Experimental Analysis of Spur Gear Pair with Geometrical and Operating Parameters

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ABSTRACT

This study presents experimental work to analyze spur gear pair with geometrical and operating parameters. The spur gear pair accommodates the correction in tooth addendum, gear backlash, and linear tip relief profile modification with three levels. As per Taguchi L9 orthogonal array, nine test spur gear pairs are precisely manufactured to analyze the combined influence of these three geometrical parameters on gear dynamics. Other basic gear design parameters and operating parameters were held at a constant level. RMS acceleration in the vertical direction to quantify the dynamic response of test gear pairs. Experimental data are analyzed by using the Taguchi method to investigate the rank of influencing parameters to minimize vibration response. Finally, the confirmation test was performed by using a simulation study to validate the experimental results. The simulation results are similar to the experimental results. Similarly, 20 experiments were conducted with different speed and load combinations to check the effect of operating conditions on gear dynamics. From this experimental study, it is observed that the rank of influencing parameters and optimum level of geometrical are varied with respect to operating conditions. It may be concluded that typically optimized spur gear pair operated at one particular combination of load and speed may not show good performance at other operating conditions. Therefore, this study suggests that the realistic range of operating conditions should be considered while selecting suitable geometrical parameters.

Keywords: Gear dynamics, Geometrical parameters, Operating parameters, Optimization, Profile relief, Orthogonal Array (OA)

1. Introduction

Geared systems are famous for generating large amounts of mesh and bearing forces under certain operating conditions. Forces acting on gear teeth amplified under resonance conditions result in a larger amplitude of vibration levels that decrease gears' fatigue life. In some constant-speed applications, it is possible to avoid resonance. Although many gear systems work across a wide speed range, some resonances are unavoided. Moreover, changes in natural frequency to avoid resonance are not possible due to the design constraints of the gear pair system. In addition, gear tooth geometry is an essential factor affecting noise and vibration [1, 2]. Researchers have developed various mathematical models to investigate gear noise and vibration [3, 4]. Some of them suggest methods to control the vibration level of gear systems. However, the relationship between the geometrical parameters of the gear tooth surface and gear vibration needs to be clearly understood when the gear system operates over a wide range of dynamic conditions.

Therefore, it is necessary to investigate the spur gear pairs' dynamic characteristics with variations in geometrical parameters and operating conditions. This study aims to develop a suitable experimental method that simulates the influence of geometrical and operating parameters. The novelty of this study is that this method can help gear designer to select suitable values of gear geometrical parameters when the gear pair systems are required to operate at a wide range of speeds and loading conditions.

Harris [5] examined mesh stiffness variation as a periodical stimulation capable of causing gear vibrations, although the gears lacked manufacturing defects. Mesh stiffness variation in engagement is crucial in analyzing gear pair dynamics even when the gears run at a constant speed and load conditions [6-8]. Benton and Seireg [7] experimentally simulated instabilities under parametric excitations. Due to limitations in the manufacturing process and design constraints, it is not practically possible to manufacture gears with an ideal involute profile; also, when the gears are operating at nominal speed and torque, the teeth elastically deform. The effect of all this is to deviate actual contact between meshing teeth from its theoretical plane of action, which causes transmission error in the motion of the gear pair. Transmission error under load is widely used for assessing the quality of gear noise and vibration [3, 9]. Tooth profile modification is an effective way to minimize gear vibration and noise[10-11]. In 1988, Lin et al. [12, 13] developed a simple spur gear transmission model and studied the effect of various factors on its dynamic behavior. Moreover, they concluded that the vibration response depends upon the operating speed and applied torque. Besides, in 1989, Lin et al. [14, 15] were the first to publish that an alteration to the tooth profile had successfully decreased load in gear analysis [16]. The best value of the amount and extent of linear tooth profile modification depends on the applied torque carried by a geared system [17].

Cai and Hayashi [18] studied the gear vibration based on the effect of profile error and contact ratio. They further claim that rotational vibration is reduced to zero by optimizing the tooth profiles of spur gears. Yoon and Rao [19] investigated the potential for reducing the maximum dynamic load by modifying teeth using the cubic spline profile. Liou et al. [20] investigated analytically the effect of the involute contact ratio of spur gear pairs without profile modification. Kahraman and Blankenship [21] evaluated a gear pair with external driving excitation, clearance, and parametric in a series of experiments. Also, they experimentally analyzed the response of tip relief and contact ratio on the dynamic response of spur gear pairs [22, 23].

Geared systems are always bound to have some backlash caused by manufacturing error or wear, or it may be due to being designed to provide better lubrication or avoid interference. Recently, researchers paid more attention to the geared systems with backlash as the excitation due to backlash may cause tooth separation. Wang [24, 25] investigated experimentally that displacement excitation caused by static transmission error and impact excitation caused by excessive backlash may create sub-harmonic components in the response of a geared system. Singh and Comparin [26, 27] proposed rattle criteria and demonstrated their application. They also discussed critical issues related to the geared systems. The importance of consideration of manufacturing errors and gear backlash in gear dynamics [28]. Wang et al. [29] discovered nonlinear vibration in geared systems with two nonlinear factors: changing mesh stiffness and backlash. Bonari and Pellicano [30] explored the existence of manufacturing flaws that increase the vibration's amplitude and cause chaotic motion throughout a wide range of rotating speeds with minimal torque. Further, more advanced dynamic models have been developed to study the effect of various geometrical factors on gear pair.

