

# 計算認知神經科學 計算視覺



# Edge



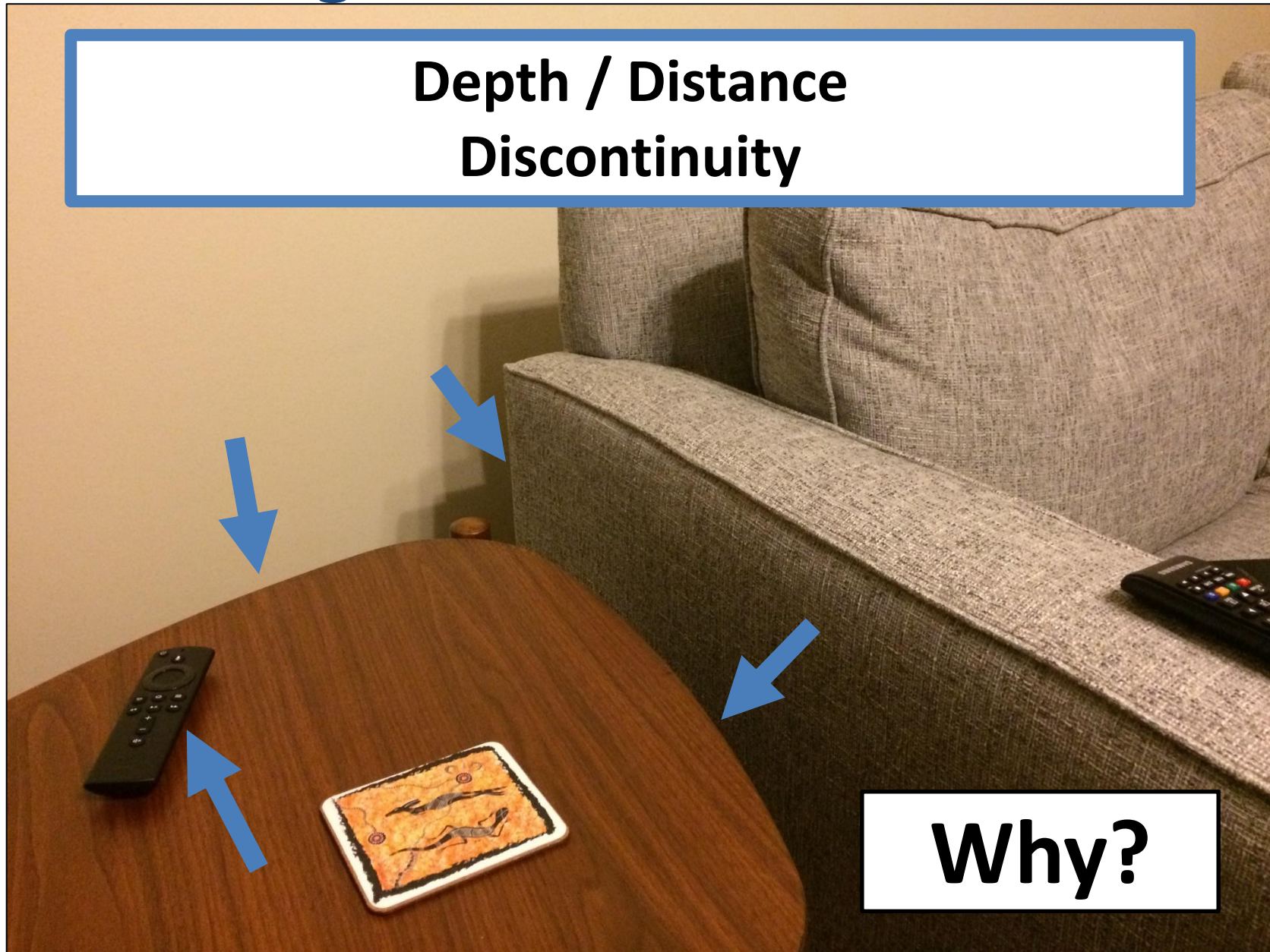


# Color often not a powerful feature



However, these are all images of people but the colors in each image are very different.

