in the present work to test the applicability of the three models.

2 MODELS OF ESTIMATION

The mean global irradiance on a tilted surface, $\bar{G}(\beta,\alpha)$, is obtained by adding the direct $\bar{I}$, the diffuse, $\bar{D}_m$, and the ground-reflected irradiance, $\bar{R}_g$, components. The models discussed here all share the same formulations for the diffuse and ground-reflected components but a different formulation for the direct component[31].

The average daily global irradiance on a tilted surface, $\bar{G}(\beta,\alpha)$, is expressed as

$$\bar{G}(\beta,\alpha) = \bar{R}\bar{G}_m = \bar{R}\bar{K}_T\bar{G}_{om} \tag{1}$$

where $\bar{R}$ is the ratio of the daily average global irradiance on a tilted surface (neglecting the effect of non-isotropical distribution of the sky component) to that on a horizontal surface for each month. The global irradiance arriving on an inclined surface, $\bar{G}(\beta,\alpha)$, is considered as comprising three components: direct, diffuse and ground-reflected irradiation; it is given by

$$\bar{G}(\beta,\alpha) = (\bar{G}_m - \bar{D}_m)\bar{R}_b + \bar{D}_m\,(1 + \cos\beta)/2 + P_g\,\bar{G}_m\,(1 - \cos\beta)/2$$

Dividing both sides by $\bar{G}_m$ we obtain

$$\bar{R} = \underbrace{\left(1 - \frac{\bar{D}_m}{\bar{G}_m}\right)\bar{R}_b}_{\text{(direct)}} + \underbrace{\frac{\bar{D}_m}{\bar{G}_m}\left(\frac{1+\cos\beta}{2}\right)}_{\text{(diffuse)}} + \underbrace{P_g\left(\frac{1-\cos\beta}{2}\right)}_{\text{(reflected)}} \tag{3}$$

where $\bar{R}_b$, is the ratio of the average direct irradiance incident on an inclined plane to that on a horizontal plane for each moth; β, is the tilt of the surface from the horizontal; P_g, is the ground reflectance; $\bar{D}_m$, is the monthly average daily diffuse irradiation on a horizontal surface; $\bar{G}_m$, is the month average daily global irradiation on a horizontal surface; $\bar{G}_{om}$, is the calculated mean daily extraterrestrial global irradiation for each month; $\bar{K}_T$, is the "clearness index" expressed as the ratio of $\bar{G}_m$ to $\bar{G}_{om}$,

$$\bar{K}_T = \bar{G}_m/\bar{G}_{om} \tag{4}$$

The monthly average extraterrestrial irradiation $\bar{G}_{om}$ is given by

$$\bar{G}_{om} = \frac{1}{(m_2 - m_1)} \sum_{n=m_1}^{m_2} (G_o)_n \tag{5}$$

where m_1 and m_2 are, respectively, the days of the year at the start and end of the month and $(G_o)_n$ is the extraterrestrial irradiation on a horizontal surface on day n of the year which is approximated by

$$(G_o)_n = \frac{24}{\pi} I_o \left[1 + 0.033 \cos\left(\frac{360n}{365}\right)\right] \times [\cos\phi \cos\delta \sin\omega_s + (\omega_s 2\pi/360) \sin\phi \sin\delta] \tag{6}$$

where I_o, is the solar constant (given in Table 2), n is the day of the year, ϕ is the latitude and δ is the declination of the sun which can be approximately expressed by

$$\delta = 23.45^{\circ} \sin [360(284+n)/365] \tag{7}$$

ω_S, is the sunset hour angle

$$\cos \omega_S = -\tan \phi \tan \delta \tag{8}$$

The values of n selected for eqns (6) and (7) to estimate mean monthly values of daily extraterrestrial irradiation and recommended by Page[28] are given in Table 1. Appropriately weighted monthly mean values of the declination (δ) are obtained and given in Table 2. The values of $\overline{G}_{om}$ for day n estimated for the latitude of Athens from eqn (6) for each month are tabulated in Table 3. $\overline{G}_{om}$ can be conveniently estimated from eqn (6) by selecting for each month, the day of the year for which the daily extraterrestrial global irradiation is nearly the same as the monthly mean value. Using the recommended days for each month (Table 1) can lead to small errors in $\overline{G}_{om}$. The term within the first set of brackets in eqn(6) accounts for the seasonal variation in irradiation intensity above the atmosphere caused by the variation in earth-sun distance due to ellipticity of the earth's orbit. The three terms in eqn(3) will be discussed separately in the sections to follow.

2.1 *The Diffuse Component*

As mentioned above, assuming isotropic diffusion, the diffuse component (the 2nd term in eqn (3)), $\dfrac{\overline{D}_m}{\overline{G}_m}\left(\dfrac{1+\cos\beta}{2}\right)$ is expressed in the same manner for all the three models applied in this study.

Since measurements of $\overline{D}_m$ are rarely available, $\overline{D}_m$ must be estimated from measurement of the average daily global irradiance. The three relationships we used to find the diffuse irradiance fraction $\overline{D}_m/\overline{G}_m$ as a function of $\overline{K}_T$ are expressed by

$$\frac{\overline{D}_m}{\overline{G}_m} = \begin{cases} 1.390 - 4.027\,\overline{K}_T + 5.531\,\overline{K}_T^{\,2} - 3.108\,\overline{K}_T^{\,3} & \text{(Liu \& Jordan, 1962)} \\ 1.00 - 1.13\,\overline{K}_T & \text{(Page, 1961)} \\ 1.26 - 1.41\,\overline{K}_T & \text{(Lalas \textit{et al.}, 1987)} \end{cases} \tag{9}$$

All these relationships were derived from experimental data. Page's correlation gives more accurate estimate of the diffuse irradiance fraction. The third equation of expression (9) is valid only for the Athens location, since they used data from Athens.

2.2 *The Ground Reflected Component*

For isotropic reflection and using a common albedo P_g the daily ground-reflected diffuse irradiance is expressed by the 3rd term in eqn(3).

$$R_g = P_g \left(\frac{1-\cos\beta}{2} \right) G_m$$

and is also calculated by the same way for all the three models.

Note that the ground reflection P_g is usually taken to be equal to 0.2 which corresponds to bare soil reflectivity[1,2]

2.3 *The Direct Component*

As already mentioned the three models discussed in this work differ in the formulation for the direct component.

