Homework Week 4

Text Questions

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1. Weiss, Exercise 7.1. Show the intermediate steps.

Sort the sequence 3, 1, 4, 1, 5, 9, 2, 6, 5 using insertion sort.

```
3| 1 4 1 5 9 2 6 5
1 3| 4 1 5 9 2 6 5
1 3 4| 1 5 9 2 6 5
1 1 3 4| 5 9 2 6 5
1 1 3 4 5| 9 2 6 5
1 1 3 4 5 9| 2 6 5
1 1 2 3 4 5 9| 6 5
1 1 2 3 4 5 6 9| 5
1 1 2 3 4 5 5 6 9|
```

2. Weiss, Exercise 7.19.

Sort 3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5 using quicksort with median-of-three partitioning and a cutoff of 3.

Top level sort (recursion 0 level):

Mof3(3,9,5) = 5, 5 == pivot.

Sort 1st, mid, and last, and partition:

```
[3 1 4 1 5 3 2 6 5 |5 9]

[3 1 4 1 (5) 3 2 6 (5) |5 9]
          i         j

[3 1 4 1 5 3 2 6 5 |5 9]
             j i

[3 1 4 1 5 3 2 5 5 6 9]

small: [3 1 4 1 5 3 2 5]

large: [5 6 9]
```

Sort small side (recursion 1 level):

Mof3(3,1,5) = 3, 3 == pivot.

Sort 1st, mid, and last, and partition:

```
[1 1 4 3 5 3 2 5]

[1 1 4 2 5 3 |3 5]

[1 1 (4) 2 5 (3) |3 5]
      i       j

[1 1 3 2 (5) 4 |3 5]
       j  i

[1 1 3 2 3 4 5 5 ]

small: [1 1 3 2]

large: [3 4 5 5]
```

Sort small of small (recursion 2 level):

Mof3(1,1,2) = 1, 1 == pivot.

Sort 1st, mid, and last, and partition:

```
[1 3 |1 2]

[1 3 |1 2]
 j i

[1 1 3 2]

small: [1 1]
large: [3 2]

insertion sort returns: [1 1 2 3]
```

Sort large of small (recursion 2 level):

Mof3(3,4,5) = 4, 4 == pivot.

Sort 1st, mid, and last, and partition:

```
[3 5 |4 5]
```

```
 j i

insertion sort returns: [3 4 5 5]
```

Insertion sort takes care of large of top (recursion 1 level):

```
insertion sort returns: [5 6 9]
```

Final array:

```
[1 1 2 3 3 4 5 5 5 6 9]
```

3. Weiss, Exercise 7.28a.

When implementing quicksort, if the array contains lots of duplicates, it may be better to perform a three-way partition (into elements less than, equal to, and greater than the pivot), to make smaller recursive calls. Assume three-way comparisons, as provided by the compareTo method.

a. Give an algorithm that performs a three-way in-place partition of an N-element subarray using only $N - 1$ three-way comparisons. If there are d items equal to the pivot, you may use d additional Comparable swaps, above and beyond the two-way partitioning algorithm. (Hint: As i and j move toward each other, maintain five groups of elements as shown below):

```
EQUAL SMALL UNKNOWN LARGE EQUAL
           i       j
```

```
int i = left, j = right - 1;
int leftPivots = 0;
int rightPivots = 0;

for( ; ; ){

  while( a[ ++i ].compareTo( pivot ) < 0 ){}
  while( a[ --j ].compareTo( pivot ) > 0 ){}

  if (a[j].compareTo(pivot) == 0){
    swapReferences(a, a[j], left + 1 + leftPivots); //swap small side pivots to left
    leftPivots++;
  }

  if (a[i].compareTo(pivot) == 0){
    swapReferences(a, a[i], right - 2 - rightPivots); //swap large side pivots to right
```

```
    rightPivots++;
  }

  if(i<j)
    swapReferences( a, i, j );
  else
    break;
}

// return right elements + pivot
for(int k = 0; k < rightPivots + 1; k++){
  swapReferences( a, i + k, right - (k+1) );
}

// return left elements
for(int k = 0; k < leftPivots; k++){
  swapReferences( a, j - k, left + (k+1) );
}
```

