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ORIGINAL PAPER

Optimization of Geometrical Features of Spur Gear Pair Teeth for Minimization of Vibration Generation

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Abstract

Purpose Predicting dynamic loads on gear teeth and noise has always been a key component of gear design. Numerous studies have examined the impact of nonlinear factors on the dynamic response of gear pair systems, including backlash and time-varying mesh stiffness of teeth. However, for the construction of quiet gears, the relationship between the dynamics and geometrical parameters of a spur gear pair is not completely understood. The literature review reveals that there is a need for further investigation in terms of effect of geometrical features playing a role in dynamic response. The novelty of this work is that, experimental study can help gear designers to identify the most influencing parameters of the system in the early stage of gear design.

Methods This study experimentally investigated the combined influence of geometrical features of gear tooth such as tooth addendum, gear backlash and linear tip relief tooth profile modification on the vibration behavior of spur gear pair. For this purpose, an experimental test setup was designed and developed for investigation. Dynamic response of spur gear pairs was measured in terms of RMS acceleration. The present study also attempted to optimize geometrical features of gear tooth through Taguchi experiment. For that, three factors with three levels were selected. The L_9 orthogonal array was chosen to conduct statistical experiment to minimize vibration of gear pair.

Results Taguchi experiments reveal that linear tip relief tooth profile modification is the most influencing parameter as compared to gear backlash and tooth addendum. It has been found from the main effect analysis that an optimal combination of parameters with their values as addendum 3.00 mm, backlash in the range of 0.150–0.175 mm and zero linear tip relief tooth profile modification resulted in inducing lowest vibrations.

Conclusion Increase in the level of backlash results in reducing the vibration response. The effect of tooth profile modification becomes ineffective due to manufacturing variance and tolerances of gear tooth faces. The optimal combination of geometrical parameters and their levels can play major role in dynamic response of spur gear pairs.

Keywords Gear vibration · Geometrical parameters · Impact hammer test · Vibration spectrum · Fast Fourier transform (FFT) · Taguchi methodology

Introduction

In industrial rotating machinery, gears are among the vital essential parts. Shafts and bearings transmit gear vibration to the surrounding structure, causing undesired noise. Dynamic tooth loads at or near resonant frequencies generate significant stresses in the gear teeth and bearings, which can lead to premature failure. At some constant speed applications, gear resonance can be avoided, and many gears operate across a wide speed range, there it is difficult to avoid all gear resonances. As a result, when designing geared systems, it is crucial to analyze the dynamic behavior of the gears. Since they are geometrically flawless and have infinite rigidity, ideal

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involute gears transmit motion uniformly. Although there are many imperfections in real-world gears, besides those are inherent in the manufacturing process. The mesh stiffness is limited and fluctuates with tooth location in the mesh cycle due to adjacent pitch errors, profile errors, misalignment, and lead errors. Recently, more advanced dynamic models developed by researchers to investigate effects of nonlinear gear parameters on the dynamic behavior of gear pair system.

Lin et al. [1] evaluated various profile changes and the ensuing dynamic loads. Velex and Maatar [2] investigated how mounting failures and form variances affected gear dynamics. Liou et al. [3] demonstrated a simulation technique for dynamic load on a spur gear transmission is impacted by the gear contact ratio. Wang et al. [4] gave a comprehensive summary on models and approaches used in deterministic gear dynamics. Parey et al. [5] developed a model of 6 DOF for gear with localized tooth defects. The model takes into account the impacts of time-varying mesh stiffness and damping, gear error-induced backlash excitation, and profile changes. Bonari and pellicano [6] presented a technique for assessing nonlinear spur gear vibrations in the presence of manufacturing flaws. Li [7] illustrated tooth variations, assembly flaws, and machining mishaps that affected loading capacity and load-sharing ratio, and utilized a finite-element method that expressly designed the transmission error of spur gears. Kim et al. [8] illustrated pressure angle and the contact ratio are time-varying variables in the dynamic model. In comparison to the prior model, the new one generates more accurate dynamic reactions. Moradi and Salarieh [9] studied spur gear pair nonlinear oscillations, with backlash nonlinearity. Wen et al. [10] examined the random dynamics of a pair of spur gears when they were excited by white noise and harmonics. The path integration method examines the model's backlash nonlinearity and time-varying mesh stiffness.

Ma et al. [11] applied addendum improvements and tooth profile modification to exhibit mesh stiffness for profile-shifted gears. The tooth profile modification curves at different levels of tooth profile are evaluated utilizing loaded static transmission error. Wang et al. [12] built a 3-DOF torsional model of a locomotive's spur gear by taking backlash, mesh stiffness, and static transmission error. Liu et al. [13] proposed a dynamic spur gear model with a time-varying contact ratio and pitch deviation on the dynamic features of a spur gear pair. Gao et al. [14] suggested an optimization technology for an involute spur gear pair and tested the optimization technique using a transmission system experiment. Ratanasumawong et al. [15] established the connection between the shape of a helical gear tooth surface and the vibration levels. The results demonstrated the feasibility of examining gear tooth surface geometry using vibration measurement. Amabili and Rivola [16] investigated the stability and steady-state responses of a pair of low-contact-ratio spur gears with a single DOF. The meshing tooth pair stiffness, damping coefficient, and gear faults are all time-varying in the model. The numerical simulation results of two pairs of spur gears steady-state responses are verified with experimental results.