The equations for the direct component calculations proposed by Liu and Jordan[1], Klein[3], and Klein-Theilacker's[4] methods are presented in the followings.

2.3.1. *Liu and Jordan's Method*

Liu and Jordan's[1] procedure suggests that the monthly mean daily direct irradiation received on an inclined surface I_m (ß, α) can be expressed by the 1st term in eqn(3), i.e.

$$I_m(\beta,\alpha) = \frac{(\bar{G}_m - \bar{D}_m)}{\bar{G}_m} \bar{R}_b = (1 - \frac{\bar{D}_m}{\bar{G}_m} \bar{R}_b) \tag{10}$$

where $\bar{R_b}$ is the ratio of the mean extraterrestrial irradiation incident on an inclined plane to that on a horizontal plane in the absence of the earth's atmosphere for each month. $\bar{R}_b$ on a horizontal surface can be written as the difference between the global and diffuse values in eqn (10), because they are more commonly measured using pyranometer. $\bar{R}_b$ is a function of the transmittance of the atmosphere and for surfaces facing directly towards the equator is calculated by

$$\bar{R}_b = \frac{\cos(\phi-\beta)\cos\delta \sin\omega'_S + (\pi/180)\,\omega'_S \sin(\phi-\beta)\sin\delta}{\cos\phi \cos\delta \sin\omega_S + (\pi/180)\omega_S \sin\varphi \sin\delta} \tag{11}$$

where ω is the solar hour angle which is 15° x (hours from solar noon), afternoon positive, mornings negative and ω'_S is the sunset hour angle for the tilted surface which is given by

$$\omega'_S = \min[\omega_S, \arccos\{-\tan(\phi-\beta)\tan\delta\}] \tag{12}$$

At this point we applied expressions (9) to calculate $\bar{D}_m/\bar{G}_m$. Generally, the values of $\bar{R}$ estimated from eqn (3) with $P_g = 0.2$ tend to agree more closely with experimental measurements when the Liu and Jordan relationship is used, as has been shown by Klein[3].

2.3.2 Klein's Method

The method developed by Liu and Jordan[1] for estimating average daily global irradiation for each calendar month on surfaces facing directly towards the equator is verified with experimental measurements and extended to allow calculation of monthly average irradiation on surfaces of a wide range of orientations. The extension to surfaces of orientation other than south involves the integration of extraterrestrial irradiation on the surfaces for the period during which the sun is both above the horizon and in front of the surface. This is divided by the mean daily extraterrestrial irradiation on a horizontal surface. The expression of the direct component in eqn (10) above always holds in the procedure of this model. The difference of this method from that of Liu and Jordan's method is due to $\bar{R}_b$ formulation. The equations for the calculations proposed by Klein as corrected by Andersen[25] and Klein[26] are:

$$\bar{R}_b = [\cos\beta \sin\delta \sin\phi) (\pi/180) (\omega_{SS}-\omega_{Sr}) - (\sin\delta \cos\phi \sin\beta \cos\alpha) (\pi/180) / (\omega_{SS}-\omega_{Sr}) + \\ + (\cos\phi \cos\delta \cos\beta) (\sin\omega_{SS}-\sin\omega_{Sr}) + (\cos\delta \cos\alpha \sin\phi \sin\beta) (\sin\omega_{SS}-\sin\omega_{Sr}) + \\ + (\cos\delta \sin\beta \sin\alpha) (\cos\omega_{SS}-\cos\omega_{Sr})] / 2[(\cos\phi \cos\delta \sin\omega_S + (\pi/180)\omega_S \sin\phi \sin\delta)] \quad (13)$$

where α is the surface azimuth with values in the range ±90° of the due south, and

$$\omega_{SS} = \min\{\omega_S, \cos^{-1}[(AB - \sqrt{A^2-B^2+1} /A^2+1)]\} \quad \text{if } \alpha < 0 \quad (14a)$$

$$\min\{\omega_S, \cos^{-1}[(AB + \sqrt{A^2-B^2+1} / (A^2+1)]\} \quad \text{if } \alpha > 0 \quad (14b)$$

$$\omega_{Sr} = -\min\{\omega_S, \cos^{-1}[(AB + \sqrt{A^2-B^2+1} /A^2+1)]\} \quad \text{if } \alpha < 0 \quad (15a)$$

$$-\min\{\omega_S, \cos^{-1}[(AB - \sqrt{A^2-B^2+1} / (A^2+1)]\} \quad \text{if } \alpha > 0 \quad (15b)$$

with $0° < \omega_S < \pm 90°$ (east positive, west negative)

$$A = \cos\varphi / ((\sin\alpha \tan\beta) + \sin\varphi/\tan\alpha)) \\ B = \tan\delta [\cos\varphi/\tan\alpha - \sin\varphi/(\sin\alpha \tan\beta)] \quad (16)$$

where angular values are in radians.

$\bar{R}$ is estimated by individually considering the direct, diffuse and reflected components of irradiance on the tilted plane. The expression for $\bar{R}_b$ assumes that the ratio of the mean component on a tilted plane to that on a horizontal plane is the same at the earth's surface as above the atmopshere.

Daily average monthly values for day n of Table 1 for Liu-Jordan and Klein models of $\bar{G}_m$, were calculated. In addition, from the daily average monthly values of $\bar{D}_m$, $\bar{G}_m$, $\bar{G}_{om}$ the values of $\bar{D}_m/\bar{G}_m$ given by eqn (9) and the $\bar{G}_m/\bar{G}_{om}$ values were calculated. These values are given in Table 3. From these values the estimates of $\bar{R}$ factors and the $\bar{G}(\beta,\alpha)$ values for tilted planes are obtained. The calculations of $\bar{G}_m$ (and $\bar{R}$, $\bar{R}_b$, $\bar{G}(\beta, \alpha)$) values can be modified in various ways in accordance with the prevailing moisture, air-pollution and/or cloudiness conditions which affect some terms of the relative equations.

Taking the minimum of the two expressions in each of eqns(14) and (15) accounts for the fact that the sun may fall behind the tilted plane during early morning or late afternoon while it is still above the horizon.

