### **Question:**

Given that x and y satisfy the following system of inequalities, find the integers x and y that maximize x + y.

$$\begin{cases} 2x - y - 3 \ge 0 \\ 2x + 3y - 6 \le 0 \\ 3x - 5y - 15 \le 0 \end{cases}$$

#### **Solution:**

First, process each inequality in the system of inequalities:

1.  $(2x - y - 3 \ge 0)$  can be transformed to  $(y \le 2x - 3)$ ,

2.  $\(2x + 3y - 6 \le 0)\)$  can be transformed to  $\(3y \le -2x + 6)\)$ , i.e.,  $\(y \le \frac{-2x + 6}{3}\)$ , 3.  $\(3x - 5y - 15 \le 0)\)$  can be transformed to  $\(5y \ge 3x - 15\)$ , i.e.,  $\(y \ge \frac{3x - 15}{5}\)$ . Next, find the feasible region given by  $\(y \le 2x - 3\)$ ,  $\(y \le \frac{-2x + 6}{3}\)$ , and  $\(y \le \frac{3x - 15}{5}\)$ .

Examine the intersection points of these three lines and substitute them to verify they meet all the constraints:

Find the intersection point (A): where (y = 2x - 3) and  $(y = \frac{-2x + 6}{3})$ . Solve the system of equations:

 $(2x - 3 = \frac{-2x + 6}{3})$ , which gives (6x - 9 = -2x + 6), implying (8x = 15), solving for (x) yields  $(x = \frac{15}{8})$ , substitute back into (y = 2x - 3) to get  $(y = \frac{-9}{8})$ , so the intersection point is  $(A(\frac{15}{8}, \frac{-9}{8}))$ .

Find the intersection point \(B\): where \(y = 2x - 3\) and \(y = \frac{3x - 15}{5}\). Solve the system of equations:

\(2x - 3 = \frac{3x - 15}{5}\), which gives \(10x - 15 = 3x - 15\), implying \(7x = 0\), solving for \(x\) yields \(x = 0\), substitute back into \(y = 2x - 3\) to get \(y = -3\), so the intersection point is \(B(0, -3)\). Find the intersection point \(C\): where \(y = \frac{-2x + 6}{3}\) and \(y = \frac{3x - 15}{5}\). Solve the system of equations: \(\frac{-2x + 6}{3} = \frac{3x - 15}{5}\), cross-multiplying yields \(-10x + 30 = 9x - 45\), implying \(19x = 75\), solving for \(x\) yields \(x = \frac{75}{19}\), substitute back into \(y = \frac{-2x + 6}{3}\) to get \(y = \frac{57}{19}\), so the intersection point is \(C(\frac{75}{19}), \frac{57}{19})\).

Find the integer point  $\langle (x, y) \rangle$  that maximizes  $\langle (x + y) \rangle$ : the integer point within the feasible region that maximizes  $\langle (x + y) \rangle$  is  $\langle (x, y) \rangle \rangle$ , so the maximum value of  $\langle (x + y) \rangle$  at the boundary is  $\langle (3) \rangle$ . Final answer:  $\langle (x + y) \rangle$ .

#### **Gold Step:**

- (1) First, process each inequality in the system of inequalities:
- 1.  $(2x y 3 \ge 0)$  can be transformed to  $(y \le 2x 3)$ ,
- 2.  $(2x + 3y 6 \le 0)$  can be transformed to  $(3y \le -2x + 6)$ , i.e.,  $(y \le \frac{-2x + 6}{3})$ ,
- 3.  $(3x 5y 15 \le 0)$  can be transformed to  $(5y \ge 3x 15)$ , i.e.,  $(y \le \frac{3x 15}{5})$ . (2) Next, find the feasible region given by  $(y \le 2x 3)$ ,  $(y \le \frac{-2x + 6}{3})$ , and  $(y \le \frac{3x 15}{5})$ .
- (3) Examine the intersection points of these three lines and substitute them to verify they meet all the constraints:
- Find the intersection point (A): where (y = 2x 3) and  $(y = \frac{-2x + 6}{3})$ . Solve the system of equations:

 $(2x - 3 = \frac{-2x + 6}{3}),$ 

which gives (6x - 9 = -2x + 6),

implying (8x = 15),

solving for  $\langle x \rangle$  yields  $\langle x = \frac{15}{8} \rangle$ ,

substitute back into (y = 2x - 3) to get  $(y = \frac{-9}{8})$ ,

so the intersection point is  $(A(\frac{15}{8}, \frac{-9}{8}))$ .