Amabili and Fregolent [17] developed a novel approach for determining modal characteristics (damping and frequency) and analogous gear errors of a spur gear pair. The test rig model is a single DOF system, and the solution is expressed based on the harmonic balance method. The developed technique can be used for identifying natural frequency, damping, and profile errors. Cai and Hayashi [18, 19] developed a new method to calculate the optimum profile modification to make the rotational vibration become zero. By addressing the variable fraction of the time-varying stiffness of meshing gears with excitation force, an analytical solution is derived. When gear pairs are in contact, the linear one-degree-of-freedom model evaluates vibration imposed by any shape of profile inaccuracy.

Umezawa et al. [20] developed a novel simulator for a spur gear pair. Model of single-degree-of-freedom with tooth meshing stiffness, constant damping, and errors measured with automatic measuring apparatus. The simulator depicts the dynamic behavior of spur gear pairs under a wide range of loads and speeds. The Runge–Kutta-Gill method was employed for numerical solutions and validated with experimental dynamic transmission error of spur gear pairs. Amer et al. [21–23] developed the nonlinear multi DOF model to represent a nonlinear mechanical system. The author used a multiple-scale technique for an approximate solution to identify the resonances of the system. Bek et al. [24, 25] investigated time history plots and resonance curves to demonstrate the effect of the physical parameters on dynamic behavior. Amer et al. [26, 27] developed a nonlinear dynamic model of double nonlinear damped spring pendulums by considering distinct parameters of a dynamic system. They examined steady-state solutions under the stability and instability criteria. In addition, claimed that this model can be used to control vibrations of vibratory systems. Amer et al. [28] illustrated Lagrange's equation based on governing equations and multiple-scale methods for gear pairs. Desavale et al. [29] illustrated the Euclidian distance-based technique with an artificial neural network for fault identification in the gearbox. Choy et al. [30] numerically modeled the dynamics of a gear transmission system with multiple-tooth damage. Wigner-Ville Distribution and the Wavelet transform employed for damage identification of gear teeth. Liu and Parker [31] investigated the impact of tooth profile variation on vibration in multi-mesh gear sets. The dynamic load distribution between each gear tooth is taken into account along with the impact of changing mesh stiffnesses, profile variations, and contact loss in the nonlinear analytical model.

The analysis of the literature shows that many parameters influence dynamic behavior in different levels and ways; in addition, they interact with each other. Numerous researchers find efforts to understand the parameters influencing gear vibration and noise to minimize it. Although the nature of the phenomena is well accepted, scientific knowledge of gear vibration is still limited. In addition, quantitative relationships between the factors and vibration resources remain unclear. Therefore, there is scope to explore the effect of a combination of factors resulting in a change of geometrical features of gear teeth profile, viz., addendum, backlash, and tooth profile modification.

The objective of the present study is to develop an experimental setup capable of simulating the influence of significant parametric variations under constant load and speed conditions. Taguchi methodology is adopted to find optimum levels of parameters that lead to a minimizing vibration. Nine various sets of gear pairs are manufactured based on Taguchi orthogonal array. For all nine such sets of gear pair, experiments were conducted to identify the level of vibration. Further analysis is carried out for the identification of optimal factors and their levels to minimize vibration.

Experimental Investigation of Dynamic Response of Spur Gear Pairs

Experimental Test Setup

In order to demonstrate the effects of variation in geometrical feature of spur gear pair on its dynamic behavior. A unique gear dynamic test setup is constructed for collecting vibration responses for various changes in geometrical features of gear pair to analyze optimal parameters.

Figure 1 shows the gear pair test setup, the drive is connected to a single-stage gearbox through flexible jaw coupling. The output shaft of the gearbox is connected to a dynamometer for loading conditions. A VFD is employed to vary the input speed of the pinion, and a rope brake dynamometer for applying the output torque by adjusting the tension in the rope.

A single-stage gearbox is made into two parts. For the analysis of test gear pairs with variations in geometrical features, various sets of gear pairs are developed. Further, it can be easily mounted or replaced in the gearbox without disturbing the final adjustment of the test setup. The split type single-stage gearbox is sufficiently rigid to withstand the dynamic load. The input and output shafts of the gearbox are supported by two similar bearings. Bearing caps are used for each bearing block with oil seals to prevent leakage from the gearbox during the performance of tests. Two spacers are

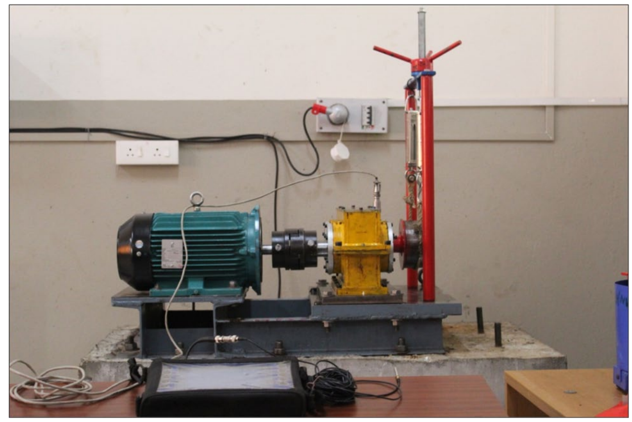

Fig. 1 Experimental setup

used for each shaft locating the exact position of the gear on the shaft. The drive unit, gearbox, and dynamometer are installed on a flat foundation plate that is rigidly attached to the foundation. The vertical and lateral alignment of the input and output shafts can also be adjusted using a separate gearbox plate. All the gearbox components are installed and aligned subsequently.

Test Procedure

Prediction of Natural Frequency by Impact Hammer Method

Based on experimental modal analysis, the fundamental natural frequency of the entire gearbox assembly is identified. Accelerometers are mounted on the lower half side plate along the horizontal direction and second on the lower half flange along the vertical direction [31]. An accelerometer of CTC made, Model-AC102-1A, No. 13522 sensitivity of 100 mV/g is used for experimentation. From the impact test, the fundamental natural frequency is 300 Hz and 140 Hz in horizontal and vertical directions shown in Fig. 2a, b.