Table 1. Recommended average day for each month and corresponding mean values of global irradiation ($\bar{G}_m$) (for Athens, latitude ϕ = 37° 58′) Greece, used for Liu-Jordan and Klein models (4th column). In the 5th column the mean monthly global irradiation used in Klein-Theilacker's model is given

Month	*Day of the year (n)*	*Date*	*$\bar{G}m$ ($MJ.m^{-2}$) Liu-Jordan and Klein*	*$\bar{G}m$($MJ.m^{-2}$) Klein-Theilacker*
J	17	Jan. 17	7.95	7.22
F	46	Feb. 15	10.32	10.01
M	75	Mar. 16	13.39	13.92
A	105	Apr. 15	17.66	18.96
M	135	May 15	20.54	22.74
J	162	June 11	24.63	25.28
J	198	July 17	25.81	25.50
A	228	Aug. 16	23.31	23.10
S	259	Sept 16	19.52	18.45
O	289	Oct. 16	12.06	12.54
N	319	Nov. 15	8.22	8.39
D	345	Dec. 15	5.98	6.49

To estimate $\bar{G}(\beta, \alpha)$, the average daily irradiance on a tilted plane within a given month:

(i) Obtain the average daily irradiation on a horizontal plane at the earth's surface, $\bar{G}_m$, from local data which may be available.
(ii) Obtain the daily irradiation on a horizontal plane at the top of the atmosphere, $\bar{G}_{om}$ for the day n given in Table 1.
(iii) Use the resulting $\bar{G}_m/\bar{G}_{om}$ to obtain $\bar{D}_m/\bar{G}_m$ (from eqn (9)) and with the tilt angle, β, it can be obtained the coefficient for diffuse irradiance, $(\bar{D}_m/\bar{G}_m)(1+\cos\beta)/2$.
(iv) Obtain the ratio $\bar{R}_b$ from equ (13).
(v) Solve eqns (1) and (2) to obtain the average daily global irradiance on the tilted plane, $\bar{G}(\beta,\alpha)$, using values from the proceeding steps.

2.3.3 Klein and Theilacker's Method

A procedure designed to take into account the effect of the atmosphere was proposed by Klein and Theilacker[4].

This method develops an algorithm for $\bar{R}$ for any surface orientation. It differs from Liu and Jordan and Klein methods in the manner in which the direct irradiation component is determined. $\bar{R}$ can be found by integrating I_T, the instantaneous irradiation on an inclined surface, and I, the instantaneous irradiation on a horizontal surface over a period of N days.

$$\bar{R} = \frac{\sum_{n=1}^{N} \int_{t_{SS}}^{t_{SS}} I_T dt}{\sum_{n=1}^{N} \int_{t_{SR}}^{t_{SS}} I dt}) \tag{17}$$

where t_{SR} and t_{SS}, the sunrise and sunset times.

It must be stated that in order to apply this procedure one has to use 300 days (No >300) at least in eqn (17) to represent the long-term average conditions accurately. This number corresponds to (at least) 10 years data for each month.

If measurements of I are available, I_T can be estimated by separately considering the direct, diffuse, and ground reflected irradiation components. Assuming diffuse and ground-reflected irradiation to be isotropic, I_T can be expressed:

$$I_T = (I - I_d)\,R_d + I_d\,(1+\cos\beta)/2 + P_g I(1-\cos\beta)/2 \tag{18}$$

where I_d is the instantaneous diffuse irradiation on a horizontal surface and R_b is the ratio of instantaneous direct irradiation on the inclined surface to that on a horizontal surface. As shown in Duffie and Beckman[23], R_b is equal to the ratio of the cosine of the solar incidence angle $\nu(\beta,\alpha)$ on the inclined surface to the cosine of the solar zenith angle $\zeta = \frac{\pi}{2} - \gamma$ (γ = solar elevation),

$$R_b = \frac{\cos\nu(\beta,\alpha)}{\cos\zeta} = \frac{A\cos\omega - B + C\sin\omega}{\cos\omega - \cos\omega_S} \tag{19}$$

where, $A = \cos\beta + \tan\phi\cos\alpha\,\sin\beta$, $B = \cos\omega_S\cos\beta + \tan\delta\sin\beta\cos\alpha$

$C = \sin\beta\sin\alpha/\cos\phi$, with the usual notation for ϕ, δ, α, ω, and ω_S (20)

Substituting eqn (18) for I_T, eqn (17) can be rewritten as

$$\overline{R} = \frac{\int_{t_{SR}}^{t_{SS}} \{(\overline{I} - \overline{I}_d)R_b + \overline{I}_d(1+\cos\beta)/2 + P_g\overline{I}\,(1-\cos\beta)/2\}\,dt}{\overline{G}_m} \tag{21}$$

where the bars indicate long-term average values of these quantities obtained by summing up I and I_d over N days and then dividing by N.

Liu and Jordan[27], and Collares-Pereira and Rabl[15] have shown that I_d and I can be related to $\overline{D}_m$ and $\overline{G}_m$, through the relations

$$\overline{I}_d = r_d\,\overline{D}_m \text{ and } \overline{I} = r_T\overline{G}_m \tag{22}$$

$$\text{where } r_d = \frac{\pi}{24d}\,[\cos\omega - \cos\omega_S] \tag{23}$$

$$r_T = (a + b\cos\omega)\,r_d \tag{24}$$

with $a = 0.409 + 0.5016\sin(\omega_S - 60°)$, $b = 0.6609 - 0.47647\sin(\omega_S - 60°)$

$$d = \sin\omega_S - \frac{\pi}{180°}\omega_S\cos\omega_S \tag{25}$$

Substituting eqns(22) through (25) into eqn(21) and integration for the period between sunrise and sunset results in the following relation for $\overline{R}$,

$$\overline{R} = \overline{D} + \overline{D}_m/\overline{G}_m\,(1+\cos\beta)/2 + P_g(1-\cos\beta)/2 \tag{26}$$

where

$$D = \begin{cases} \max\{0, G(\omega_{SS}, \omega_{SR})\} & \text{if } \omega_{SS} \geq \omega_{SR} \\ \max\{0, [G(\omega_{SS}, \omega_s) + G(\omega_s, \omega_{SR})]\} & \text{if } \omega_{SR} > \omega_{SS} \end{cases} \tag{27}$$