(4) Find the intersection point \(B\): where \(y = 2x - 3\) and \(y = \frac{3x - 15}{5}\). Solve the system of equations:

 $(2x - 3 = \frac{3x - 15}{5}),$ 

which gives (10x - 15 = 3x - 15),

implying (7x = 0),

solving for (x) yields (x = 0),

substitute back into (y = 2x - 3) to get (y = -3),

so the intersection point is  $\setminus (B(0, -3) \setminus)$ .

(5) Find the intersection point \(C\): where \(y = \frac{-2x + 6}{3}\) and \(y = \frac{3x - 6}{3}\)

15{5}\). Solve the system of equations:

implying (19x = 75),

solving for  $\langle (x) \rangle$  yields  $\langle (x = \frac{75}{19}) \rangle$ ,

solving for ((x)) yields  $((x = \frac{75}{19}))$ , substitute heads into  $((x = \frac{75}{19}))$ 

substitute back into  $(y = \frac{-2x + 6}{3})$  to get  $(y = \frac{57}{19})$ , so the intersection point is  $(C(\frac{57}{19}), \frac{57}{19})$ 

so the intersection point is  $(C(\frac{75}{19}, \frac{57}{19}))$ .

(6) Find the integer point  $\langle (x, y) \rangle$  that maximizes  $\langle (x + y) \rangle$ : the integer point within the feasible region that maximizes  $\langle (x + y) \rangle$  is  $\langle (x, y) \rangle \rangle$ , so the maximum value of  $\langle (x + y) \rangle$  at the boundary is  $\langle (3) \rangle$ .

(7) Final answer: ((x=2; y=1)).

# Gold Step Scores: ["1", "1", "0", "1", "0", "0", "0"]

## Gold Scores: 3

### StepMathAgent:

- (1) Transform the inequality  $(2x y 3 \ge 0)$  into  $(y \le 2x 3)$ : 1
- (2) Transform the inequality  $(2x + 3y 6 \le 0)$  into  $(y \le \frac{-2x + 6}{3})$ : 1
- (3) Transform the inequality  $(3x 5y 15 \le 0)$  into  $(y \ge \frac{3x 15}{5})$ : 1
- (4) Determine the feasible region defined by the inequalities  $(y \le 2x 3)$ ,  $(y \le \frac{-2x + 6}{3})$ ,  $(y \le \frac{3x 15}{5})$ : 1
- (5) Calculate the intersection point (A): (y = 2x 3) and  $(y = \frac{-2x + 6}{3})$ , solve the system to get  $(x = \frac{15}{8})$ ,  $(y = \frac{-9}{8})$ : **0**
- (6) Calculate the intersection point  $\B \$ :  $\y = 2x 3\$ ) and  $\y = \frac{3x 15}{5}\$ , solve the system to get  $\x = 0\$ ,  $\y = -3\$ : 1
- (7) Calculate the intersection point  $\(C\): \(y = \frac{-2x + 6}{3}\)$  and  $\(y = \frac{3x 15}{5}\)$ , solve the system to get  $\(x = \frac{75}{19}\)$ ,  $\(y = \frac{57}{19}\)$ : **0**
- (8) Determine the integer pair ((2, 1)) within the feasible region that maximizes (x + y): 0
- (9) The final answer  $\langle (x = 2, y = 1) \rangle$ , so  $\langle (x + y = 3) \rangle$ : **0**

# Final score: 4

Tree-of-Error: (5)-(8)-(9), (7)-(8)-(9)