Acquiring Vibration Spectrum

The operating speed of 1200 rpm and output torque of 5.6 N-m is set as constant for experimental test and further analysis. A sweep test was conducted to identify a minimum number of locations for mounting accelerometers on gear box housing. The accelerometers are mounted on bearing block in vertical, horizontal, and axial direction as shown in Fig. 3a–c. A total of 12 points on four bearings are selected to measure the vibration of the gear pair system. The results of two tests conducted under two different operating conditions revealed that bearing block number 4 presents a higher side response in the vertical direction. As a result, this position was identified for installing the accelerometer

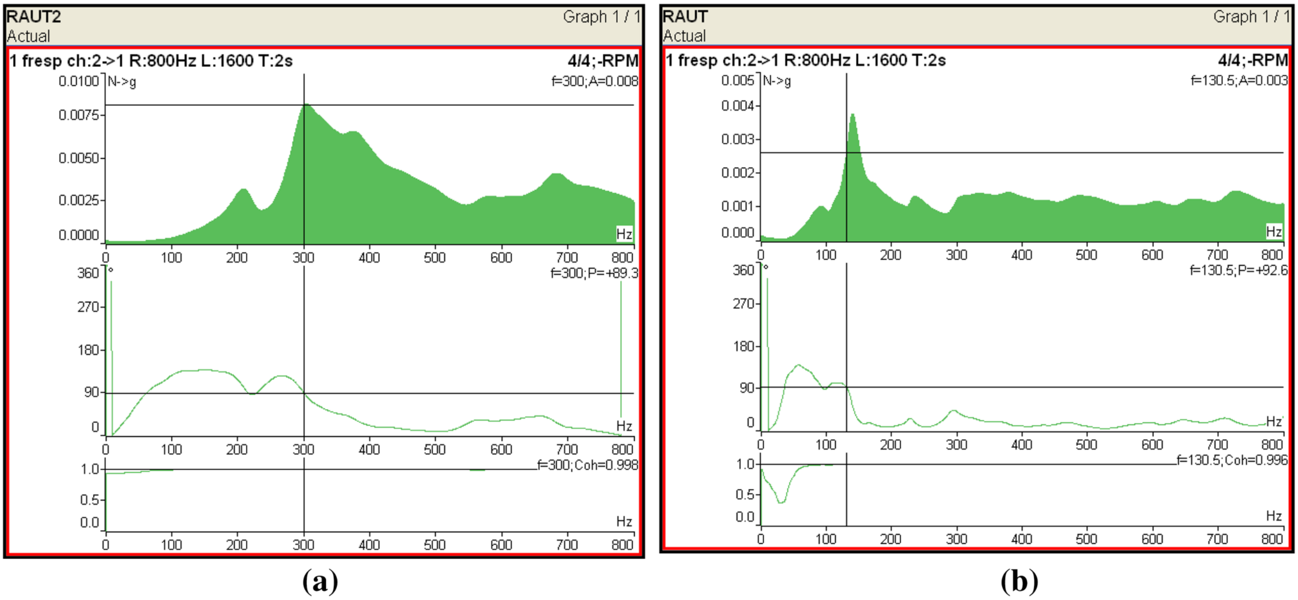

Fig. 2 Natural frequency by experimental modal analysis: **a** horizontal direction, **b** vertical direction

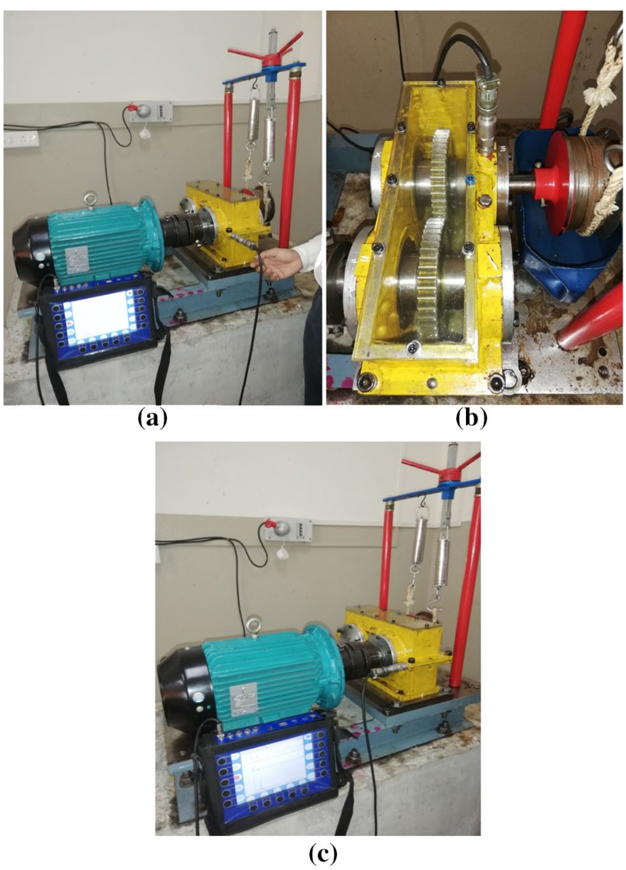

Fig. 3 Acceleration direction on test setup: **a** horizontal, **b** vertical, **c** axial

during testing. In addition, the noise level of the gearbox was measured using a microphone installed at 75 cm to 100 cm in front of the test setup. The microphone of GRAS 40PH-10CCP made with frequency range 10 Hz to 20 kHz, a dynamic range of 33 dB to 135 dB, and a sensitivity of 100 mV/Pa is used.