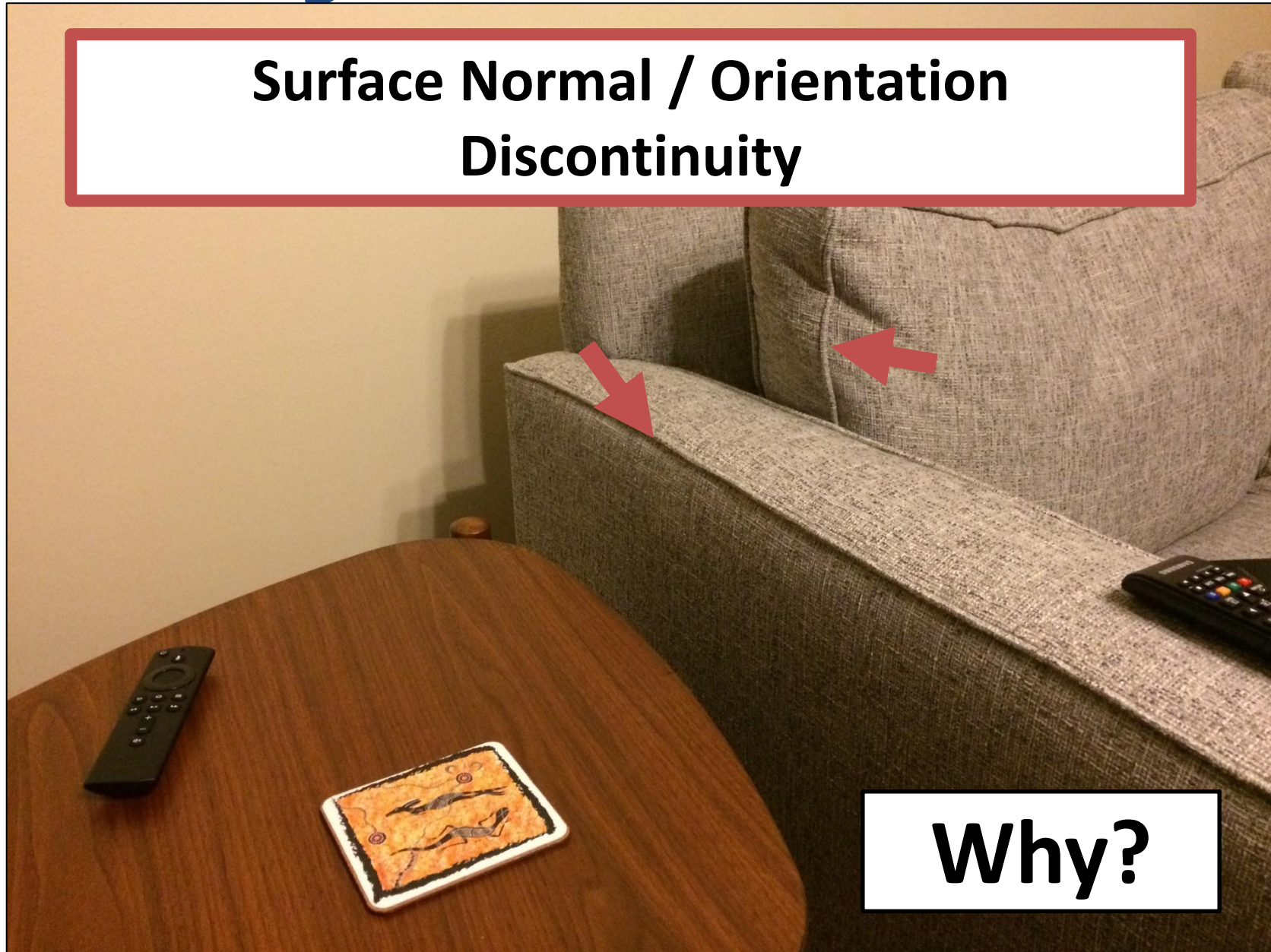
# Where do Edges Come From?