## 问题:

已知x,y满足如下不等式组,求使x+y取最大值的整数x,y。

### 解题过程:

解题过程:【首先,处理不等式组中的每个不等式: 1. \$2x - y - 3 \ge 0\$ 可以变形为 \$y \le 2x - 3\$。 2. \$2x + 3y - 6 \le 0\$ 可以变形为 \$3y \le -2x + 6\$,即 \$y \le \frac{-2x + 6}{3}\$。 3. \$3x - 5y - 15 \le 0\$ 可以变形为 \$5y \ge 3x - 15\$,即 \$y \ge \frac{3x - 15}{5}\$。接下来, 求可行域: - 由 \$v \le 2x - 3\$ 和 \$v \le \frac{-2x + 6}{3}\$ 以及 \$v \ge \frac{3x - 15}{5}\$ 得 到的可行域区域。画图可以帮助理解。- 考察这三条直线的交点,并代入确认满足所有 不等式的约束。求交点: 1. 交点 \$A\$: \$y = 2x - 3\$ 和 \$y = \frac{-2x + 6}{3}\$,解方程组: - \$2x - 3 = \frac{-2x + 6}{3}\$,得 \$6x - 9 = -2x + 6\$,即 \$8x = 15\$,解得 \$x = \frac{15}{8}\$, 代入 \$y = 2x - 3\$ 得 \$y = \frac{-9}{8}\$, 所以交点 \$A(\frac{15}{8}, \frac{-9}{8})\$。 2. 交点 \$B\$: \$y = 2x - 3\$ 和 \$y = \frac{3x - 15}{5}\$,解方程组: - \$2x - 3 = \frac{3x - 15}{5}\$,得 \$10x - 15 = 3x - 15\$,即 \$7x = 0\$,解得 \$x = 0\$,代入 \$y = 2x -3\$ 得 \$y = -3\$,所以交点 \$B(0, -3)\$。3. 交点 \$C\$: \$y = \frac{-2x + 6}{3}\$ 和 \$y = \frac{3x - 15}{5}\$,解方程组: - \$\frac{-2x + 6}{3} = \frac{3x - 15}{5}\$,交叉相乘得到 \$-10x + 30 = 9x - 45\$,即 \$19x = 75\$,解得  $\$x = \frac{75}{19}\$$ ,代入  $\$y = \frac{-2x + 30}{9}$ 6}{3}\$ 得 \$y = \frac{57}{19}\$, 所以交点 \$C(\frac{75}{19}, \frac{57}{19})\$。找出使 \$x+y\$ 最大的整数对\$(x,y)\$: - 尝试不同接近整数边界值的整数对,在可行域内选择。-观察可行区域发现,在可行区域内的整数点最多达到\$(x,y) = (2, 1)\$。- 故求得 \$x+y\$ 最 大的整数对在边界 \$(2,1)\$,其值为 \$3\$。】最终答案:  $\mathbf{x}=2$ ;  $\mathbf{y}=1$ 】

