ENGINEERING MATHEMATICS

UNIT–I : MATRICES AND DETERMINANTS

(As per DCET C20 Diploma Curriculum — Marks: 4)

DCET Exam Note:

Syllabus Coverage:

- Matrix types and algebra (Addition, Subtraction, Multiplication)
- Determinants of 2x2 and 3x3 matrices
- Cramer's Rule (2 and 3 variables)
- Adjoint and Inverse (2x2 and 3x3)
- Eigen values (2x2 matrices only)

1. MATRICES

Definition

A matrix is a rectangular arrangement of numbers or symbols in rows and columns, enclosed within brackets.

If a matrix has m rows and n columns, it is said to be of order $m \times n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Element Notation: a_{ij} where i = row number, j = column number

Important

In DCET examinations, **always identify the order first** - it's the crucial first step for most matrix problems.

Illustration 1.1

Find the order of the matrix:

$$A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & 4 & 6 \end{bmatrix}$$

Solution:

$$\text{Rows} = 2, \quad \text{Columns} = 3$$

$$\therefore \text{Order} = 2 \times 3$$

Illustration 1.2

Write the order of the matrix:

$$B = \begin{bmatrix} 4 & 6 & 8 & 10 \end{bmatrix}$$

Solution:

$$\text{Rows} = 1, \quad \text{Columns} = 4$$

$$\therefore \text{Order} = 1 \times 4$$

2. TYPES OF MATRICES

Type	Condition	Example
Row Matrix	Only one row	$[2\ 4\ 6]$
Column Matrix	Only one column	$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
Square Matrix	$m = n$	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
Zero/Null Matrix	All elements = 0	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Diagonal Matrix	Square with $a_{ij} = 0$ for $i \neq j$	$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
Identity Matrix	Diagonal elements = 1, others = 0	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Table 1: Types of Matrices (Important for DCET)

DCET Exam Note:

Remember: Identity matrix (I) is crucial for:

- Finding inverse: $AA^{-1} = I$
- Eigen values: $|A - \lambda I| = 0$

3. ALGEBRA OF MATRICES

3.1 Addition & Subtraction

Formula:

Condition: Same order ($m \times n$)

Rule: $(A+B)_{ij} = a_{ij} + b_{ij}$

Illustration 3.1

Add: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

Solution:

$$A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

3.2 Scalar Multiplication

$$k \cdot A = k \cdot \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

3.3 Matrix Multiplication

Formula:

Condition: Columns of A = Rows of B

If A is $m \times n$ and B is $n \times p$, then AB is $m \times p$

$(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

Illustration 3.2

Multiply: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

Solution:

$$AB = \begin{bmatrix} (1 \times 2 + 2 \times 1) & (1 \times 0 + 2 \times 2) \\ (3 \times 2 + 4 \times 1) & (3 \times 0 + 4 \times 2) \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

Important

Matrix multiplication is **NOT commutative**: $AB \neq BA$
But it's associative: $(AB)C = A(BC)$ and distributive: $A(B + C) = AB + AC$

4. DETERMINANTS

Formula:

For 2×2 matrix:

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

For 3×3 matrix (Sarrus Rule):

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - ceg - bdi - afh$$

Illustration 4.1

Evaluate: $\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$

Solution:

$$(2 \times 4) - (3 \times 1) = 8 - 3 = 5$$

Illustration 4.2

Evaluate: $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix}$

Solution:

$$
\begin{aligned}
&= 1(1 \times 0 - 4 \times 6) - 2(0 \times 0 - 4 \times 5) + 3(0 \times 6 - 1 \times 5) \\
&= 1(0 - 24) - 2(0 - 20) + 3(0 - 5) \\
&= -24 + 40 - 15 = 1
\end{aligned}
$$

5. SINGULAR & NON-SINGULAR MATRICES

Definition

- **Singular:** $|A| = 0$ (No inverse exists)
- **Non-singular:** $|A| \neq 0$ (Inverse exists)

Illustration 5.1

Check if $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is singular.

Solution:

$$|A| = (1 \times 4) - (2 \times 2) = 4 - 4 = 0$$

$\therefore$ A is singular (inverse doesn't exist).

6. CRAMER'S RULE

Formula:

For system $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$:

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D} \quad (\text{if } D \neq 0)$$

Illustration 6.1

Solve: $2x + y = 5$, $x - y = 1$

Solution:

$$D = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3$$

$$D_x = \begin{vmatrix} 5 & 1 \\ 1 & -1 \end{vmatrix} = -5 - 1 = -6$$

$$D_y = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 2 - 5 = -3$$

$$x = \frac{-6}{-3} = 2, \quad y = \frac{-3}{-3} = 1$$

7. ADJOINT AND INVERSE

Formula:

For 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\mathrm{Adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad A^{-1} = \frac{1}{|A|}\mathrm{Adj}(A) \quad (\text{if } |A| \neq 0)$$

Illustration 7.1

Find inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:

$$|A| = 4 - 6 = -2, \quad \mathrm{Adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

8. EIGEN VALUES (2×2 ONLY)

Concept

Eigen values (λ) satisfy $A\mathbf{x} = \lambda\mathbf{x}$
Found by solving characteristic equation: $|A - \lambda I| = 0$

Step-by-Step Method

1. Form $A - \lambda I$
2. Find determinant $|A - \lambda I|$
3. Set $= 0$ and solve for λ

Illustration 8.1

Find eigen values of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Solution:

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = (2-\lambda)^2 - 1 = \lambda^2 - 4\lambda + 3$$

$$\lambda^2 - 4\lambda + 3 = 0 \Rightarrow (\lambda - 3)(\lambda - 1) = 0$$

$$\therefore \lambda = 1, 3$$

Practice Problems (DCET Pattern)

DCET Exam Note:

Solve these typical DCET questions:

1. Find order: $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ [Ans: 2×3]
2. Add: $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ [Ans: $\begin{bmatrix} 3 & 5 \\ 2 & 2 \end{bmatrix}$]

3. Multiply: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ [Ans: $\begin{bmatrix} 4 & 7 \\ 10 & 15 \end{bmatrix}$]
4. Find $|A|$ for $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ [Ans: 2]
5. Solve using Cramer's rule: $x + y = 5$, $2x - y = 1$ [Ans: $x = 2, y = 3$]
6. Find A^{-1} for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ [Ans: $\frac{1}{5} \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$]
7. Find eigen values: $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ [Ans: 2, 4]

Quick Revision Points

- Order: rows × columns
- Add/subtract: same order only
- Multiply: col(A) = row(B)
- $|2 \times 2|$: $ad - bc$
- Singular: $|A| = 0$
- $A^{-1} = \frac{1}{|A|}\text{Adj}(A)$
- Eigen: Solve $|A - \lambda I| = 0$
- I = Identity matrix

DCET Exam Note:

DCET Exam Tips:

- Always check matrix dimensions first
- For multiplication, verify compatibility
- For inverse, check $|A| \neq 0$
- Eigen values only for 2×2 in DCET
- Practice time management: 1-2 minutes per mark