$$G(\omega_1,\omega_2) = \frac{1}{2d}\{(\frac{bA}{2} - a'B)(\omega_1 - \omega_2)\frac{\pi}{180} + (a'A - bB)(\sin\omega_1 - \sin\omega_2) - a'C(\cos\omega_1 - \cos\omega_2) + \frac{bA}{2}(\sin\omega_1\cos\omega_1 - \sin\omega_2\cos\omega_2) + \frac{bC}{2}(\sin^2\omega_1 - \sin^2\omega_2)\} \tag{28}$$

and $$a' = a - \frac{\overline{D}_m}{\overline{G}_m} \tag{29}$$

The sunrise and sunset hour angles, ω_{SR} and ω_{SS}, are found by setting the numerator of eqn(19) equal to zero and solving for ω. A quadratic equation results, giving two values for ω which must be within $\pm\omega_S$. The signs of ω_{SR} and ω_{SS} are affected by the surface orientation

$$|\omega_{Sr}| = \min\{\omega_s, [\arccos(AB + C\sqrt{A^2 - B^2 + C^2} / (A^2+C^2)]\} \tag{30}$$

$$\omega_{SR} = \begin{cases} -|\omega_{SR}| & \text{if } (A>0 \text{ and } B>0) \text{ or } A \geq B \\ +|\omega_{SR}| & \text{otherwise} \end{cases} \tag{31}$$

$$|\omega_{SS}| = \min\{\omega_s, [\arccos(AB - C\sqrt{A^2-B^2+C^2} / (A^2+C^2)]\} \tag{32}$$

$$\omega_{SS} = \begin{cases} +|\omega_{SS}| & \text{if } (A>0 \text{ and } B>0) \text{ or } A \geq B \\ -|\omega_{SS}| & \text{otherwise} \end{cases} \tag{33}$$

Table 2. Monthly declination and mean monthly values of solar constant (Io) used in this study[28]

Month	*Declination (degrees)*	*Io(w.m^{-2})*
J	−20.71	1411
F	−12.81	1401
M	−1.80	1382
A	9.77	1358
M	18.83	1337
J	23.07	1325
J	21.16	1322
A	13.65	1333
S	2.89	1353
O	−8.72	1377
N	−18.37	1397
D	−22.99	1409

Depending on the surface orientation and time of year, the sun may rise and set on an inclined surface once, twice, or not at all. The algorithm given by eqns (21) through (33) will result in an appropriate value of $\bar{R}$ in all of these situations.

A simpler algorithm can be derived for surfaces oriented directly toward the equator. In this case

$$\bar{R} = \frac{\cos(\phi-\beta)}{d\cos\phi}\left[\left(a - \frac{\bar{D}_m}{\bar{G}_m}\right)\left(\sin\omega's - \frac{\pi}{180}(\omega's)\cos\omega''s\right) + \right.$$

$$\left.\frac{b}{2}\left((\omega'_s)\frac{\pi}{180} + \sin\omega'_s(\cos\omega'_s - 2\cos\omega''_s)\right)\right] +$$

$$\frac{\bar{D}_m}{\bar{G}_m}\left(\frac{1+\cos\beta}{2}\right) + P_g\frac{(1-\cos\beta)}{2} \tag{34}$$

where $\omega''_s = \arccos(-\tan(\varphi - \beta)\tan\delta)$, $\omega'_s = \min(\omega'_s, \omega''_s)$ (35)

3 RESULTS AND DISCUSSION

Unfortunately, long-term measurements of radiation data on both inclined and horizontal surfaces are not available to permit an evaluation of the $\bar{R}$ algorithms described in section 2. However, several methods have been developed for the estimation of daily or monthly average global irradiation on tilted surfaces based on horizontal surface data. Three methods for estimating $\bar{R}_T$ (eqn 10, 13, 26) are used. All three algorithms rely on a number of assumptions, e.g. that irradiation is symmetrically distributed about solar noon, and that diffuse and ground-reflected irradiations are isotropic. There are differences among the three algorithms in the manner in which the direct irradiation component is treated. The Liu and Jordan method assumes that $\bar{R}_b$ can be estimated as the ratio of monthly-average extraterrestrial irradiation on the inclined surface to that on a horizontal surface, neglecting the fact that the long-term average ratio of instantaneous direct to extraterrestrial irradiation varies with time of day. The Klein-Theilacker

Table 3. Daily average monthly values $\bar{G}_m$, $\bar{D}_m$, $\bar{G}_{om}$ (MJ.m^{-2}), $\bar{D}_m/\bar{G}_m$ and $\bar{G}_m/\bar{G}_{om}$ obtained by Klein-Theilacker's model using Liu and Jordan's approach for the diffuse component calculation (at Athens).

Month	$\bar{G}_m$	$\bar{D}_m$	$\bar{G}_{om}$	$\bar{D}_m/\bar{G}_m$	$\bar{G}_m/\bar{G}_{om}=\bar{K}_T$
J	7.22	3.23	17.70	0.45	0.408
F	10.01	4.32	23.16	0.43	0.432
M	13.92	5.60	30.03	0.40	0.464
A	18.96	6.69	36.33	0.35	0.522
M	22.74	7.28	40.28	0.32	0.565
J	25.28	7.33	41.80	0.29	0.605
J	25.50	6.97	40.66	0.27	0.627
A	23.10	6.44	37.28	0.28	0.620
S	18.45	5.72	31.94	0.31	0.578
O	12.54	4.67	25.19	0.37	0.498
N	8.39	3.56	19.07	0.42	0.440
D	6.48	2.99	16.04	0.46	0.405

algorithm given by equations (17) to (33) uses this information. All algorithms require an estimate of the ($\bar{D}_m/\bar{G}_m$) fraction which is related to $\bar{K}_T = \dfrac{\bar{G}_m}{\bar{G}_{om}}$. The Liu and Jordan correlation is used in the results which are given in Table 3. The alternative methods of calculating $\bar{R}$ are compared in Table 4. The methods mentioned have been used to surfaces of arbitrary tilt and azimuth. Results are tabulated and are presented below and are compared among the three methods. To verify the applicability of the methods used in this paper and to compare their results, we have calculated incident

Table 4. Comparison of calculated values of $\bar{R}$ for a vertical surface facing south ($\beta = 90°$, $\alpha = 0$) ϕ = 37° 58′N, using Liu and Jordan's procedure for the diffuse component calculation