# Where do Edges Come From?

**Surface Normal / Orientation  
Discontinuity**

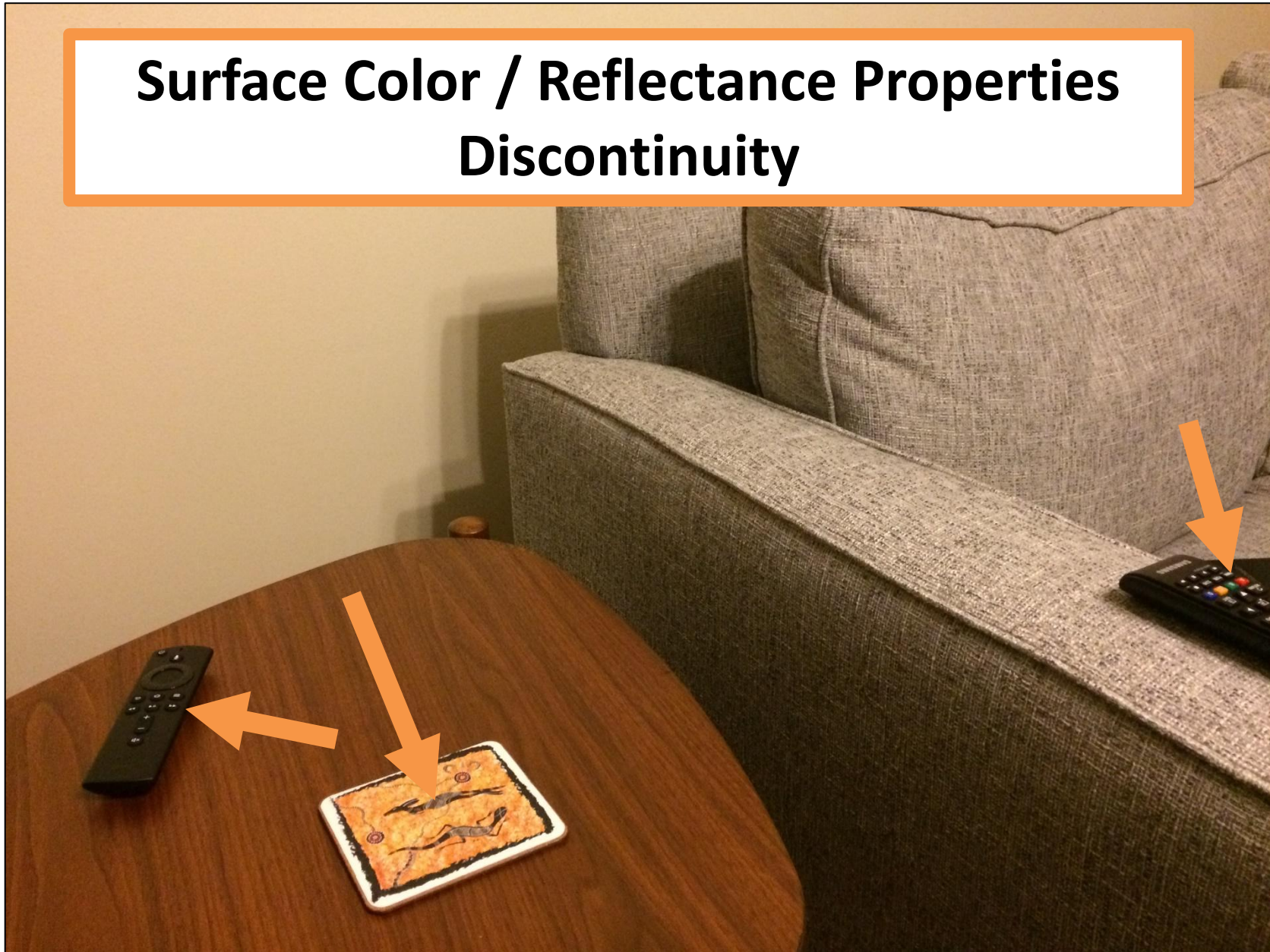


**Why?**



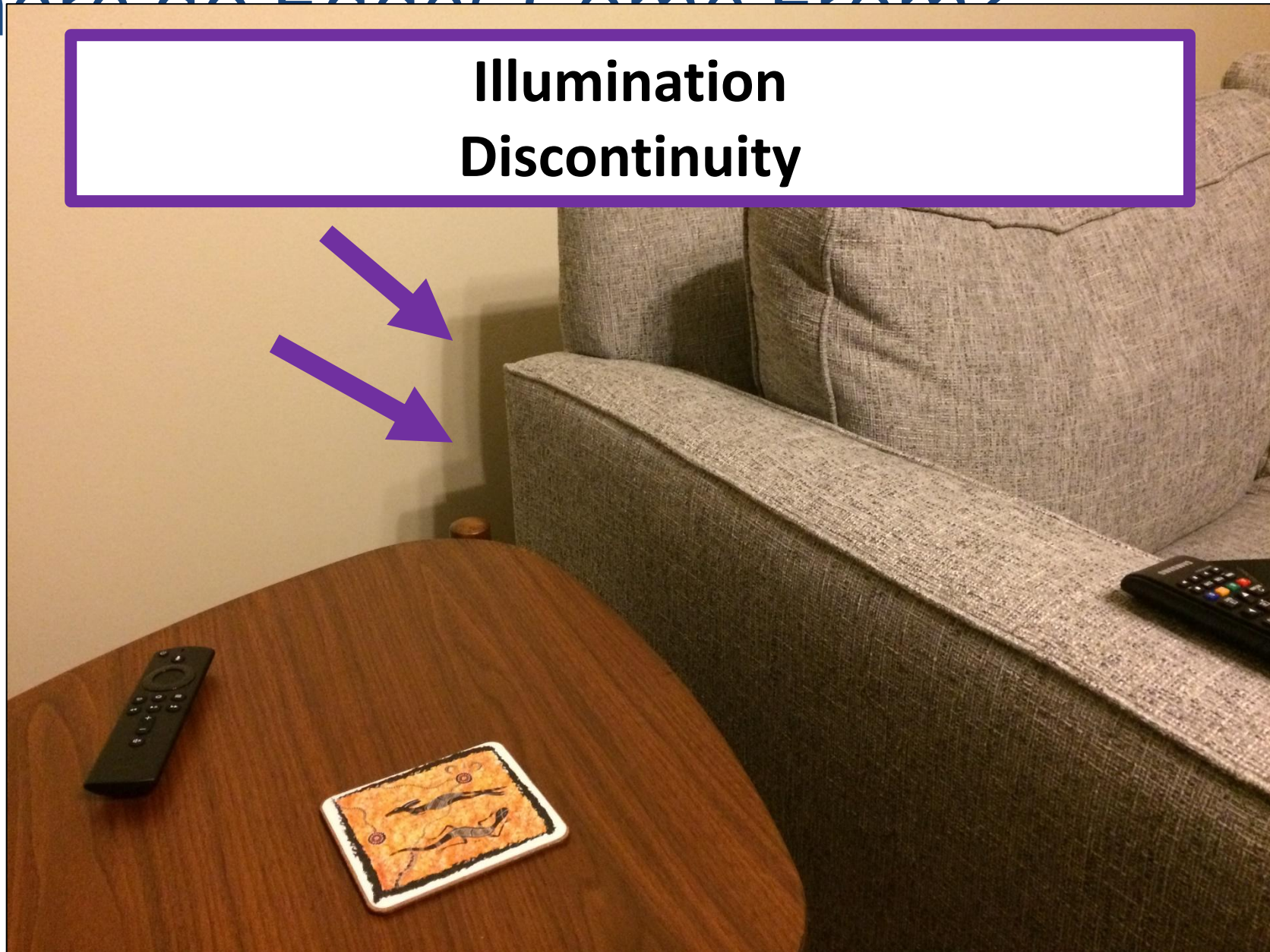
# Where do Edges Come From?