The frequency spectra are analyzed by processing the time signals through an FFT analyzer. FFT of Adash 4400 VA4Pro is used to analyze digitized signals obtained from sensors. The data acquisition system of Adash 4400 VA4Pro is a 4-channel vibration analyzer with up to 90 kHz frequency range which is based on a unique digital signal processing board (DSP). At a sample rate of 4096 Hz, the vibration signals are recorded. The frequency range of 10–1600 Hz is considered to capture first tooth mesh frequency with side bands. A constant absolute bandwidth of 1 Hz is used to distinguish the sidebands of interest with leakage and bin errors inherent to experimental spectra analysis. The signal duration is 1 s, and 1600 analyzer lines are required to illustrate the 4096-point transform. The amplitude of the reference signal is determined after filtering using the raw observed vibration signal root-mean-square (RMS) value.

Frequency–Response Plots of Spur Gear Pairs

Three trial tests are run to anticipate the effect of gear pair geometrical features (like addendum, backlash, and tooth profile alteration) on dynamic behavior. Three gear pairs are manufactured with changes in addendum, backlash, and

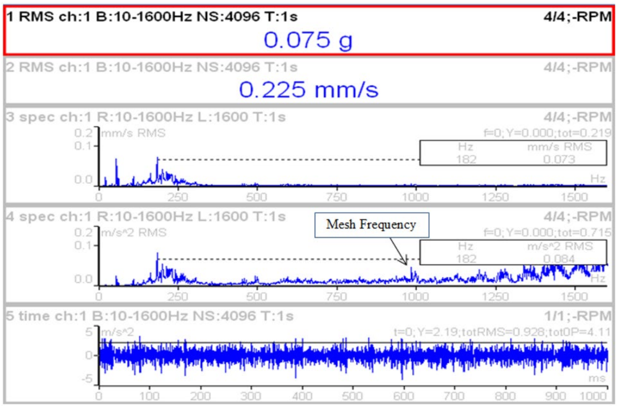

Fig. 4 Frequency–response plot of sample test gear pair No. 1

Table 1 Vibration responses of sample three gear pairs

Gear pair no.	Designation	RMS acceleration (g)
1	Sample test gear pair 1	0.075
2	Sample test gear pair 2	0.073
3	Sample test gear pair 3	0.094

tooth profile modification (only linear tip relief) by keeping other parameters constant.

(a) Sample test gear pair 1

The gear pair with addendum 1.1 times module and addendum for 20^0 full depth involute system is 1.0 times module. In this case, the remaining gear design parameters are at standard levels. Module = 3.00 mm, number of teeth = 50, face width = 20 mm, minor diameter = 140.680 mm, base diameter = 140.954 mm, pitch diameter = 150 mm, circular tooth thickness = 4.649 to 4.662 mm and backlash lies between 0.100 and 0.125 mm. The gear pair has a standard involute profile (without tooth profile modification). Frequency–response plot for the gear pair is shown in Fig. 4.

(b) Sample test gear pair 2

The gear pair with a backlash of 0.150 to 0.175 mm with a standard backlash of 3.00 mm module and 150 mm center distance of 0.100 to 0.125 mm [15]. The remaining gear design parameters are at standard levels. The response of sample test gear pair 2 in terms of RMS acceleration (g) with a change in backlash is shown in Table 1.

c) Sample test gear pair 3

The gear pair with long tip relief profile modification may be characterized by two parameters [1]. First, the amount of relief at the tip of the gear tooth, $\Delta = 15–20$ microns, and second the length of modification in mm or roll angle, Ln = 2.647 mm or $\alpha = 21.99^0$. The response of sample test gear pair 2 in terms of RMS acceleration (g) with a change in long tip relief profile modification are shown in Table 1.

Methodology of Optimization of Geometrical Features of Gear Profiles to Minimize Dynamic Response

By selecting the appropriate level of geometrical features, such as tooth addendum, gear backlash, and tooth profile modification, the objective is to lower the vibration level of the spur gear pair.

Selection of Parameters and Their Levels

Three factors tooth addendum, gear backlash, and tooth profile modification for a spur gear pair are considered and summarized in Table 2

Where, Ln is normalized extent of tooth profile modification. The L_9 orthogonal array (OA) is set and array of experiments is listed in Table 3, based on the number of factor and levels.

Table 2 Three parameters and their levels

Levels	Parameters			
	addendum (A), mm	Backlash (B) (mm)	Linear tip relief tooth profile modification (C)	
			Amount (Δ), µm	Extent (L), mm
First level	2.7	0.050–0.075	0	L = 0 (zero relief)
Second level	3.0	0.100–0.125	15–20	L = Ln/2 (short relief)
Third level	3.3	0.150–0.175	15–20	L = Ln (long relief)

Table 3 Taguchi L_9 orthogonal array (OA)

Experiment (or trial) number	Parameters		
	Tooth addendum	Backlash	(C) Tooth profile modification
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2

Design and Manufacturing of Nine Spur Gear Pairs as per OA

The test gear pairs of unity ratio with 20° involute gear tooth systems are selected. The material of test gear pair is SAE 8620. The gear tooth parameters selected are as follows:

Module: 3 mm
Pressure angle: 20°
Face width: 20 mm
Number of teeth: 50
Center distance: 150 mm

Three levels of three factors are considered and same are given in Table 4. Table 5 shows the detailed dimensions determined from nine gear pair sets.