Month	$\bar{K}_T$ *(L-J)* - Klein*	$\bar{R}$ *Eqns(3) and (10) (L-J)**	$\bar{R}$ *Calculated from eqns (3) and (13) (Klein)*	$\bar{K}_T$ *(Klein-Theilacker)*	$\bar{R}$ *Calculated from eqns (26) or (34) (K-T)**
J	0.449	1.56	1.26	0.408	1.399
F	0.446	1.18	1.02	0.432	1.110
M	0.446	0.82	0.80	0.464	0.816
A	0.486	0.57	0.64	0.522	0.580
M	0.510	0.43	0.53	0.565	0.432
J	0.588	0.36	0.46	0.605	0.368
J	0.635	0.37	0.48	0.627	0.387
A	0.625	0.49	0.59	0.620	0.503
S	0.611	0.73	0.76	0.578	0.725
O	0.479	1.05	0.94	0.498	1.025
N	0.431	1.40	1.16	0.440	1.338
D	0.373	1.55	1.25	0.405	1.513

(L-J)* Liu and Jordan
(K-T)* Klein - Theilacker

Table 5. Calculation of daily average global irradiance ($\bar{G}$) on a surface tilted β = 37° 58′ from horizontal in Athens (ϕ = 37° 58′N) (with α = 0 and α = ±15°), estimated by Klein's model using Liu and Jordan's approach for the diffuse component calculation

Month	$\bar{R}_b$	$\bar{R}_b$	$\bar{R}$	$\bar{R}$	$\bar{G}(\beta,\alpha)$ $\alpha = 0$ (MJ.M^{-2}. day^{-1})	$\bar{G}(\beta,\alpha)$ $\alpha = 15°$ (MJ.m^{-2}.day^{-1})
J	1.779	1.745	1.433	1.413	11.39	11.23
F	1.524	1.500	1.281	1.267	13.22	13.07
M	1.297	1.287	1.150	1.144	15.40	15.32
A	1.133	1.133	1.063	1.063	18.77	18.77
M	1.027	1.030	1.000	1.002	20.53	20.57
J	0.982	0.986	0.976	0.979	24.05	24.12
J	1.002	1.006	0.994	0.997	25.67	25.73
A	1.086	1.088	1.054	1.056	24.58	24.62
S	1.225	1.219	1.152	1.148	22.49	22.41
O	1.428	1.409	1.241	1.230	14.97	14.83
N	1.691	1.660	1.367	1.350	11.24	11.09
D	1.878	1.841	1.411	1.393	8.45	8.34

global irradiance on variously oriented tilted surfaces for Greece. Athens is selected for having rather long-term measurements of total and diffuse global irradiation and smooth changes in the amount and structure of global irradiation during the year. The total global irradiation values for the n days of Table 1, averaged over the 37 years (1953-1989) have been used in the relationships of Liu-Jordan and Klein's methods. Besides, the mean monthly values for the same period have been used in the formulae of Klein-Theilacker's procedure. In addition, monthly average daily values of global irradiation were taken as input data, and separated into direct and diffuse components using Liu-Jordan's, Page's and Lalas's correlation for the calculation of $\bar{D}_m$ component. $\bar{R}$ factors were then calculated in three ways. e.g. using Liu-Jordan's, Klein's, and Klein's-Theilacker's methods (eqns. (17) to (33)).

The most interesting and most characteristic results of the calculations are given in Tables (4) to (9). Estimates of monthly averages of daily irradiance incident on a surface tilted 38° from the horizontal facing south in Athens (ϕ =38°N) are obtained and compared with those incident on surfaces oriented 15° west of south; the results are displayed in Table 5.

The daily average irradiation incident on a horizontal surface for each month $\bar{G}_m$ is obtained from Table 3. The mean daily extraterrestrial global irradiation on a horizontal surface, $\bar{G}_{om}$, for each month can be determined from eqn(6) using the days of the year in Table 1 or from Table 3.

The ratio $\bar{G}_m/\bar{G}_{om}$ determines $\bar{K}_T$ for each month which can be used to calculate $\bar{D}_m/\bar{G}_m$ from eqns(9). In Table 4 there appear two different columns of values for K_T: the first column results from Liu-Jordan and Klein models, while the second one is obtained by using the Klein-Theilacker model. The differences are due to the different global irradiation values utilized in the models (see Table 1). The values of $\bar{R}$ for vertical surfaces oriented towards south are also given in Table 4.

Table 6. Monthly values of $\overline{G}(\beta,\alpha)$ ($MJ.m^{-2}$), $\overline{R}_T$ calculated by Klein – Theilacker method, at β=30°, with azimouth α=90°, 60°, 45°, 30°, 0, –30°, –45°, -60°, –90°, for Athens data (1953–1989), by using Liu – Jordan diffuse irradiation relationships

Azimouth (α)	J		F		M		A		M		J		J		A		S		O		N		D	
	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$	$\overline{G}(\beta,\alpha)$	$\overline{R}_T$
90°	6.94	0.961	9.50	0.949	13.09	0.940	17.74	0.936	21.24	0.934	23.61	0.934	23.82	0.934	21.62	0.936	17.36	0.941	11.89	0.948	8.05	0.959	6.27	0.966
60	8.55	1.183	11.15	1.114	14.56	1.046	18.85	0.994	21.76	0.957	23.75	0.939	24.17	0.948	22.70	0.982	19.12	1.036	13.80	1.100	9.83	1.171	7.86	1.212
45	9.27	1.283	11.87	1.185	15.17	1.089	19.25	1.015	21.86	0.961	23.66	0.936	24.16	0.948	23.03	0.997	19.80	1.073	14.61	1.164	10.62	1.266	8.58	1.323
30	9.85	1.363	12.43	1.241	15.62	1.122	19.52	1.029	21.87	0.961	23.50	0.929	24.06	0.944	23.21	1.005	20.30	1.100	15.24	1.215	11.25	1.341	9.16	1.413
0.0	10.36	1.434	12.94	1.292	16.02	1.150	19.69	1.038	21.80	0.958	23.29	0.921	23.89	0.937	23.28	1.008	20.70	1.122	15.80	1.260	11.82	1.409	9.67	1.491
–30	9.85	1.363	12.43	1.241	15.62	1.122	19.52	1.029	21.87	0.961	23.50	0.929	24.06	0.944	23.21	1.005	20.30	1.100	15.24	1.215	11.25	1.341	9.16	1.413
–45	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
–60	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
-90	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–

$\bar{R}$ are calculated from values of $\bar{R}_b$ for each month for both the south (eqn. 17) and the 15° west of south (eqn. 13) surfaces titled at the angle of Athens latitude (Table 5). The average daily irradiance on each of the surfaces is the product $\bar{R}\bar{G}_m=\bar{G}(\beta,\alpha)$ for each month.