**Surface Color / Reflectance Properties  
Discontinuity**



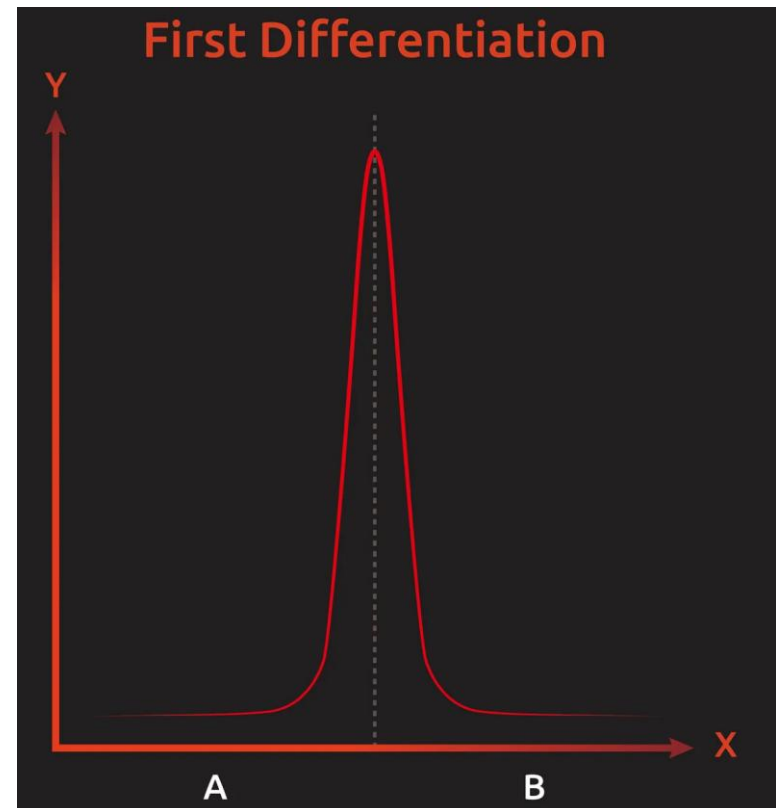
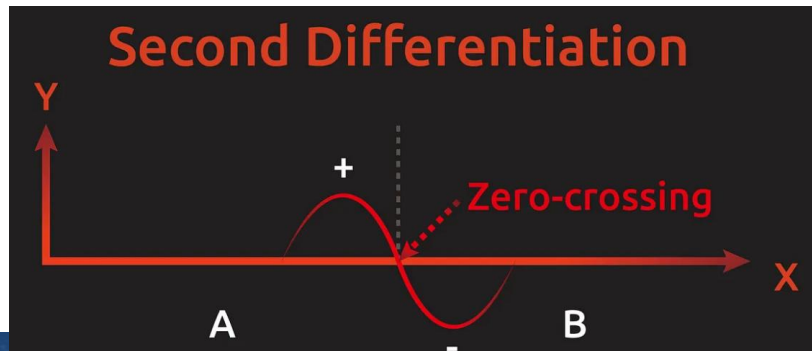
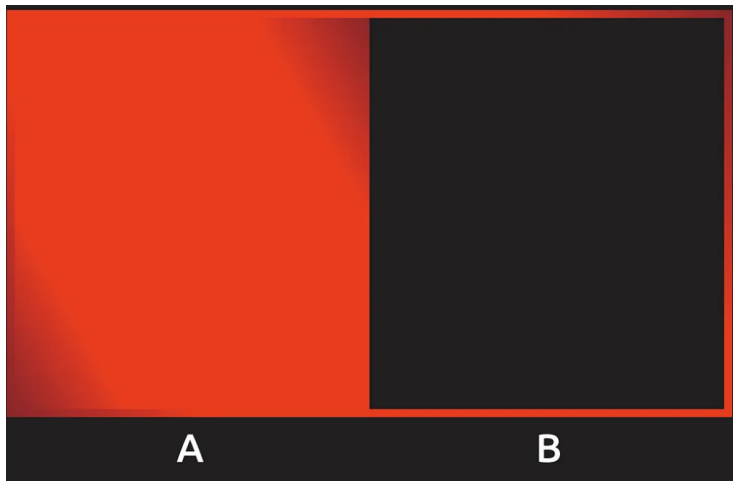


# Where do Edges Come From?



# 邊緣(edge)

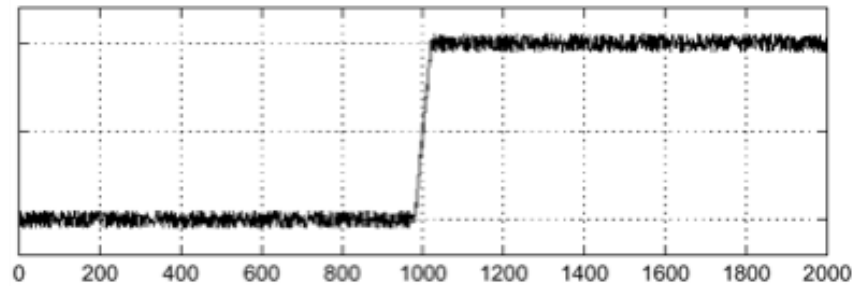
在 A、B 兩個區域內的像素灰階值是非常相近的，而兩個區域之間的灰階值會出現「**急劇變化 (Abrupt change)**」，可以把該區域之灰階值做一次微分，並繪製成二維示意圖來觀察





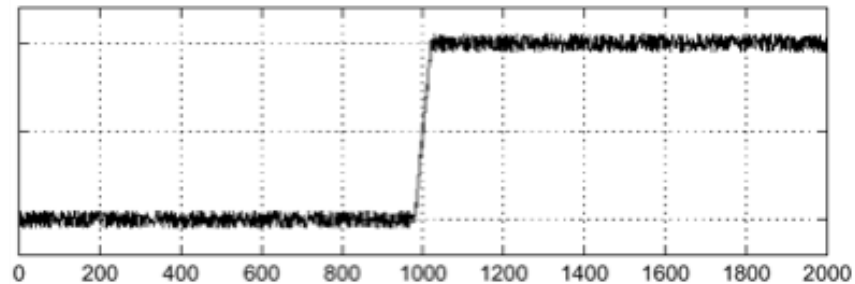
# How do you find the edge of this signal?