4. Weiss, Exercise 9.1.

Find a topological ordering for the graph in Figure 9.81.

```
   A 2 1  1  0 0  -  - - - - -
   B 1 1  1  1 1  0  - - - - -
   C 3 3  3  3 3  3  2 1 1 0 -
   s 0 -  -  - -  -  - - - - -
   D 2 1  0  - -  -  - - - - -
   E 4 4  3  2 1  0  0 - - - -
   F 2 2  2  2 2  2  2 1 0 - -
   t 3 3  3  3 3  3  3 3 2 1 0
   G 1 0  -  - -  -  - - - - -
   H 1 1  0  0 -  -  - - - - -
   I 2 1  1  1 1  1  1 0 - - -
Enq:  s G D,H A - B,E - I F C t
Deq:  s G  D  H A  B  E I F C t

Ordering: [s,G,D,H,A,B,E,I,F,C,t]
```

5. Weiss, Exercise 9.10a.

Explain how to modify Dijkstra's algorithm to produce a count of the number of different minimum paths from v to w.

Djikstra's algorithm keeps track of the length of the shortest path to a given vertex. You could keep track of another value as well, the number of shortest paths that are equal to that length.

When you know v's shortest path, you can mark it as known. You have kept track of the number of shortest paths to v. Then, for every adjacenct vertex w to v you can add the number of shortest paths to w (through v or not) to the count of shortest paths to v.

If the shortest path to w through v is less than the length of the existing path, then replace the count of w with the count of v. If it's the same length, then add the count of v to the count of w.

6. Weiss, Exercise 9.15.

a. Find a minimum spanning tree for the graph in Figure 9.84 using both Prim's and Kruskal's algorithms.

Vertices:

(A,B) 3 Accepted

(B,C) 10 Rejected

(D,A) 4 Accepted

(A,E) 4 Rejected

(E,B) 2 Accepted

(B,F) 3 Accepted

(F,C) 6 Rejected

(C,G) 1 Accepted

(D,E) 5 Rejected

(E,F) 11 Rejected

(F,G) 2 Accepted

(D,H) 6 Rejected

(H,E) 2 Accepted

(E,I) 1 Accepted

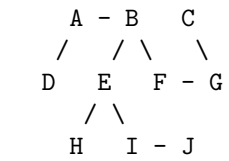
(I,F) 3 Rejected

(F,J) 11 Rejected

(J,G) 8 Rejected

(H,I) 4 Rejected

(I,J) 7 Accepted



b. Is this minimum spanning tree unique? Why?

No, not unique, you can connect the vertices in more than one configuration.

7. Weiss, Exercise 9.38a and 9.38b.

You are given a set of N sticks, which are lying on top of each other in some configuration. Each stick is specified by its two endpoints; each endpoint is an ordered triple giving its x, y, and z coordinates; no stick is vertical. A stick may be picked up only if there is no stick on top of it.

a. Explain how to write a routine that takes two sticks a and b and reports whether a is above, below, or unrelated to b. (This has nothing to do with graph theory.)

Compute the ranges of the two sticks on the x and y axes:

$R(a_{start}(x), a_{end}(x)) = R_{ax}$ $R(a_{start}(y), a_{end}(y)) = R_{ay}$

$R(b_{start}(x), b_{end}(x)) = R_{bx}$ $R(b_{start}(y), b_{end}(y)) = R_{by}$

If either of:

1) the intersection of the x-ranges of a and b
2) the inersection of the y-ranges of a and b

are zero, then the two sticks are not in the same place.

If the intersection of both ranges != 0, then calculate the point at which the two sticks cross (intersection of the two lines in the x-y planes) and the stick with the higher z-value at that point is on top.

b. Give an algorithm that determines whether it is possible to pick up all the sticks, and if so, provides a sequence of stick pickups that accomplishes this.

Can do this with Kruskal's algorithm. Every stick is a vertex in its own disjoint set. The graph is "directed" in that a stick on top of another outranks the one below it.

If stick A is on the top of the stack (all things below it), then A is the set of picked up sticks. If stick B is below A, then add B to the set of A. If a stick has nothing on top of it, it can be added to the set of A, reppeat until no sticks remain (essentially topological sort it). If there is no cycle, you can pick up all the sticks, otherwise not.