The results of monthly calculations of $\bar{G}(\beta,\alpha)$ and $\bar{R}_b$ by Klien-Theilacker's method (eqns 17 to 33) for a tilt of 30° and for α =0°, ±30°, ±45°, ±60°, ±90° are given in Table 6. It can be seen that symmetrical values for $\bar{G}(\beta,\alpha)$, $\bar{R}_b$ are obtained for equal angles east and west of south as shown in Table 6 and this gives equal values for the irradiance totals at equal angles east and west of south. This method will therefore give more accurate results in this respect than the method of Klein. The Liu and Jordan correlation for $\bar{D}_m$ is used in the results of $\bar{G}(\beta,\alpha)$ and $\bar{R}_b$ estimates in Table 6.

In Table 7, the influence of the different diffuse component calculation methods on $\bar{R}_b$ and $\bar{G}(\beta,\alpha)$, is presented. $\bar{R}$ is calculated from different values of $\bar{R}_d$ for each month for α=0°, β=30°. The average daily global irradiance on each of the surfaces is always the product of $\bar{R}$ $\bar{G}(\beta,\alpha)$, for each month. Values of $\bar{R}_b$ have been calculated from [1] in three different ways using eqns (11), (13) and (26) corresponding to the following possible determination of $\bar{G}(\beta,\alpha)$: (i) Liu and Jordan's calculated values; (ii) Page's calculated values; (iii) Lalas *et al.* calculated values. Table 7 summarizes these results. The use of the three above mentioned correlations for the preliminary evaluation of $\bar{G}(\beta, \alpha)$ has influence on the accuracy of the model.

In Table 8 estimated monthly mean daily global irradiance sample results for $\bar{R}_b$ and $\bar{R}$ are given for surfaces oriented towards south with inclinations $\beta = \phi$, $\beta = \phi - 8°$, $\beta = \phi - 15°$ and $\beta = \phi - 30°$, where ϕ is the latitude of the station in degrees. The results presented in Table 8 show that the three models give more or less the same results and the accuracy of the estimated data from the three models are of the same order. The models agree quite well with each other.

In Table 9, the total global irradiance on three tilted surfaces, (β = 30°,45°,60°), computed from the three models discussed earlier are compared for the latitude of Athens. Global irradiation values are estimated for α = 0. The Liu and Jordan correlation for $\bar{G}(\beta,\alpha)$ is used in the results of this Table. $\bar{R}$ is calculated from values of $\bar{R}_b$ for each month. The agreement of the results of three models is seen to be not as good as those between Tables 7 and 8.

4 CONCLUSIONS

The utilization of solar energy will depend on simple computational methods, such as Liu and Jordan, Klein, Klein-Theilacker etc. The main features of the present investigations are as follows:

1. The isotropic model by Liu and Jordan and the analytical methods of Klein and Klein-Theilacker are equally accurate for locations like Athens (Greece) and anyone of the models can be employed for estimating monthly mean daily global irradiance on tilted surfaces. This may also be true to other locations in middle latitudes regions having the same climatic conditions.
2. Liu and Jordan correlation is recommended because of its simplicity for estimating intercepted by tilted surfaces irradiance for locations with climate conditions similar to these in Greece.
3. Among all the inclinations studied here, a tilt angle equal to the latitude of the place collects the maximum energy round the year.

Table 7. Comparison of $\overline{R}_b$, $\overline{R}_T$ and $\overline{G}(\beta,\alpha)$ monthly values obtained from different calculation methods using the diffuse irradiation calculation relationships for Athens (α=0, β=30°, $\overline{G}(\beta,\alpha)$ in KJ.m^{-2})

(1*)	Liu and Jordan eqns. (1), (3) and (10)									Klein eqns. (1), (3) and (13)								
(2+)	Liu – Jordan			Page			Lalas et al.			Liu – Jordan			Page			Lalas et al.		
Month	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$
J	1.92	1.53	12124.3	1.92	1.45	11518.5	1.92	1.32	10.462.5	1.67	1.38	10.956.7	1.67	1.32	10504.7	1.67	1.22	971.68
F	1.60	1.34	13776.8	1.60	1.28	13240.8	1.60	1.19	12307.4	1.46	1.26	12251.4	1.46	1.21	12525.7	1.46	1.14	11784.4
M	1.30	1.16	15509.3	1.30	1.13	15131.1	1.30	1.28	14472.6	1.28	1.15	15376.2	1.28	1.12	15015.8	1.28	1.07	14388.2
A	1.07	1.03	18200.8	1.07	1.02	18037.0	1.07	1.80	17739.8	1.15	1.08	19043.8	1.15	1.06	18786.8	1.15	1.04	18320.5
M	0.93	0.94	19388.8	0.93	0.95	19392.7	0.93	0.95	19400.0	1.06	1.03	21104.9	1.06	1.02	20943.8	1.06	1.00	20635.2
J	0.87	0.90	22264.2	0.87	0.91	22312.9	0.87	0.91	22455.4	1.03	1.01	24888.9	1.03	1.01	24815.5	1.03	1.00	24601.1
J	0.90	0.92	23763.5	0.90	0.92	23777.3	0.90	0.92	23852.2	1.04	1.02	26471.6	1.04	1.02	26492.3	1.04	1.01	26200.3
A	1.01	1.00	23302.5	1.01	1.00	23270.5	1.01	0.99	23125.3	1.11	1.07	25026.5	1.11	1.07	24940.9	1.11	1.05	24602.9
S	1.20	1.13	22136.3	1.20	1.13	22013.2	1.20	1.10	21558.4	1.22	1.15	22493.6	1.22	1.15	22358.5	1.22	1.12	21859.3
O	1.48	1.28	15407.3	1.48	1.24	14948.2	1.48	1.17	14125.2	1.39	1.22	14751.1	1.39	1.19	14367.4	1.39	1.13	13679.7
N	1.81	1.45	11882.7	1.81	1.38	11303.5	1.81	1.26	10294.8	1.60	1.32	10884.5	1.60	1.27	10446.1	1.60	1.18	9687.7
D	2.04	1.51	9011.8	2.04	1.41	8464.9	2.04	1.24	7432.1	1.75	1.36	8135.0	1.75	1.29	7731.3	1.75	1.16	6969.1