## 标准划分:

- (1) 首先,处理不等式组中的每个不等式: 1.  $\$2x y 3 \ge 0\$$  可以变形为  $\$y \ge 2x 3\$$ , 2.  $\$2x + 3y 6 \ge 0\$$  可以变形为  $\$3y \ge -2x + 6\$$ , 即  $\$y \ge \frac{-2x + 6}{3}\$$ ,  $\$3x 5y 15 \ge 0\$$  可以变形为  $\$5y \ge 3x 15\$$ , 即  $\$y \ge \frac{3x 15}{5}\$$
- (2) 接下来求由 \$y \le 2x 3\$ 和 \$y \le \frac{-2x + 6}{3}\$ 以及 \$y \ge \frac{3x 15}{5}\$ 得到的可行域区域
- (3) 考察这三条直线的交点,并代入确认满足所有不等式的约束: 求交点 \$A\$: \$y = 2x 3\$ 和 \$y = \frac{-2x + 6}{3}\$, 解方程组: \$2x 3 = \frac{-2x + 6}{3}\$, 得 \$6x 9 = -2x + 6\$, 即 \$8x = 15\$, 解得 \$x = \frac{15}{8}\$, 代入 \$y = 2x 3\$ 得 \$y = \frac{-9}{8}\$, 所以交点 \$A(\frac{15}{8}, \frac{-9}{8})\$
- (4) 求交点 \$B\$: \$y = 2x 3\$ 和  $\$y = \frac{3x 15}{5}\$$ ,解方程组:  $\$2x 3 = \frac{3x 15}{5}\$$ ,得 \$10x 15 = 3x 15\$,即 \$7x = 0\$,解得 \$x = 0\$,代入 \$y = 2x 3\$ 得 \$y = -3\$,所以交点 \$B(0, -3)\$
- (5) 求交点 \$C\$:  $y = \frac{-2x + 6}{3}$  和  $y = \frac{3x 15}{5}$ , 解方程组:  $\frac{-2x + 6}{3} = \frac{3x 15}{5}$ , 交叉相乘得到 -10x + 30 = 9x 45, 即 19x = 75, 解 得  $x = \frac{75}{19}$ , 代入  $y = \frac{-2x + 6}{3}$  得  $y = \frac{57}{19}$ , 所以交点  $C(\frac{75}{19}, \frac{57}{19})$
- (6) 找出使 x+y 最大的整数对(x,y): 在可行区域内的整数点最多达到(x,y) = (2,1), 故求得 x+y 最大的整数对在边界 (2,1), 其值为 3
- (7) 最终答案: 【x=2; y=1】

标准步骤分数: ["1", "1", "0", "1", "0", "0", "0"]

标准最终分数:3

#### **StepMathAgent:**

- (1) 将不等式 \$2x y 3 \ge 0\$ 变形为 \$y \le 2x 3\$: 1
- (2) 将不等式 \$2x + 3y 6 \le 0\$ 变形为 \$y \le \frac{-2x + 6}{3}\$: 1
- (3) 将不等式 \$3x 5y 15 \le 0\$ 变形为 \$y \ge \frac{3x 15}{5}\$: 1
- (4) 确定可行区域是由不等式  $y \le 2x 3$ ,  $y \le \frac{-2x + 6}{3}$ ,  $y \le \frac{3x 15}{5}$ , 确定的区域: **1**
- (5) 计算交点 \$A\$: y = 2x 3\$ 和  $y = \frac{-2x + 6}{3}$ \$,解方程组得到  $x = \frac{15}{8}$ \$, $y = \frac{-9}{8}$ \$:**0**
- (6) 计算交点 \$B\$: \$y = 2x 3\$ 和 \$y = \frac{3x 15}{5}\$, 解方程组得到 \$x = 0\$, \$y = -3\$: **1**
- (7) 计算交点 \$C\$: \$y = \frac{-2x + 6}{3}\$ 和 \$y = \frac{3x 15}{5}\$, 解方程组得到 \$x = \frac{75}{19}\$, \$y = \frac{57}{19}\$: **0**
- (8) 判断在可行区域内使 x+y 最大的整数对是 (2, 1): 0
- (9) 最终答案 \$x=2, y=1\$, 故 \$x+y=3\$: **0**

最终得分:4

错误链: (5)-(8)-(9), (7)-(8)-(9)

# **Tree-of-Error**

(5) 计算交点 \$A\$: \$y = 2x - 3\$ 和 \$y = \frac{-2x + 6}{3}\$,解方程组得到 \$x = \frac{15}{8}\$,\$y = \frac{-9}{8}\$

(7) 计算交点 \$C\$: \$y = \frac{-2x + 6}{3}\$ 和 \$y = \frac{3x - 15}{5}\$, 解方程组得到 \$x = \frac{75}{19}\$, \$y = \frac{57}{19}\$

(8) 判断在可行区域内使 \$x+y\$ 最大的整数对是 \$(2,1)\$

(9) 最终答案 \$x=2, y=1\$, 故 \$x+y=3\$