Nine test gear pairs are manufactured with the help of a gear hobbing machine to achieve quality grade number 6. Special manufacturing techniques are used to create the differences in tooth addendum levels, gear backlash, and linear tip relief profile adjustment. Changing the length of the tooth addendum in test gears is achieved by controlling only the outside diameter of test gears. The addendum ratio (AR) approach modifies addendums. A hob cutter with greater whole depth than the standard is used for the operation. The backlash is controlled by varying the tooth thickness of gear teeth. The tooth thickness is reduced by sinking the cutter dipper into the blank to the predetermined tolerance. After machining test gear pairs, they are hardened up to 58–60 HRC and case depth up to 0.8 to 1.0 mm to improve fatigue strength. Finally, tooth profile modification (only tip relief) operations are performed on a gear grinding machine. Figure 5 shows measurement of pitch circle diameter run-out. The backlash in gear pairs were measured on roll tester machine as shown in Fig. 6. Finally, accuracy of tooth profile quality grades was checked by plotting K-chart. Tooth profile accuracy was tested with the help of profile testing equipment as shown in Fig. 7. The nine spur gear pairs built by the deduced dimensions listed in Table 5 are shown in Fig. 8.

Table 4 Three levels of basic design parameters of spur gear pair

Sr. no.	Parameters	Unit	First level	Second level	Third level
1	Module	mm	3.0	3.0	3.0
2	Number of teeth	–	50	50	50
3	Pressure angle	deg	20	20	20
4	Face width	mm	20	20	20
5	Centre distance	mm	150	150	150
6	Root circle diameter	mm	140.680	140.680	140.680
7	Base circle diameter	mm	140.954	140.954	140.954
8	Pitch circle diameter	mm	150	150	150
9	Outside circle diameter	mm	155.4	156	156.6
10	Addendum	mm	2.7	3.0	3.3
11	Dedendum	mm	4.66	4.66	4.66
12	Theoretical contact ratio	–	1.59	1.75	1.91
13	Whole depth of tooth	mm	7.36	7.66	7.96
14	Backlash	mm	0.05–0.075	0.100–0.125	0.150–0.175
15	Normal circular tooth thickness	mm	4.667–4.674	4.662–4.649	4.637–4.624
16	Linear tip relief profile modification				
	(a) Amount, Δ	μm	Unmodified	15–20	15–20
	(b) Extent, L	mm	Zero relief (ZR) L = 0	Short relief (SR) L = Ln/2	Long relief (LR) L = Ln

Table 5 Design parameters for nine spur gear pairs

Sr. no.	Parameters	Unit	Gear pair 1 $A_1B_1C_1$	Gear pair 2 $A_1B_2C_2$	Gear pair 3 $A_1B_3C_3$	Gear pair 4 $A_2B_1C_2$	Gear pair 5 $A_2B_2C_3$	Gear pair 6 $A_2B_3C_1$	Gear pair 7 $A_3B_1C_3$	Gear pair 8 $A_3B_2C_1$	Gear pair 9 $A_3B_3C_2$
1	Module	mm	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
2	Number of teeth	–	50	50	50	50	50	50	50	50	50
3	Pressure angle	Deg	20	20	20	20	20	20	20	20	20
4	Gear ratio	–	1:1	1:1	1:1	1:1	1:1	1:1	1:1	1:1	1:1
5	Face width	mm	20	20	20	20	20	20	20	20	20
6	Centre distance	mm	150	150	150	150	150	150	150	150	150
7	Root circle diameter	mm	140.680	140.680	140.680	140.680	140.680	140.680	140.680	140.680	140.680
8	Base circle diameter	mm	140.954	140.954	140.954	140.954	140.954	140.954	140.954	140.954	140.954
9	Pitch circle Diameter	mm	150	150	150	150	150	150	150	150	150
10	Outside (tip) diameter	mm	155.4	155.4	155.4	156	156	156	156.6	156.6	156.6
11	Theoretical involutes contact ratio		1.59	1.59	1.59	1.75	1.75	1.75	1.91	1.91	1.91
12	Addendum	mm	2.7	2.7	2.7	3.0	3.0	3.0	3.3	3.3	3.3
13	Dedendum	mm	4.66	4.66	4.66	4.66	4.66	4.66	4.66	4.66	4.66
14	Whole depth of tooth	mm	7.36	7.36	7.36	7.66	7.66	7.66	7.96	7.96	7.96
15	Circular pitch	mm	9.424	9.424	9.424	9.424	9.424	9.424	9.424	9.424	9.424
16	Base pitch	mm	8.856	8.856	8.856	8.856	8.856	8.856	8.856	8.856	8.856
17	Backlash	mm	0.05–0.075	0.100–0.125	0.150 – 0.175	0.05–0.075	0.100–0.125	0.150–0.175	0.05–0.075	0.100–0.125	0.150–0.175
18	Normal circular tooth thickness	mm	4.678–4.674	4.662–4.649	4.637–4.624	4.687–4.674	4.662–4.649	4.637–4.624	4.687–4.674	4.662–4.649	4.637–4.624
19	Linear tip relief profile modification										
	Amount (delta), Δ	um	Unmodified	15–20	15–20	15–20	15–20	Unmodified	15–20	Unmodified	15–20
	Extent, L	mm	Zero relief ZR	1.044 $L=L_n/2$ SR	2.088 $L=L_{n,}$ LR	1.324 $L=L_n/2$, SR	2.467 $L=L_{n,}$ LR	Zero relief ZR	3.195 $L=L_{n,}$ LR	Zero relief ZR	1.598 L = Ln/2 SR
	Roll angle, α	deg	zero	24.82	22.57	24.86	21.99	zero	21.42	zero	24.93

Fig. 5 Measurement of PCD run-out

Fig. 6 Measurement of backlash

Fig. 7 Measurement of PCD tooth profile

Experimental Tests of Spur Gear Pairs

For analysis, nine trials were carried out under the condition of constant torque T = 5.6 N-m and speed $N = 1200$ rpm. The dynamic response of all nine gear pairs is measured in terms of RMS acceleration. The frequency–response plot for gear pair number 1 ($A_1B_1C_1$) is shown in Fig. 9. Similarly, remaining eight gear pairs are tested experimentally and vibration levels are compiled in Table 6.