Ratanasumawong [31] illustrated different tooth forms to analyze the vibration characteristics of helical gears. Surface geometry like bias-in-modification, pressure angle, and lead crowing evaluated the relation among meshing components and surface geometries of helical gears. Moradi and Salarieh [32] investigated the backlash nonlinearity due to the nonlinear oscillation of spur gear pairs based on the multiscale method. Ghosh and Chakraborty [33] optimize analytical methods to study the profile error and pitch effect on optimal modification.

Lin and He [34] studied the static transmission error of gear pairs with assembly errors, machining errors, and tooth modifications. Liu et al. [35] studied the transmission stationarity of the spur gear pair, which is impacted by the pitch deviation induced by the machining error. The meshing features of the spur gear pair with the pitch deviation are analyzed to examine the system's motion characteristics and time-varying contact ratio. Gao et al. [36] established a tooth profile model and optimization method for the involute gear.

From the literature review analysis, it is clear that operating parameters and geometrical factors potentially affect the overall dynamic performance of the gear pair system. However, the interaction between the spur gear pair's operating parameters and geometrical features must be clarified, consistent, and complete. In short, the operating parameters' role in selecting the best value of geometrical parameters is not clearly understood in designing a gear pair system for minimum vibration level. Still, several issues require clarification and further investigation. There is an extension to examine the impact of geometrical parameters resulting in a transition of gear tooth surface geometric features, viz., addendum, backlash, and linear tip relief tooth profile modification. Further, to investigate the effect of operating parameters on optimizing geometrical parameters of gear pair that resulted in minimum vibration level.

Therefore, this experimental study aims to systematically analyze operating conditions' influence on optimizing the geometrical factors for minimum vibration level. The present paper attempts to clarify the role of operating conditions in optimizing the geometrical factors of spur gear pair.

2. Experimental Test Set up

A novel dynamic test setup was designed and constructed, as shown in Fig. 1 (a). The setup consists of an induction motor [Make: Crompton and Grieves, Power: 5 HP], single-stage gearbox and rope dynamometer. The flexible coupling is used to connect the motor shaft to the input shaft of the gearbox. Variable frequency drive (VFD) controls and sets the gearbox's operating speed to any desired value. The split-type gearbox facilitates easy mounting and replacing of the gear pair during performing tests. A simple rope brake dynamometer is used to apply the torque on the gearbox's output shaft. Each test spur gear was rigidly mounted on the input and output shaft of the gearbox. A bearing block supported each shaft. Two identical size deep groove ball bearing (SKF 6310) was used for each shaft. Bearing caps were used to fix the bearing position in the bearing block and the shaft position in the gearbox casing. At the end of the output shaft, a rope drum pulley is mounted to apply the load on the gearbox.

The drive motor, split-type gearbox, and rope brake dynamometer are mounted on the ground, and the foundation plate is perfectly aligned with alignment equipment. Variations in vibratory response can be attained only by varying geometrical parameters and operating parameters in this type of test setup. A sweep test was performed beforehand to set the accelerometer's position during testing. Bearing block number 4 was the final location along the vertical direction selected for mounting the accelerometer on the casing, as shown in Fig. 1 (b).

(a)

(b)

Fig. 1 (a) Experimental test rig, (b) Accelerometer in vertical position

2.1 Data Acquisition and Instrumentation

The vibration signals, particularly in frequency and time domain, are captured and analyzed in real-time using an Fast Fourier Transform (FFT) analyzer. The frequency spectra were examined through an FFT. The vibration signals were measured at a sampling frequency of 4096 Hz, stored in a tape recorder, and consequently analyzed in an FFT analyzer. To at least catch the first tooth mesh frequency and side bands, the frequency range was 1600 Hz. The sidebands of interest are distinguished, as well as any sideband leakage and bin errors, using a standard constant bandwidth of 1 Hz.

The length of the signal was set to one second. The frequency range and the quantity of analyzer lines employed are vital factors. In this experiment, 1600 analyzer lines were used to display a 4096-point transform. After filtering, the root-mean-square (RMS) value of the raw observed vibration signal is used to determine the amplitude of the reference signal. The frequency spectra were averaged using four linear averages, and the time-domain vibration signal was converted to the frequency-domain using narrow-band filters.

3. Methodology to conduct experiments

From the literature analysis, it is clear that many studies discussed the nonlinear vibration of gear pair systems with single gear parameters such as contact ratio or backlash. However, in actual practice, gear systems consist of many nonlinear parameters, and their combined effects are not negligible. This experimental study analyzed the combined effect of these nonlinear parameters and operating conditions using the Taguchi method. Further, this study also investigates the combined effect of these parameters with varying operating conditions.