Table 7. *Cont.*

Klein-Theilacker eqns. (1), (26) or (34)								
Liu – Jordan			Page			Lalas et al.		
$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$	$\overline{R}_b$	$\overline{R}$	$\overline{G}(\beta,\alpha)$
–	1.434	10358.9	–	1.353	9770.4	–	1.203	8728.8
–	1.292	12938.2	–	1.239	12403.7	–	1.146	11473.3
–	1.150	16016.8	–	1.123	15692.0	–	1.076	14982.5
–	1.038	19688.7	–	1.031	19540.7	–	1.015	19247.4
–	0.958	21795.7	–	0.958	21798.6	–	0.959	21805.7
–	0.921	23287.3	–	0.923	23328.1	–	0.928	23467.6
–	0.937	23896	–	0.938	23907.8	–	0.941	23983.8
–	1.008	23285.3	–	1.006	23249.9	–	1.00	23103.4
–	1.122	20692.9	–	1.112	20521.5	–	1.086	20046.2
–	1.260	15803.8	–	1.225	15362.9	–	1.159	15543.5
–	1.409	11817.5	–	1.339	11236.3	–	1.219	10225.1
–	1.491	9674.5	–	1.40	9080.1	–	1.237	8023.9

(1*): Model applied for the $\overline{R}_b$ calculations.
(2+): Method used for the diffuse radiation calculations

Table 8. Comparison of $\bar{R}_b$, $\bar{R}$ monthly values from different calculation methods for $\phi-\beta=0°$, $\phi-\beta=8°$, $\phi-\beta=15°$, $\phi-\beta=30°$ ($\phi=38°$, $\alpha=0$), $\bar{G}(\beta,\alpha)$ was calculated by the three different models (Liu and Jordan, Klein and Klein Theilacker). $\bar{D}_m$ was calculated by the Liu and Jordan relationships.

	$\phi-\beta=0°$					$\phi-\beta=8°$					$\phi-\beta=15°$					$\phi-\beta=30°$				
Month	*(L–J)**		*Klein*		*(K–T)+*	*(L–J)**		*Klein*		*(K–T)+*	*(L–J)**		*Klein*		*(K–T)+*	*(L–J)**		*Kelin*		*(K–T)+*
	$\bar{R}_b$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}_b$	$\bar{R}$	$\bar{R}$
J	2.09	1.61	1.78	1.43	1.51	1.92	1.52	1.67	1.38	1.43	1.75	1.43	1.55	1.31	1.35	1.28	1.16	1.21	1.12	1.13
F	1.69	1.38	1.52	1.28	1.33	1.60	1.34	1.46	1.26	1.29	1.50	1.28	1.39	1.22	1.24	1.19	1.11	1.16	1.09	1.09
M	1.32	1.62	1.30	1.15	1.56	1.30	1.16	1.28	1.15	1.15	1.26	1.14	1.24	1.13	1.13	1.11	1.06	1.11	1.06	1.05
A	1.04	1.01	1.13	1.06	1.02	1.07	1.03	1.15	1.08	1.04	1.08	1.04	1.14	1.08	1.05	1.05	1.03	1.07	1.04	1.03
M	0.87	0.90	1.03	1.00	0.92	0.93	0.95	1.06	1.03	0.96	0.97	0.97	1.07	1.04	0.99	1.00	1.00	1.05	1.03	1.01
J	0.80	0.85	0.98	0.98	0.87	0.87	0.90	1.03	1.01	0.92	0.92	0.94	1.05	1.03	0.96	0.99	0.99	1.04	1.02	1.00
J	0.83	0.87	1.00	0.99	0.89	0.90	0.92	1.04	1.03	0.94	0.94	0.95	1.06	1.04	0.97	0.99	1.00	1.04	1.03	1.00
A	0.96	0.97	1.09	1.05	0.98	1.01	1.00	1.11	1.07	1.01	1.03	1.02	1.11	1.08	1.02	1.03	1.02	1.06	1.04	1.02
S	1.19	1.23	1.23	1.15	1.17	1.20	1.13	1.22	1.15	1.12	1.18	1.12	1.20	1.14	1.11	1.08	1.06	1.09	1.06	1.05
O	1.54	1.31	1.43	1.24	1.29	1.47	1.28	1.39	1.22	1.26	1.40	1.23	1.33	1.19	1.22	1.16	1.10	1.13	1.08	1.09
N	1.96	1.52	1.69	1.37	1.47	1.81	1.45	1.60	1.32	1.41	1.66	1.37	1.49	1.27	1.33	1.25	1.14	1.19	1.11	1.13
D	2.24	1.60	1.59	1.41	1.57	2.04	1.51	1.75	1.36	1.49	1.84	1.41	1.61	1.30	1.40	1.32	1.16	1.24	1.12	1.15

(L-J)*: Liu-Jordan, (K-T)*: Klein-Theilacker.

Table 9. Comparison of $\overline{R}_b$,$\overline{R}$ and $\overline{G}(\beta,\alpha)$ values obtained from different calculation methods. Liu and Jordan diffuse correlation is used in the results which are included in this Table (α=0, ϕ=37°58′, $\overline{G}(\beta,\alpha)$, in KJ.m^{-2})