Fig. 8 Nine test spur gear pairs

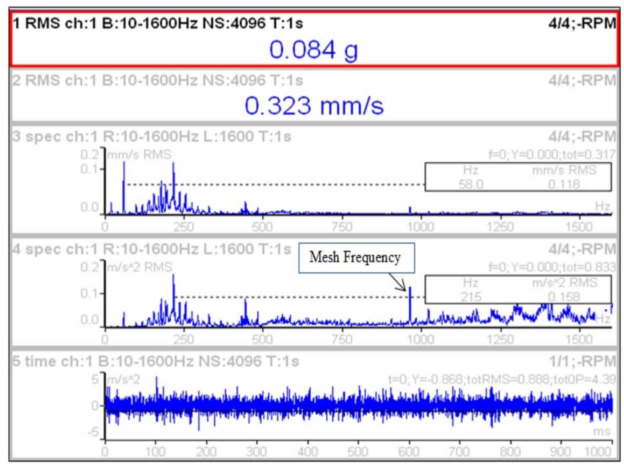

Fig. 9 Frequency–response plot for gear pair 1 ($A_1B_1C_1$)

Experimental Results Analysis as per Taguchi Methodology

The MINITAB software is used for calculating S/N ratios using smaller—the—better criterion. Table 7 lists the trials 1 to 9 measured experimental RMS (g) results along with the matching S/N ratios.

Analysis of RMS Level

The rank of influencing factors is determined from the responses for means and S/N ratios given in Tables 8 and

Table 6 Experimental results of nine spur gear pairs by Taguchi approach

Expt. or trial no.	Gear pair designation	Parameters			RMS acceleration level (g)			Mean RMS (g)
		A Tooth addendum (mm)	B Backlash (mm)	C Linear tip relief profile modification	R_1	R_2	R_3	
1	$A_1B_1C_1$	2.7	0.05–0.075	Zero relief	0.083	0.084	0.084	0.084
2	$A_1B_2C_2$	2.7	0.100–0.125	Short relief	0.150	0.153	0.155	0.153
3	$A_1B_3C_3$	2.7	0.150–0.175	Long relief	0.075	0.078	0.079	0.077
4	$A_2B_1C_2$	3.0	0.05–0.075	Short relief	0.095	0.097	0.100	0.097
5	$A_2B_2C_3$	3.0	0.100–0.125	Long relief	0.093	0.094	0.094	0.094
6	$A_2B_3C_1$	3.0	0.150–0.175	Zero relief	0.072	0.073	0.074	0.073
7	$A_3B_1C_3$	3.3	0.05–0.075	Long relief	0.215	0.216	0.218	0.216
8	$A_3B_2C_1$	3.3	0.100–0.125	Zero relief	0.073	0.075	0.076	0.075
9	$A_3B_3C_2$	3.3	0.150–0.175	Short relief	0.100	0.102	0.105	0.102

Where, R_1, R_2, R_3 are RMS (g) of each trial 3 repetitions; gear parameters A, B, C.

Table 7 Experimental results, mean and S/N ratios

Expt. or trial no.	Parameters			RMS acceleration level (g)			Mean RMS acceleration level (g)	S/N ratio η (dB)
	A	B	C	R_1	R_2	R_3		
1	1	1	1	0.083	0.084	0.084	0.084	21.548
2	1	2	2	0.150	0.153	0.155	0.153	16.324
3	1	3	3	0.075	0.078	0.079	0.077	22.320
4	2	1	2	0.095	0.097	0.100	0.097	20.232
5	2	2	3	0.093	0.094	0.094	0.094	20.568
6	2	3	1	0.072	0.073	0.074	0.073	22.732
7	3	1	3	0.215	0.216	0.218	0.216	13.270
8	3	2	1	0.073	0.075	0.076	0.075	22.536
9	3	3	2	0.100	0.102	0.105	0.102	19.797

Table 8 Ranks based on means response

Parameter designation	Parameter	Average values of mean (g)			Delta (δ)	Rank
		Level 1	Level 2	Level 3		
A	Tooth addendum	0.104	0.088	0.131	0.043	3
B	Backlash	0.132	0.107	0.084	0.048	2
C	Linear tip relief profile modification	0.077	0.117	0.129	0.052	1

Table 9 Ranks based on signal-to-noise (S/N) ratios

Parameter designation	Parameter	Average values of S/N ratios (dB)			Delta (δ)	Rank
		Level 1	Level 2	Level 3		
A	Tooth addendum	20.034	21.177	18.534	2.643	3
B	Backlash	18.350	19.809	21.586	3.236	2
C	Linear tip relief profile modification	22.272	18.784	18.689	3.583	1

9, respectively. Delta (δ) values in Tables 8 and 9 show the variations in mean and S/N ratios within the levels. Similarly, variation in mean and S/N ratio affects (δ) which directly influence dynamic response of spur gear pairs. In Tables 8 and 9, linear tip relief tooth profile modification ranked 1st, backlash and tooth addendum ranked 2nd and