3.1 Taguchi Methodology (Design of Experiments)

Three selected parameters with their three levels are shown in Table 1. These parameters focus on difficulties in maintaining the entire contact surfaces of the tooth profile of mating gears as an involute profile to fulfill the condition of the law of gearing. Maintaining contact between only the involute portion of the profile is achieved by adjusting either addendum or tooth profile modifications as non-standard gears to avoid interference-related problems.

Table 1 Three parameters with three levels

Levels	Parameters			
	A Addendum (mm)	B Backlash (mm)	C Linear Tip Relief Profile Modification	
			Amount-(Δ), μm	Extent -(L), mm
First	2.6	0.051-0.074	0	L = 0 (ZR)
Second	3.1	0.101-0.124	15-20	L = Ln/2 (SR)
Third	3.4	0.151-0.176	15-20	L = Ln (LR)

Where, Ln is normalized tooth profile modification, ZR, SR, LR refers zero, short and long relief.

Tolerance of 25 μm for backlash and 5 μm for linear tip amount is provided due to manufacturing limitations. The levels of parameters for both pinion and gear are kept the same. Based on several factors and their levels, an L9 orthogonal array is selected to design experiments, as shown in Table 2.

Table: 2 Orthogonal Array

Trial	Parameters

number	(A) Tooth addendum	(B) Backlash	(C) Linear Tip Relief Tooth profile modification
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

3.2 Design of Test Spur Gear Pairs

Nine test spur gear pairs are designed for the trials using the Taguchi L9 orthogonal array. Standard design parameters of test spur gear pairs of unity ratio design used for the experimental study are shown in Table 3.

Table 3 Fundamental variables of spur gear pair

S. N.	Variables	Units	Value
1	Module	mm	3
2	Teeth in number	-	50
3	Pressure angle	deg.	21
4	Face width	mm	21
5	Centre distance	mm	152
6	Root-circle diameter	mm	142.320
7	Base-circle diameter	mm	142.421
8	Pitch-circle diameter	mm	152
10	Addendum	mm	Variable
11	Backlash	mm	Variable
12	Linear tip relief profile modification		Variable

Using the values of design parameters given in Table 3 and Taguchi's L9 orthogonal array (OA) shown in Table 2, nine spur gear pairs are designed. 9 test gear pairs are specified and designed by three letters (A, B, C) and three suffix numbers (1, 2, 3), respectively.

3.3 Production of Nine Test Spur Gear Pairs

Test gear pairs produced in quality grade 6 using a gear hobbing machine. The material for the test gear pair was SAE 8620. Variations in geometrical features such as addendum, backlash, and linear tip relief profile modifications were used for testing gear pairs. All the dimensions of test gear pairs were measured precisely. Finally, the quality of the gear pair tooth profile was checked by plotting the K-chart shown in Fig. 2 with profile tester. Nine test spur gear pairs are shown in Fig. 3.

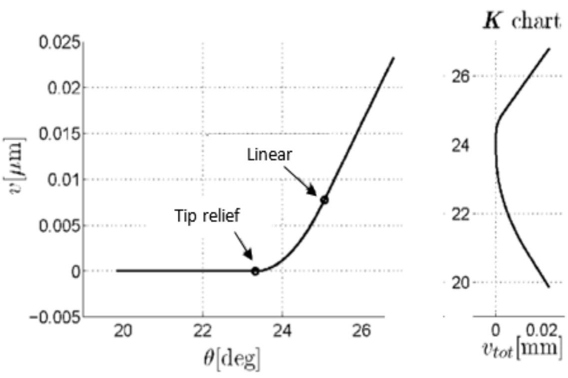


Fig: 2 k-chart	Fig. 3 Nine test spur gear pairs

3.4 Test matrix

According to Taguchi L9 orthogonal array (OA) nine trials (tests) were performed at constant torque and speed condition to investigate the effect of selected factors on dynamic behaviour of spur gear pair system. Also, in order to investigate effect of operating parameters (speed & torque) on gear pair dynamic behaviour, nine set of spur gear are tested at different operating conditions. For that purpose, total 20 experiments were performed, with different combination of load and speed as shown in Table 4.

Table 4 Operating parameters for conducting 20 experiments

Torque (T), N-m	0	4.9	5.6	5.95	7.0
Speed, (N), rpm	900	1200	1500	1800	-

4. Test Results and Data Analysis

4.1 Taguchi L9 orthogonal array (OA) experiments

In experiment No. 1, all nine trials were performed for constant torque T = 0 N-m with operational speed N = 900 rpm conditions. Root mean square (RMS) acceleration was used to assess the nine gear pairs' dynamic response. Each gear combination is tested three times to increase measurement accuracy. The sample frequency-response plot measured by an accelerometer for gear pair number 1 ($A_1B_1C_1$) is shown in Fig. 4. In similar manner nine experiments are conducted and their results are compiled in Table 5.