intensity plot



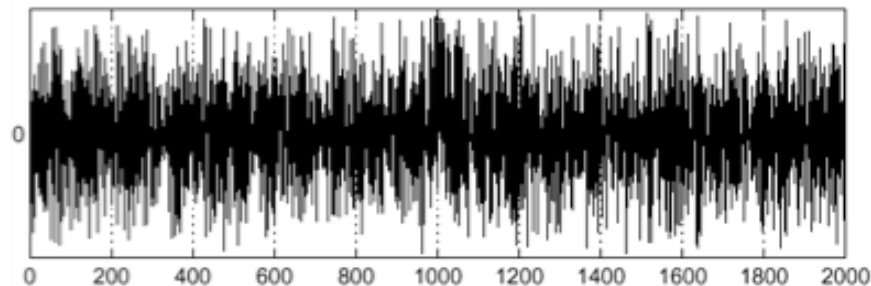
# How do you find the edge of this signal?

intensity plot



Using a derivative filter:

derivative plot



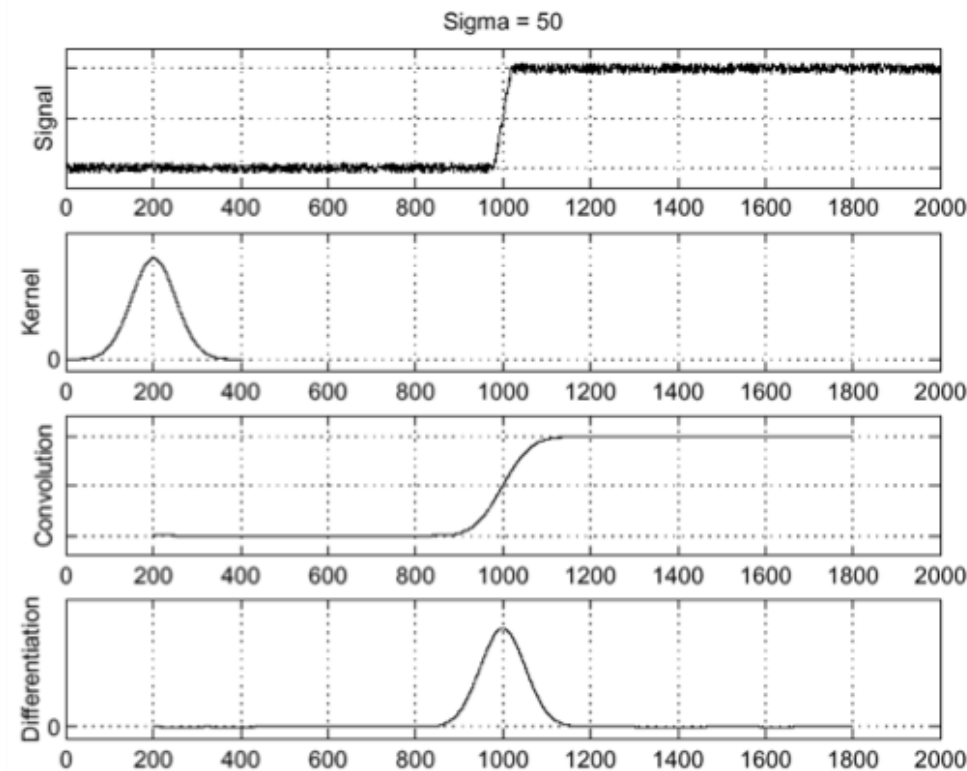
What's the problem here?



# Differentiation is very sensitive to noise

When using derivative filters, it is critical to blur first!

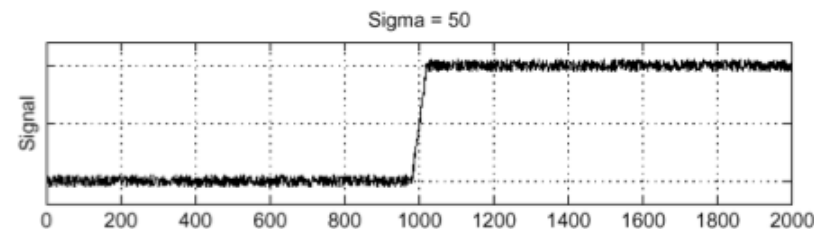
input  
Gaussian  
blurred  
derivative of  
blurred



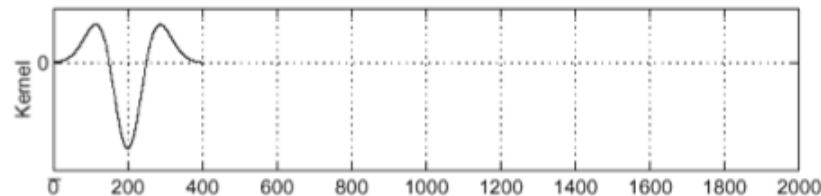
# Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