Month	*Tilt(β)*	$\overline{R}_b$- *(L.-J.)**	$\overline{R}_b$- *Klein*	$\overline{R}$- *(L.-J)**	$\overline{R}$- *Klein*	$\overline{R}$- *(K.-T.)+*	$\overline{G}(\beta,\alpha)$ - *(L.-J.)**	$\overline{G}(\beta,\alpha)$ *-Klein*	$\overline{G}(\beta,\alpha)$ - *(K.-T.)+*
Jan.	30°	1.92	1.67	1.53	1.38	1.43	12124.3	10956.7	10358.9
	45°	2.20	1.85	1.67	1.46	1.55	13283.3	11632.0	11180.6
	60°	2.33	1.90	1.72	1.47	1.58	13703.8	11681.4	11401.8
Feb.	30°	1.60	1.46	1.34	1.26	1.29	13776.8	12951.4	12938.2
	45°	1.75	1.55	1.40	1.29	1.35	14471.9	13304.5	13485.7
	60°	1.77	1.54	1.40	1.26	1.33	14398.3	12968.6	13329.8
Mar.	30°	1.30	1.28	1.16	1.15	1.15	15509.3	15376.2	16016.8
	45°	1.32	1.29	1.15	1.14	1.15	15428.9	15240.6	15938.7
	60°	1.25	1.22	1.09	1.07	1.08	14579.3	14348.7	15060.2
Apr.	30°	1.07	1.15	1.03	1.08	1.04	18200.8	19043.8	19688.2
	45°	1.00	1.10	0.97	1.04	0.98	17167.4	18310.1	18652.4
	60°	0.86	0.98	0.87	0.94	0.88	15338.5	16673.4	16718.2
May	30°	0.93	1.06	0.94	1.03	0.96	19388.8	21104.9	21795.7
	45°	0.81	0.98	0.86	0.96	0.87	17548.7	19780.7	19800.3
	60°	0.64	0.83	0.73	0.85	0.75	15000.7	17478.3	16934.1
June	30°	0.87	1.03	0.90	1.01	0.92	22264.2	24888.9	23287.3
	45°	0.73	0.93	0.80	0.93	0.82	19674.1	23017.5	20684.5
	60°	0.55	0.76	0.66	0.81	0.68	16342.9	19952.5	17229.1
July	30°	0.90	1.04	0.92	1.03	0.94	23763.5	24471.6	23891.6
	45°	0.77	0.95	0.82	0.95	0.84	21149.6	24632.4	21401.2
	60°	0.59	0.79	0.69	0.83	0.71	17673.8	21482.7	17981.8
Aug.	30°	1.01	1.10	1.00	1.07	1.01	23302.5	25026.5	23285.3
	45°	0.91	1.05	0.93	1.02	0.94	21573.5	23870.5	21664.4
	60°	0.76	0.92	0.81	0.92	0.82	18819.7	21447.9	18988.1
Sept.	30°	1.19	1.22	1.14	1.15	1.12	22136.3	22493.6	20699.9
	45°	1.17	1.21	1.11	1.14	1.10	21694.7	22193.7	20285.3
	60°	1.07	1.11	1.03	1.06	1.02	20100.4	20703.5	18810.6
Oct.	30°	1.48	1.39	1.28	1.22	1.26	15407.3	14751.1	15803.8
	45°	1.57	1.44	1.32	1.24	1.30	15913.5	14985.6	16285.9
	60°	1.55	1.40	1.29	1.20	1.27	15577.2	14440.7	15902.6
Nov.	30°	1.81	1.60	1.45	1.32	1.41	11882.7	10884.5	11817.5
	45°	2.05	1.75	1.56	1.39	1.51	12819.8	11408.0	12672.1
	60°	2.14	1.77	1.59	1.38	1.53	13060.4	11331.4	12841.5
Dec.	30°	2.04	1.75	1.51	1.36	1.49	9011.8	8135.2	9679.4
	45°	2.37	1.96	1.65	1.44	1.63	9861.9	8622.0	10563.3
	60°	2.54	2.04	1.70	1.45	1.69	10181.9	8663.3	10878.3

(L.-J)*: Liu and Jordan.
(K.-T.)+: Klein-Theilacker.

4. The results presented here can serve as a useful reference of radiation data on inclined surfaces for future solar energy applications in Greece.

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NOMENCLATURE

$\bar{G}_m$	Average daily global irradiation on a horizontal plane at earth's surface
$\bar{D}_m$	Monthly mean of daily diffuse irradiation
$\bar{G}_{om}$	Average monthly global irradiance on a horizontal plane above the atmophere (mean daily global extraterrestrial irradiation)
$\bar{G}(\beta,\alpha)$	Average daily global irradiance on a tilted plane of slope ß and azimouth α
I_o	Solar constant (W.m-2) (see Table taken from Page, 1986[28])
$I_m(\beta,\alpha)$	Monthly mean of daily direct irradiation on plane of slope ß and azimouth α
$\bar{K}_T$	"Clearness index", ratio of average daily global to average daily extraterrestrial global irradiation ($\bar{G}_m/\bar{G}_{om}$).
$\bar{n}$	Day of the year
$\bar{R}$	Ratio of average daily global irradiance on the tilted plane to that on a horizontal plane at earth's surface
$\bar{R}_b$	Ratio of average daily direct irradiance on the tilted plane to that on a horizontal plane at earth's surface
β	Inclination angle of the plane with respect to the horizontal
α	Azimuth angle of the normal to the plane, measured in the horizontal plane from due south, positive westward, negative estward
δ	Solar declination
P_g	Reflectance of the ground
ϕ	Latitude
λ	Longitude
ω	Solar hour angle, 15° x(hours from solar noon), afternoons positive
ω_S	Sunset hour angle (hours from solar noon)
ω_{ST}	Sunrise hour angle for the tilted plane
ω_{SS}	Sunset hour angle for the tilted plane
$v(\beta,\alpha)$	Solar incidence of an inclined surface

$\zeta=\frac{\pi}{2}-\gamma$	Solar zenith angle
γ	Solar elevation
I	Instantaneous global irradiation on a horizontal surface
I_d	Instantaneous diffuse irradiation on a horizontal surface
I_T	Instantaneous global irradiation on an inclined surface
N	Number of days sufficient to represent long-term average distributions
r_d	Long-term average ratio of instantaneous diffuse irradiance
r_t	Long-term average ratio of instantenous to daily global irradiation
t	Time
t_{SR}	Sunrise time for irradiation on an inclined surface
t_{SS}	Sunset time for irradiation on an inclined surface
a	Variable defined by equation (25)
a'	Variable defined by equation (29)
A	Variable defined by equation (16) or (20) (depending on the model used)
b	Variable defined by equation (25)
B	Variable defined by equation (16) or (20) (depending on the model used)
C	Variable defined by equation (20)
d	Variable defined by equation (25)
D	Variable defined by equation (27)
G	Function defined by equation (28)