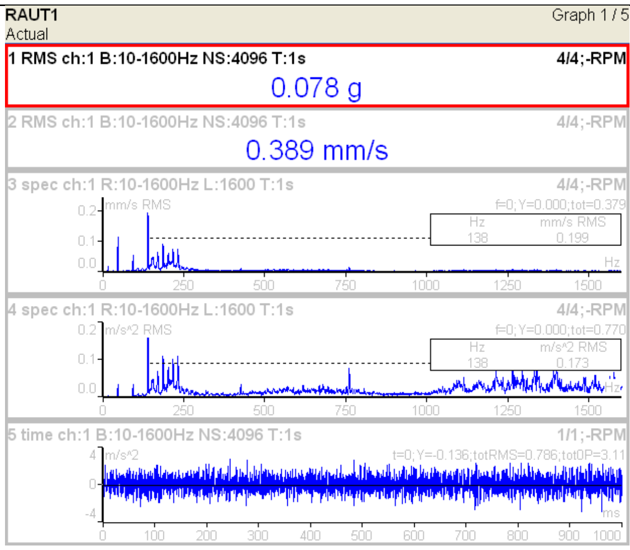


Fig. 4 Sample Frequency plot for gear pair 1 ($A_1B_1C_1$)

Table 5 Experimental results of the Taguchi L9 orthogonal array (OA) experiments

Trial No.	Gear Pair Designation	Parameters			Acceleration RMS (g)			Mean RMS (g)
		A	B	C	R_1	R_2	R_3	
1	$A_1B_1C_1$	2.7	0.05-0.075	ZR	0.077	0.078	0.080	0.078
2	$A_1B_2C_2$	2.7	0.100-0.125	SR	0.087	0.087	0.088	0.087
3	$A_1B_3C_3$	2.7	0.150-0.175	LR	0.075	0.077	0.079	0.077
4	$A_2B_1C_2$	3.0	0.05-0.075	SR	0.062	0.061	0.060	0.061
5	$A_2B_2C_3$	3.0	0.100-0.125	LR	0.083	0.084	0.085	0.084
6	$A_2B_3C_1$	3.0	0.150-0.175	ZR	0.088	0.087	0.089	0.088
7	$A_3B_1C_3$	3.3	0.05-0.075	LR	0.072	0.074	0.073	0.073
8	$A_3B_2C_1$	3.3	0.100-0.125	ZR	0.073	0.072	0.072	0.072
9	$A_3B_3C_2$	3.3	0.150-0.175	SR	0.070	0.074	0.072	0.072

4.2 Experimental Data analysis

The Taguchi method expresses the scatter around a target value using the signal-to-noise ratio (S/N). When noise elements are present, the S/N ratio measures the fluctuation within the experiment [37-38]. Taguchi effectively used this technique to determine the best condition from the studies. The better the S/N ratio, regardless of whether the quality characteristic is smaller-the-better or higher-the-better, relates to the minor variation of the output characteristics within the target value. As a result, the experiment's goal is to have the most significant feasible S/N ratio [39].

This research aims to reduce the vibration level of spur gear pairs; therefore, the "smaller- the – better" characteristic is selected, which is the logarithmic function given as

$$\left(\frac{S}{N}\right)_{SB} = -10Log_{10}\left(\frac{1}{R}\sum_{i=1}^{R} {Y_i}^2\right) \quad (1)$$

Table 6 displays the S/N ratios for trials 1 through 9 calculated using the MINITAB software.

Table 6 Experimental results, mean and S/N ratios

Trial No.	Parameters			RMS (g)			Mean RMS (g)	S/N η (dB)
	A	B	C	R_1	R_2	R_3		
1	1	1	1	0.077	0.078	0.080	0.078	22.1581
2	1	2	2	0.087	0.087	0.088	0.087	21.2096
3	1	3	3	0.075	0.077	0.079	0.077	22.2702
4	2	1	2	0.062	0.061	0.060	0.061	24.2934
5	2	2	3	0.083	0.084	0.085	0.084	21.5144
6	2	3	1	0.088	0.087	0.089	0.088	21.1103
7	3	1	3	0.072	0.074	0.073	0.073	22.7335
8	3	2	1	0.073	0.072	0.072	0.072	22.8534
9	3	3	2	0.070	0.074	0.072	0.072	22.8534

The delta (Δ) rate presented in response Tables 7 and 8 were used to establish the rank. The maximum value of delta indicates a more significant contribution of that factor on the dynamics of spur gear pairs.