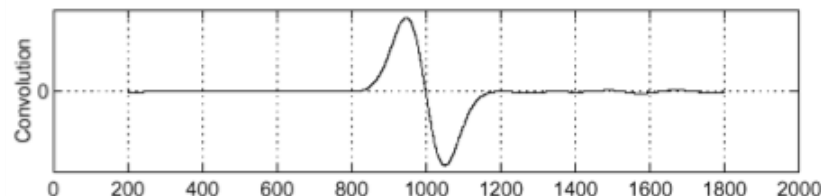
input



Laplacian of  
Gaussian



output

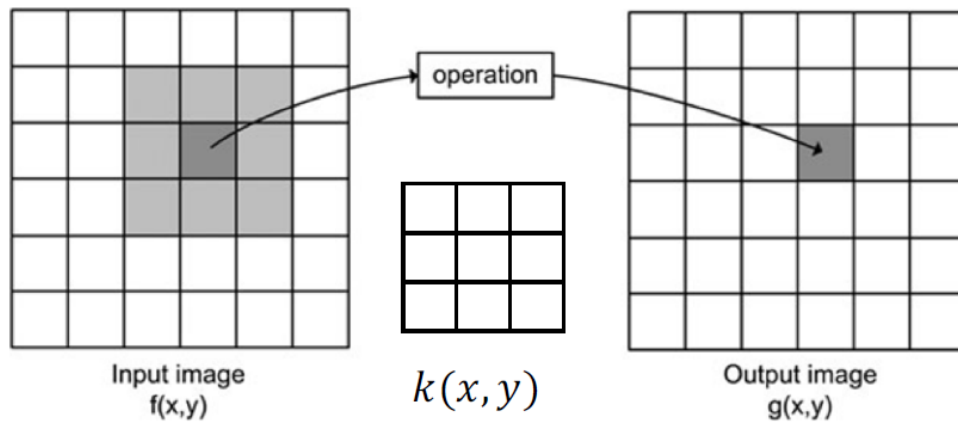


“zero crossings” at edges



# Image filtering: Convolution operator

## Important Filter: Sobel operator



$$k(x,y) =$$

|   |   |    |
|---|---|----|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

Image Credit: <http://what-when-how.com/introduction-to-video-and-image-processing/neighborhood-processing-introduction-to-video-and-image-processing-part-1/>

# The Sobel filter

|   |   |    |
|---|---|----|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

Sobel filter

=

|   |
|---|
| 1 |
| 2 |
| 1 |

What filter  
is this?

\*

|   |   |    |
|---|---|----|
| 1 | 0 | -1 |
|---|---|----|

1D derivative  
filter



# The Sobel filter

$$\begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline \end{array}$$

Sobel filter                      Blurring                      1D derivative filter

# The Sobel filter

Horizontal Sobel filter:

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

What does the vertical Sobel filter look like?



# The Sobel filter

Horizontal Sobel filter:

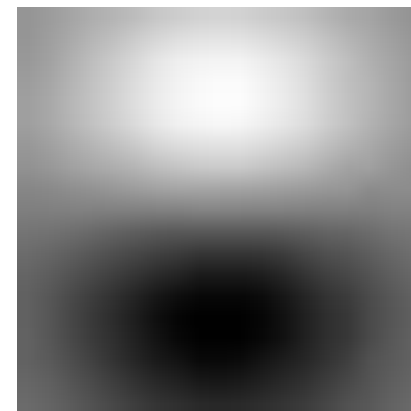
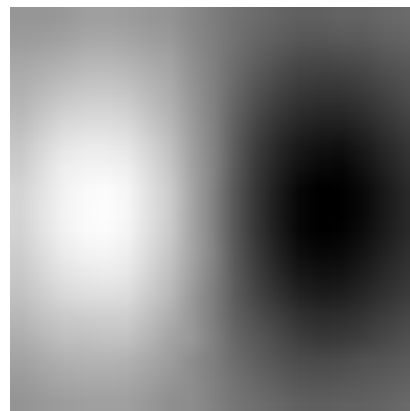
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Vertical Sobel filter:

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

# Filters We've Seen

Gaussian  
Derivative



Sobel  
Filter

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

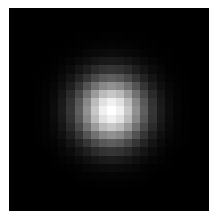
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

**Why would anybody use the bottom filter?**



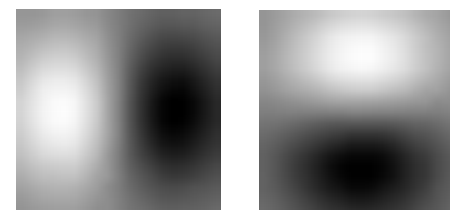
# Filters We've Seen

Smoothing



Gaussian

Derivative



Deriv. of gauss

Example

Goal

Remove noise

Find edges

Only +?