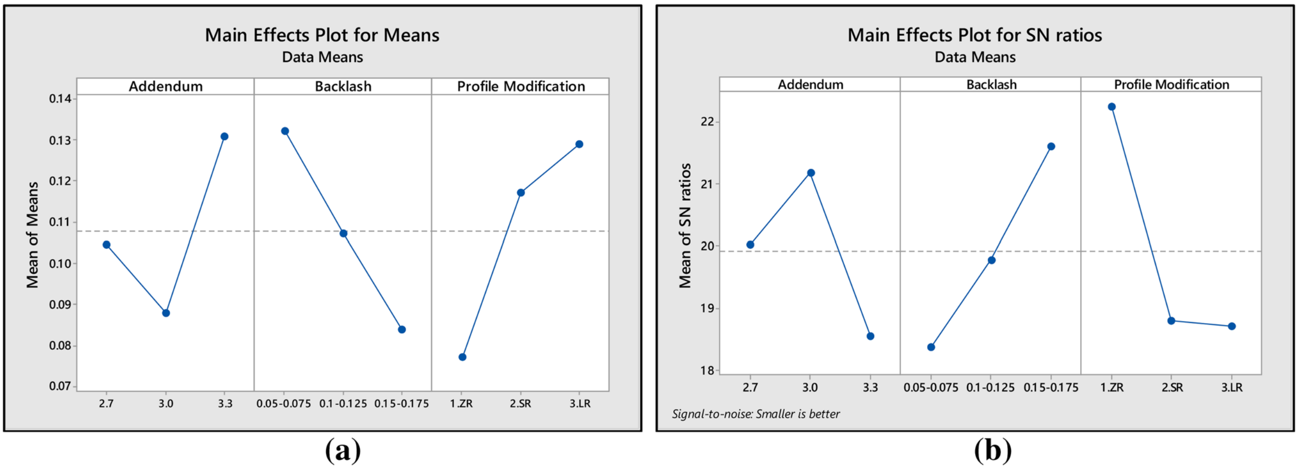

Fig. 10 Main effects plot **a** means, **b** S/N ratios

Table 10 Control parameters and their optimum level

Parameter designation	Parameter	Optimum level number	Optimum level value	Optimum parameter designation
A	Tooth addendum	2	3.00 mm	A_2
B	Backlash	3	0.150–0.175 mm	B_3
C	Linear tip relief profile modification	1	Zero relief (standard involute profile)	C_1

3rd, respectively, in terms of influence of dynamic response of spur gear pairs.

Main Effects Plot

The variation in mean acceleration response and S/N ratio with various parameters and their levels are plotted as main effect plots shown in Fig. 10a, b. The smaller the better criteria applied for mean effect plots of acceleration means response to optimize the parameters. A target value scatter is expressed by the S/N ratio. A high S/N ratio indicates that the signal outweighs the random effects of the noise factors by a significant amount. The higher the better criteria for the S/N ratio is chosen for optimal settings. From these plots, the tooth addendum (A), backlash (B), and linear tip relief tooth profile modification (C) should be at the 2nd, 3rd, and 1st levels for minimum dynamic response. Therefore, $A_2B_3C_1$ is the optimum combination of control parameters with their levels to result in minimum dynamic response. The optimal value of the control parameters is shown in Table 10.

Confirmation Test for Optimal Combination of Parameters and Their Levels

The optimal combination of factors and levels corresponding to the 6th experiment was designed based on Taguchi methodology. The confirmatory test is re-conducted for the optimal combination of parameters and their levels of spur gear pair identified. Figure 11 shows the frequency–response plot obtained from the experimental test of a spur gear pair 6 ($A_2B_3C_1$). It is seen from the plot that the value of the acceleration of gear pair set number 6 is 0.073 g, which is the lowest. Table 11 depicts researchers developed different

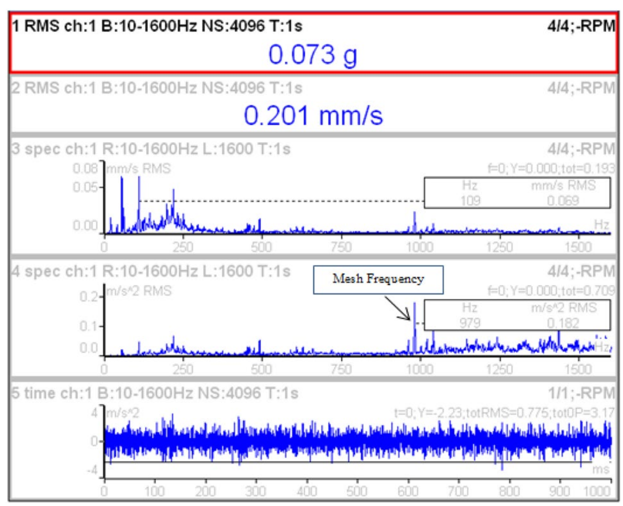

Fig. 11 Frequency–response plot for gear pair 6 ($A_2B_3C_1$)

Table 11 Comparison of dynamic response of spur gear pair by different researchers