Table 7 Response table for means

Designation	Parameter	Mean (g)			Delta (Δ)	Rank
		Level -1	Level -2	Level -3		
A	Tooth Addendum	0.08067	0.07767	0.07233	0.00833	2
B	Backlash	0.07067	0.08100	0.07900	0.01033	1
C	Linear tip relief profile modification	0.07933	0.07333	0.07800	0.00600	3

Table 8 Response table for (S/N) ratios

Designation	Parameter	S/N ratios (dB)			Delta (Δ)	Rank
		Level -1	Level -2	Level -3		
A	Tooth Addendum	21.88	22.31	22.81	0.93	2
B	Backlash	23..06	21.86	22.08	1.20	1
C	Linear tip relief profile modification	22.04	22.79	22.17	0.74	3

Based on the delta (Δ) rate, the backlash is the most influencing parameter among the three significant parameters selected for the present study that affect the dynamic behavior of spur gear pairs.

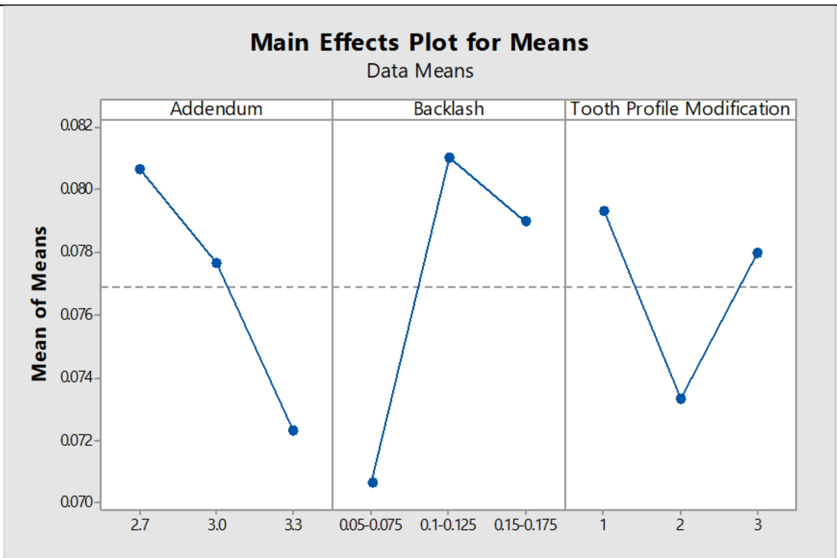


Fig. 5 Main effects plot for means

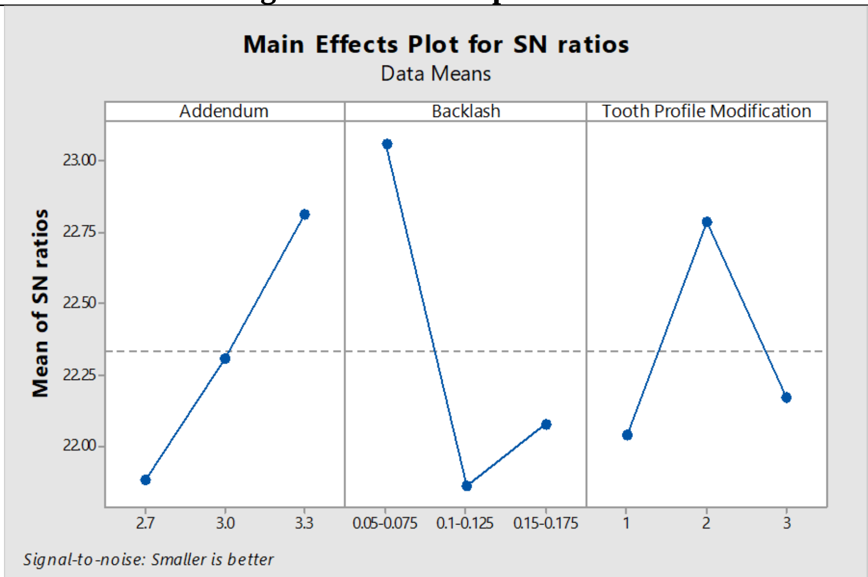


Fig. 6 Main effects plot for S/N ratios

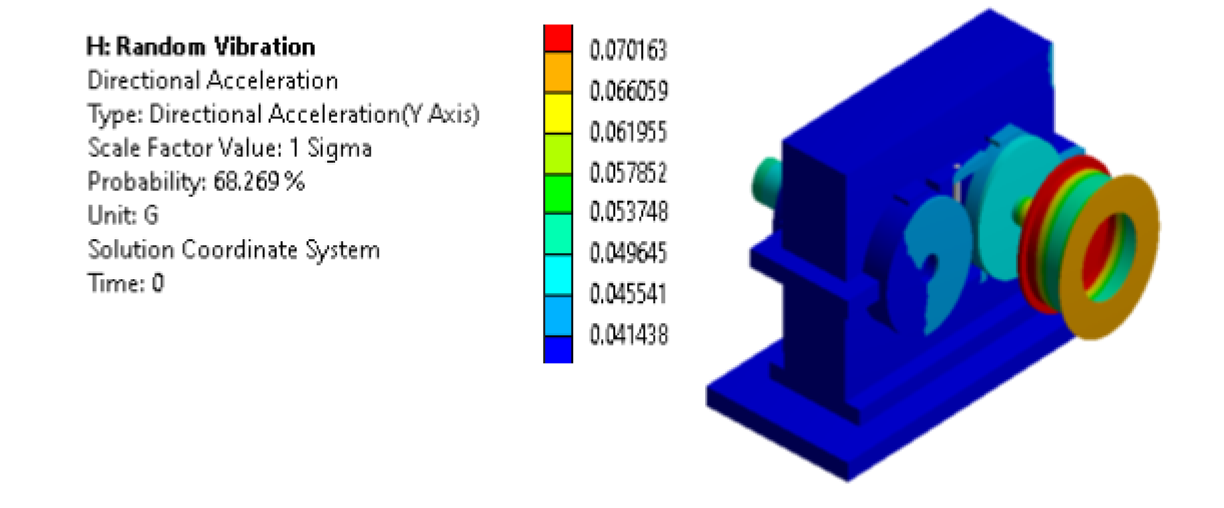
In Figs. 5 and 6, the main effect plot depicts the distinction in mean response (g) and S/N ratio with different factors and levels. The smaller, the better criteria used to optimize the parameters for mean effect plots of means response (g). The S/N ratio represents the scatter of a target value. A large S/N ratio shows that the signal significantly exceeds the random noise sources. The third, first, and second levels, respectively, should be maintained for the backlash (B), tooth addendum (A) and linear tip relief tooth profile alteration (C). For optimal setting, it is recommended that the selected value should have a maximum S/N ratio [39]. As a result, $A_3B_1C_2$ is the best combination of spur gear pairs for achieving the lowest dynamic response in terms of RMS. Table 9 shows the optimum levels of control parameters.