Yes

No

Sums to

1

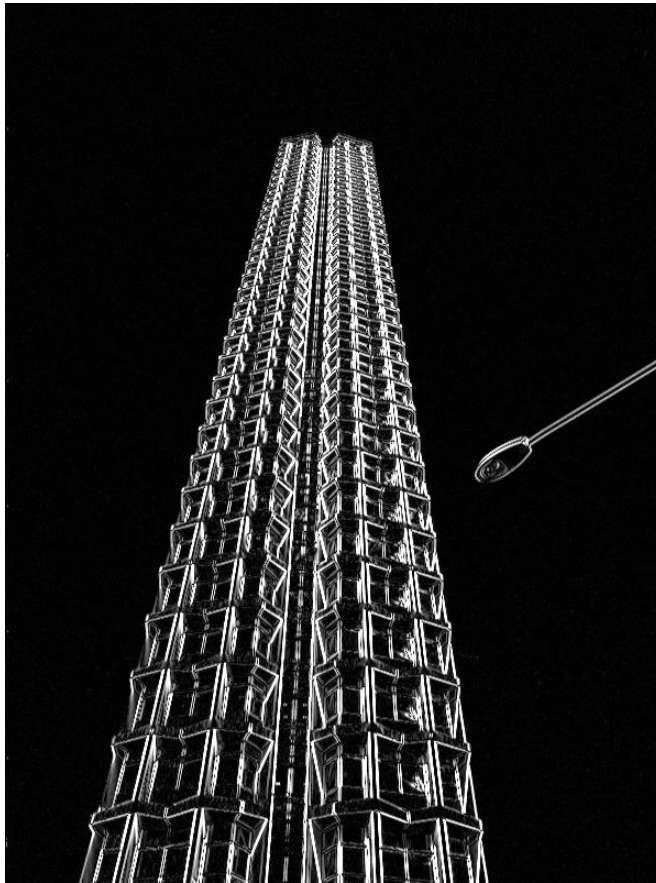
0

**Why sum to 1 or 0, intuitively?**

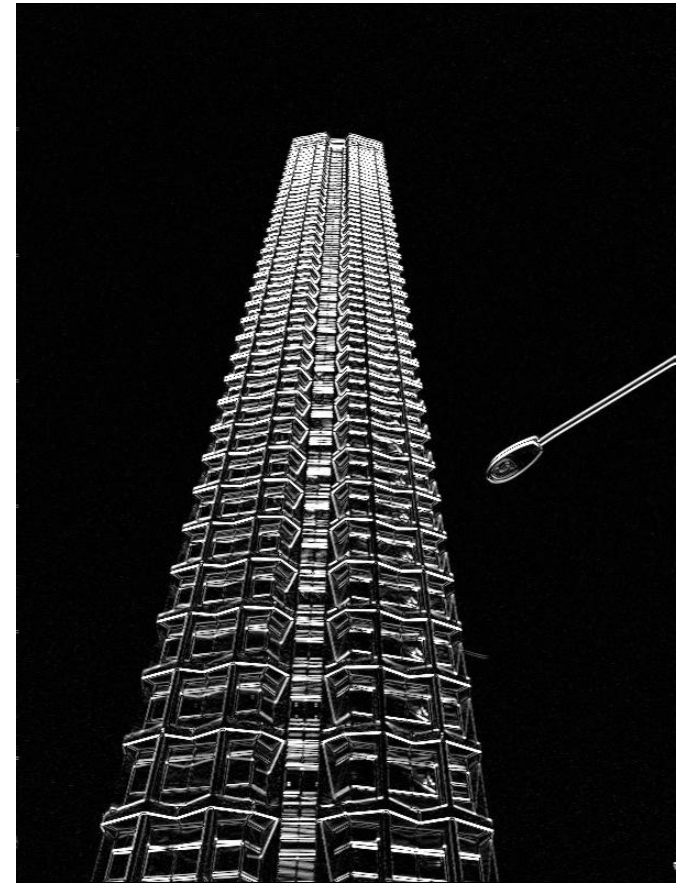
# Sobel filter example



original



which Sobel filter?

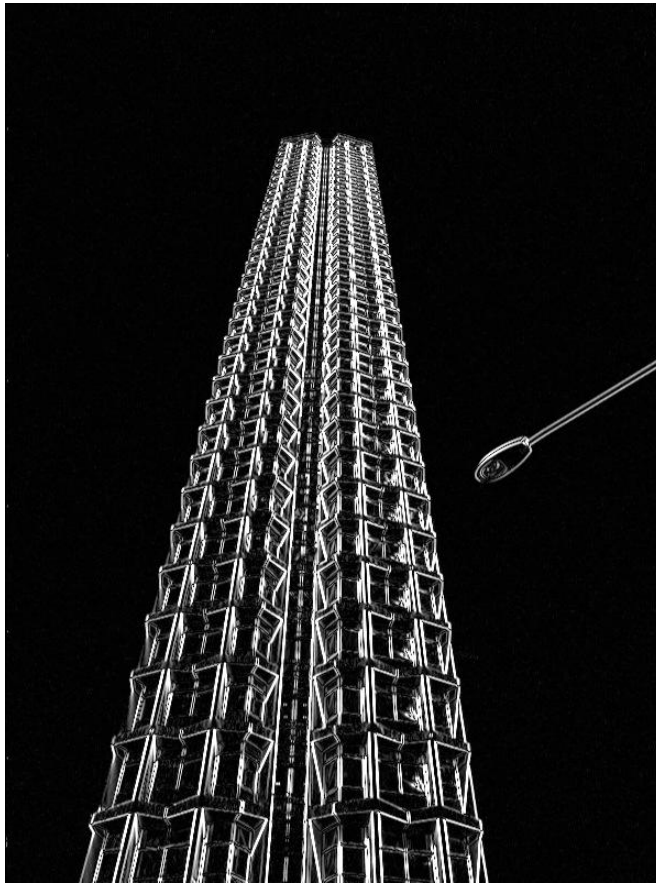


which Sobel filter?