Sr. no.	Authors	Model developed	Effects considered	Method	Results
1	Parey et.al [5]	6-dof gear dynamic model including localized tooth defects	Time-varying mesh stiffness and damping, backlash, gear errors, profile modification	Empirical mode decomposition (EMD), calculate kurtosis and crest factor value	Amplitude of acceleration signals increases with the defect size
2	Moradi and Salarieh [9]	SDOF model with backlash nonlinearity	Dynamic variables like stiffness coefficient, the system damping amplitude of engine excitation torque, and the detuning parameter	The investigation of the main, sub, and super harmonic resonances using the multiple-scale method	Show interesting nonlinear behavior under sub and super harmonic resonances due to backlash
3	Wen et al. [10]	SDOF model	Nonlinearity of the backlash and time-varying mesh stiffness	Path integration method and results compared with that of Monte Carlo simulation	Found good agreement between path integration and Monte Carlo simulation
4	Liu et al. [13]	Dynamic model with pitch deviation	System and operating parameters	Explored transmission stationarity by dynamic meshing forces	Gear pair leads to complex periodic motions due to pitch deviation
5	Proposed study	Experimental method	Addendum, backlash and linear tooth profile modification	Nine sets of spur gear pairs manufactured as per Taguchi orthogonal array. Measured data are analyzed by S/N ratio method	Predicted optimum level of parameters to minimize dynamic response

models, solution techniques and the effects included to predict dynamic response of spur gear pair. They used different performance parameters such as kurtosis and crest factor, and dynamic transmission error as a measurement of performance parameters. However, when a gear pair is operated at high speed, the number of parameters acts simultaneously on the dynamic performance of gear pair system. Therefore, present study investigated the combined influence of geometrical parameters on the dynamic behavior of gear pair system.

Results and Discussion

In the present research, the effect of a combination of geometrical factors on dynamic response is experimentally investigated using the Taguchi method. The obtained results from means and S/N ratio provide a lower dynamic response in terms of the optimal combination of factors and their levels as $A_2B_3C_1$. These factors are the standard value of tooth addendum is 3.00 mm, the highest value of backlash is 0.150 to 0.175 mm, and the standard involute profile results in the best combination to get the lower RMS level (g) for constant speed and load condition. The optimum combination of factors and levels corresponds to trial number 6 in OA, whose experimental value of RMS found to be 0.073 g, which is the lowest among the nine sets tested.

The variations ranking from response tables indicates the first dominant factor as linear tip relief as part of tooth profile modification. A significant change in the profile lowers the contact ratio and raises the dynamic load and vibrations.

A spur gear vibration can be decreased by appropriately altering the tooth surface form at the design stage [1]. However, Tables 8 and 9 and Fig. 10 show that the acceleration level increases with the level of tooth modification. The amount (Δ) and extent (L or α) of linear tooth profile modification have a major impact on the spur gear pairs dynamic performance. These results may be due to manufacturing errors and permitted tolerance. Manufacturing tooth profile alterations in the manner suggested by the theoretical approach are impractical. A modified tooth profile differs slightly from the optimum standard in practice. Errors do not have deterministic natures, and even gears with the same tolerance may experience various forms of excitation [6]. Within the same tolerance, slightly different profile inaccuracies can result in a variation of oscillation amplitude that is not insignificant.

Gear backlash is the second dominant factor, seen that the average mean response of spur gear pair decreases with an increase in gear backlash. The dynamic response is low at level 3 (0.150–0.175 mm) as compared with level 1 (0.050–0.075 mm) and level 2 (0.100–0.125 mm). The gear pair with little backlash may lead to interference due to manufacturing errors, which result in a higher amplitude of vibration. The purpose of a higher level of backlash is to avoid jamming and to guarantee that no teeth are in simultaneous contact on both sides. In unloaded gear drives, however, significant backlash can result in collisions and tooth separation. Vibration and noise difficulties arise as a result of such impacts. The 3rd dominant factor is tooth addendum, which has less effect on dynamic response than linear tip relief tooth profile modification and gear backlash. Table 8 estimates that the average response of the spur gear pair is minimum at level 2 (standard addendum = 3.00 mm), and it is maximum at a higher level 3 (addendum = 3.3 mm). The variation in the length of the tooth addendum has a significant effect on contact ratio and meshing stiffness and directly on the gear dynamics [3]. The dynamic load is decreased in low-contact-ratio gears (CR < 2.0) by raising the contact ratio.

Conclusion

An attempt to investigate the effect of various factors arising out of modification in the geometrical profile of gear pair, viz., addendum, backlash, and profile modification on the dynamic response of gear pair. It is estimated that linear tip relief tooth profile modification contributes significantly to vibration generation followed by backlash and addendum. From the mean effect plots and S/N ratio, it is observed that the optimal factor and their level combination is $A_2B_3C_1$. This translates standard value of tooth addendum (i.e., one time module = 3.00 mm), higher level of backlash in the range of 0.150 to 0.175 mm and standard involute profile results the best combination to get lower dynamic response for constant speed and load. The present research states that an optimal combination of geometrical parameters and their levels can play a major role in the dynamic response of spur gear pairs. There is a possibility of optimally selecting geometrical features of the gear tooth profile to minimize vibration generated during operations. An increase in the level of linear tip relief profile modification affects the increase in the vibration levels. Normally, tooth profile modification is one of the effective ways to minimize the variation in static transmission error (STE) and mesh stiffness of the gear pair. Due to manufacturing variances even to the level of a few micrometers, the effect of profile modification may become significant. Therefore, while applying a geometrical modification of the gear tooth profile, it is essential to check its quality in the manufacturing, assembly, and alignment process. The experimental study shows that geometrical features of a gear pair influence the overall dynamic behavior of the system. Taguchi method relatively requires lesser number of experiments to study entire parameter space. This developed technique helps gear designers to identify the most influencing parameters of the system in the early stage of gear design.

The development of a nonlinear dynamic model by considering time-varying geometrical parameters of gear pair reveals the future scope. The prognosis and estimated tarrying life of the gear can be determined using the gear diagnosis procedure. In addition, this work can be expanded to include additional soft computing techniques for rotating systems gear fault identification and experimental investigation.

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Data Availability Data used in the experiments cannot be shared due to ethical and research lab restrictions.

Declarations

Conflict of Interest No conflict of interest exists. We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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