Table 9 Optimum level and parameters

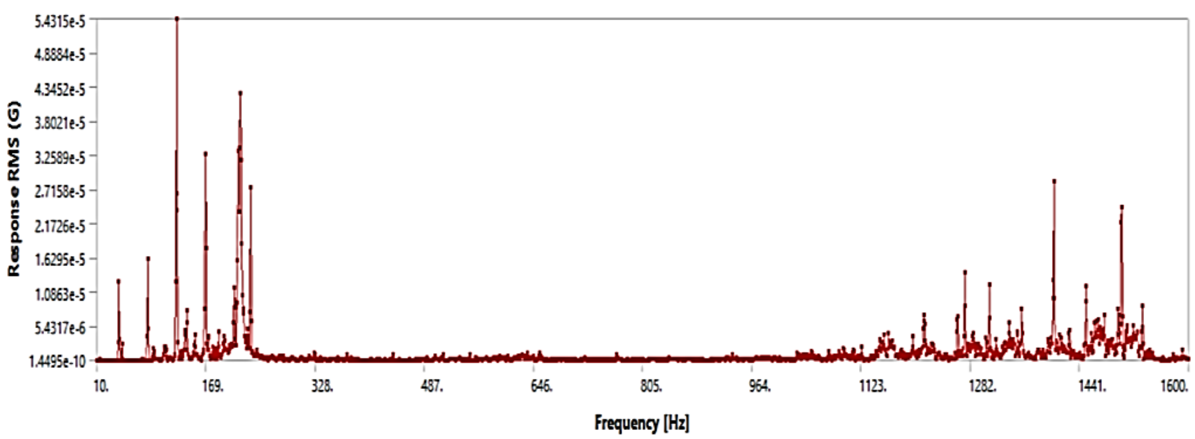
Designation	Parameter	Optimal Level	Optimal Level (mm)	Optimal Designation
A	Tooth	3	3	A_3

	Addendum			
B	Backlash	1	0.151 – 0.176	B_1
C	Linear tip relief profile modification	2	"0" relief	C_2

In this research investigation, a simulation experiment was carried out as a confirmation experiment employing a combination of the previously assessed elements and levels. A discrete model's set of algebraic equations is created from a set of differential equations using the finite element method. After imposing boundary constraints, these algebraic equations solved for the solution values at the mesh points. The equation solver resolves the discrete equations related to the finite element mesh. For modal analysis, Block Lanczos solver and random vibration solver are used for analysis. The rms acceleration (g) value is output from the random vibration solver. The test is conducted for analysis at constant torque T = 0 N-m and speed N = 900 rpm.



(a)



(b)

Fig. 7 (a) Gear pair simulation, (b) Frequency plot of gear pair ($A_3B_1C_2$)

Fig. 7 (a) and (b) shows the frequency plot attained by performing a simulation experiment. The RMS value of the simulation study is 0.070 g and 0.20 mm/s, which is approximately the lowest compared to the response values shown in Table 6. The results obtained using the simulation study are slightly higher because the actual working condition is somewhat different from those of a simulation study. From this, the experimental results are very similar to those of a simulation study.

This experiment shows that the selected factor for this experimental research plays a significant role in the dynamic response of the gear pair system. Furthermore, it is interesting to investigate the effect of operating conditions on its dynamic behavior. For that purpose, 20 Taguchi L9 orthogonal array experiments are conducted for different speed and loading conditions combinations. For analysis purposes, each experiment was conducted at constant operating conditions. The results obtained by performing 20 experiments are summarized and given in Table 10.

Table 10 Results obtained from 20 different combination of speed and torque values

Taguchi L9 Expt. No.	Operating condition (Torque, N-m, Speed, rpm)	Rank of Influencing Factors	Optimum combination
1	T = 0, N= 900	Tooth addendum = 2 Backlash = 1 Profile Modification = 3	$A_3B_1C_2$
2	T = 0, N= 1200	Tooth addendum = 2 Backlash = 1 Profile Modification = 3	$A_2B_1C_3$
3	T = 0, N= 1500	Tooth addendum = 2 Backlash = 3 Profile Modification = 1	$A_2B_2C_2$
4	T = 0, N= 1800	Tooth addendum = 1 Backlash = 2 Profile Modification = 3	$A_2B_1C_3$
5	T = 5.0, N= 900	Tooth addendum = 1 Backlash = 3 Profile Modification = 2	$A_2B_3C_1$
6	T = 5.0, N= 1200	Tooth addendum = 2 Backlash = 1 Profile Modification = 3	$A_2B_1C_1$
7	T = 5.0, N= 1500	Tooth addendum = 2 Backlash = 1 Profile Modification = 3	$A_2B_2C_1$
8	T = 5.0, N= 1800	Tooth addendum = 3 Backlash = 2 Profile Modification = 1	$A_2B_1C_1$
9	T = 5.5, N= 900	Tooth addendum = 1 Backlash = 3 Profile Modification = 2	$A_2B_3C_1$
10	T = 5.5, N= 1200	Tooth addendum = 2 Backlash = 3 Profile Modification = 1	$A_2B_3C_1$

11	T = 5.5, N= 1500	Tooth addendum = 2 Backlash = 3 Profile Modification = 1	$A_1B_2C_1$
12	T = 5.5, N= 1800	Tooth addendum = 3 Backlash = 2 Profile Modification = 1	$A_2B_1C_1$
13	T = 6.0, N= 900	Tooth addendum = 1 Backlash = 3 Profile Modification = 2	$A_2B_3C_1$
14	T = 6.0, N= 1200	Tooth addendum = 3 Backlash = 2 Profile Modification = 1	$A_2B_3C_1$
15	T = 6.0, N= 1500	Tooth addendum = 3 Backlash = 2 Profile Modification = 1	$A_2B_3C_1$
16	T = 6.0, N= 1800	Tooth addendum = 2 Backlash = 3 Profile Modification = 1	$A_1B_3C_1$
17	T = 7.0, N= 900	Tooth addendum = 1 Backlash = 3 Profile Modification = 2	$A_1B_2C_1$
18	T = 7.0, N= 1200	Tooth addendum = 1 Backlash = 3 Profile Modification = 2	$A_2B_3C_1$
19	T = 7.0, N= 1500	Tooth addendum = 3 Backlash = 2 Profile Modification = 1	$A_2B_2C_1$
20	T = 7.0, N= 1800	Tooth addendum = 1 Backlash = 2 Profile Modification = 3	$A_1B_2C_1$

Table 11 shows that researchers developed a variety of models, methods, and factors considered to predict the dynamic response of gear pairs. Some of them used experimental methods to investigate geometrical parameters' effects on the gear pair system's dynamic behavior.

Table 11 Comparisons spur gear pair dynamic responses

Sr. No.	Researchers	Models Proposed	Effects consider	Influence Factors	Solution Techniques	Conclusions
1	Lin and Parker [40]	Two- and Multi-mesh gear system models	Parametric instability.	Mesh stiffness, Contact ratio, Mesh phasing	Perturbation method and numerical integration techniques	Instability regions reduced by selecting contact ratio.
2	Li [41]	FEM	Machining errors (ME), assembly errors(AE) and tooth profile modification (TPM) on loading capacity, load-sharing	ME, AE and TPM	Finite elemet analysis	ME changes waveform and amplitude of TE, however, AE and TPM changes only amplitude of

			ratio and transmission error			TE.
3	Maliha et. al [42]	Nonlinear dynamic model	Dynamic model and combined influences of several individual effects.	Shaft and bearing external static torque, mesh stiffness variation, gear backlash, and TPM	Multi-harmonic balance method	Shaft and bearing flexibility should be considered in gear dynamic study for accurate prediction of dynamic loads.
4	Bruyere and Velex [43]	3D Gear model	Outlined optimal transformations in light of transmission error oscillations.	Transmission error, center distance variation, linear tip relief	Perturbation method	Profile modification is more suitable for precise ground gears. Effect of center distance significantly reduces the performance of tip relief.
5	Wang et. al [44]	FEM	Investigated effect of backlash caused by gear eccentricity and variation in load on dynamic transmission error (DTE).	Load, speed, gear backlash	LS-DYNA 3D software	Change in direction of load causes jumps in DTE curves.
6	Li et. al [45]	FEM	Dynamic model built using gear compatibility conditions.	Load, speed, lead crown modification	FEM	The suggested model more accurately predicts the gear system characteristics.
7	Proposed study	Experimental method	Studied the effect of operating conditions for optimization of geometric parameters of spur gear pair.	Addendum, backlash, tip relief PM, speed, torque	Taguchi method	Optimum levels of geometric parameters of spur gear pair changes with the change in operating conditions.

5. Results and Discussion

Using the Taguchi approach, the mutual impact of these geometric parameters on dynamic responsiveness is experimentally investigated. The dynamic response of a spur gear pair is measured using the Taguchi method using means and S/N ratio to account for different combinations of geometrical parameters. A higher S/N ratio implies that the spur gear pair has less dynamic responsiveness. The optimal level is determined by analyzing the means and the S/N ratio. The means and S/N ratio response tables are applied to rate the influencing factors. The response tables for means and S/N ratio and main effects plot for means and S/N ratio are shown in Table 7 and 8 and Figs. 5 and 6, respectively, for experiment No. 1. The results obtained from such 20 experiments are shown in Table 10.

From the response Tables 7 and 8 for means and S/N ratio, it can be seen that the rank of influencing parameters for experiment No. 1 is as follows: addendum has second rank, backlash first rank, and profile modification has third rank. Also, from the main effects plot for means and S/N ratio Figs. 10 and 11, it can be seen that the level of vibration response is lower at level 3 of 1st factor addendum, in case of second factor backlash level 1 is minimum and for third factor profile modification level 2 is lowest. Hence, an optimal sequence of factors and their levels is $A_3B_1C_2$. The results obtained from means and S/N ratio analysis provide an optimal combination of spur gear parameters for minimum vibration response in terms of the optimal sequence of factors and their levels as $A_3B_1C_2$. The optimal sequence of factors and their levels are as the higher value of tooth addendum 1.1 times module = 3.3 mm, the lowest value of backlash 0.050 to 0.075 mm, and tooth with short profile correction, resulting in the finest sequence to get the minimum RMS (g) level for selected operating condition 900 rpm and zero loads.

Unfortunately, this optimum sequence of factors and levels did not coincide to trials 1-9 in OA in Table 6. Manufacturing spur gear pairs with such a combination of factors and levels is impossible. Therefore, a confirmation test was conducted by using a simulation study for the same operating condition speed 900 rpm and zero load. The vibration response obtained from this simulation study is 0.07 g, the lowest among the nine sets tested. Therefore, experimental results are promising in agreement with the simulation study. Response Tables 7 and 8 show that gear backlash is the first dominant factor; this may be due to the presence of backlash with transmission errors resulting in tooth separation in case of no loaded or lightly loaded gear systems, which results in higher vibration levels [24, 28, 32]. Tooth addendum is the second dominant factor, as it is directly related to the contact ratio and mesh stiffness variation during the mesh cycle. Profile modification has less effect as compared with backlash and tooth addendum.

The rank of influencing parameters and the optimal sequence of factors and their levels change when the nine gear sets are operated with various speed and loading conditions combinations. The results obtained from 20 Taguchi experiments are shown in Table 10. These results show that operating condition plays a significant role in gear vibration response. These 20 trials reveal that tooth profile modification, as opposed to tooth addendum and backlash, is the most influential parameter. An optimum combination of factors and their levels is $A_2B_3C_1$.

6. Conclusion

In this study, the impact of operating conditions and several factors—including backlash, profile modification and addendum —on the geometrical profile of the gear pair's vibration response is investigated. In this regard, 20 Taguchi L9 experiments are conducted at various combinations of operating speed and loading conditions.

The first experiment was conducted at a speed of 900 rpm and zero load conditions. From Taguchi's analysis, it is seen that the rank of influencing parameters is as follows: backlash first rank, addendum second rank, and profile modification third rank. Moreover, the optimum combination of factors and their levels is $A_3B_1C_2$. From this experiment, it is observed that geometrical parameters significantly influence the gear pair's vibration response. Similarly, 20 Taguchi L9 experiments were conducted at various operating conditions, and the results are compiled in Table 10. From these results, it is observed that the rank of influencing parameters and optimum levels of parameters for lower vibration response are varied to operating conditions. From these 20 experiments, the rank of influencing parameters are as follows: profile modification first rank, addendum second rank, and backlash third rank. Also, the optimum combination of factors and their level is $A_2B_3C_1$. From this study, operating speed and loading conditions play significant roles in the design of quiet gear systems. It is only possible to compare gear designs if the operating conditions are varied over their realistic operating ranges.

The future scope is revealed by developing a non-linear dynamic model that considers time-varying features of a gear pair. The gear diagnosis process can be used to determine the prognosis and anticipated remaining life of the gear. Furthermore, this approach can be developed to incorporate new soft computing methods for gear fault detection and experimental investigation in rotating systems.

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