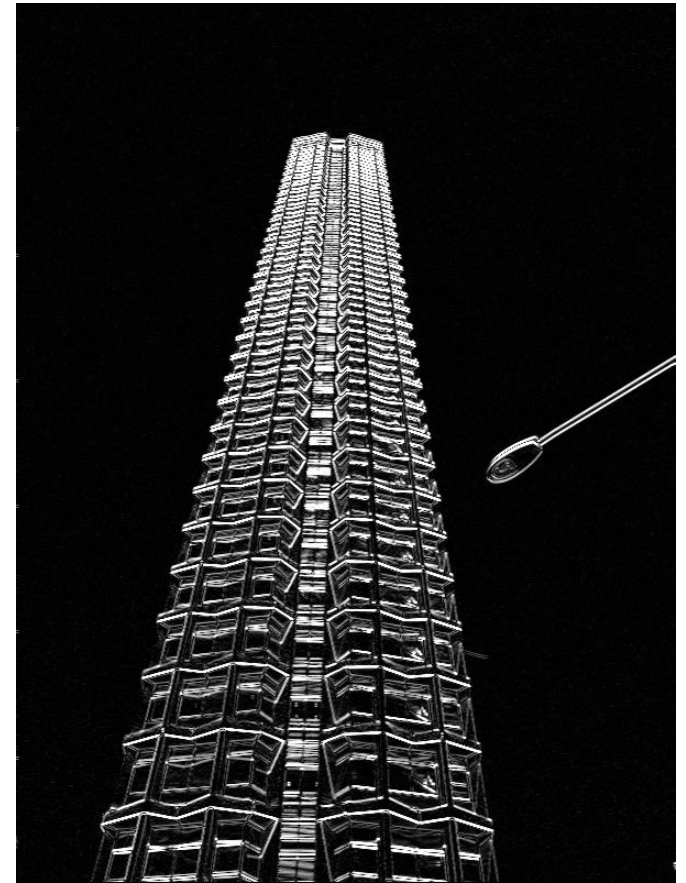
# Sobel filter example



original



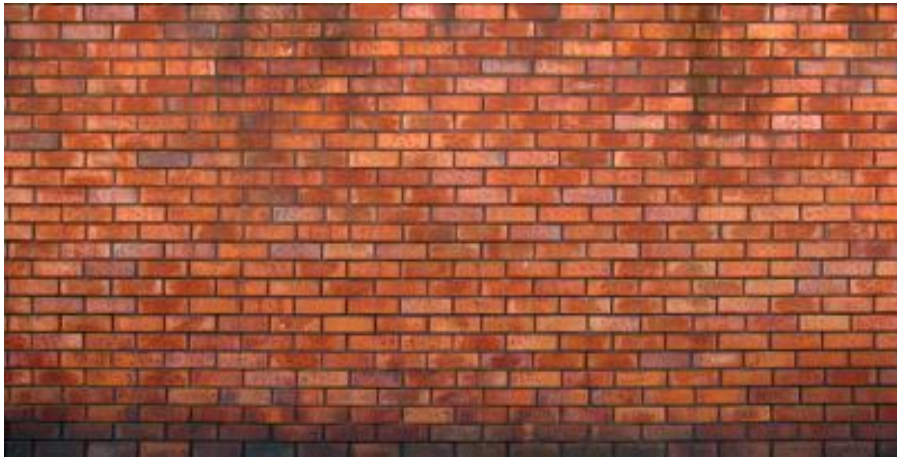
horizontal Sobel filter



vertical Sobel filter



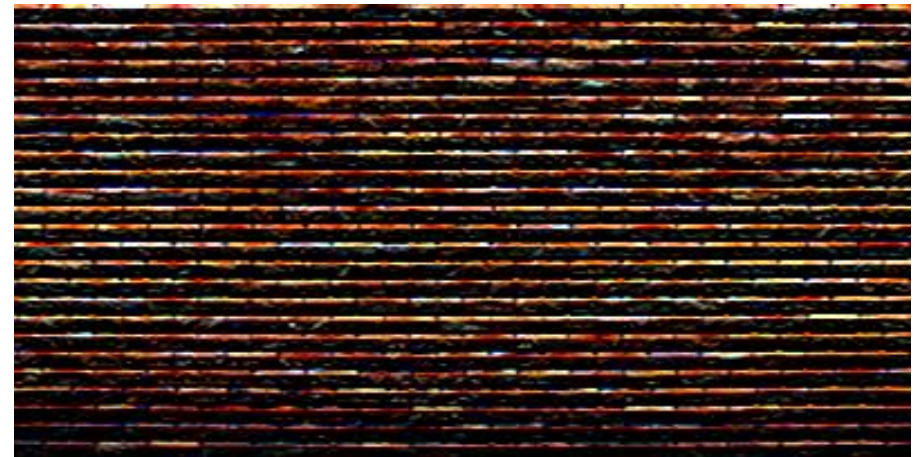
# Sobel filter example



original



horizontal Sobel filter



vertical Sobel filter

# Several derivative filters

Sobel

|   |   |    |
|---|---|----|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

|    |    |    |
|----|----|----|
| 1  | 2  | 1  |
| 0  | 0  | 0  |
| -1 | -2 | -1 |

Scharr

|    |   |     |
|----|---|-----|
| 3  | 0 | -3  |
| 10 | 0 | -10 |
| 3  | 0 | -3  |

|    |     |    |
|----|-----|----|
| 3  | 10  | 3  |
| 0  | 0   | 0  |
| -3 | -10 | -3 |

Prewitt

|   |   |    |
|---|---|----|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

|    |    |    |
|----|----|----|
| 1  | 1  | 1  |
| 0  | 0  | 0  |
| -1 | -1 | -1 |

Roberts

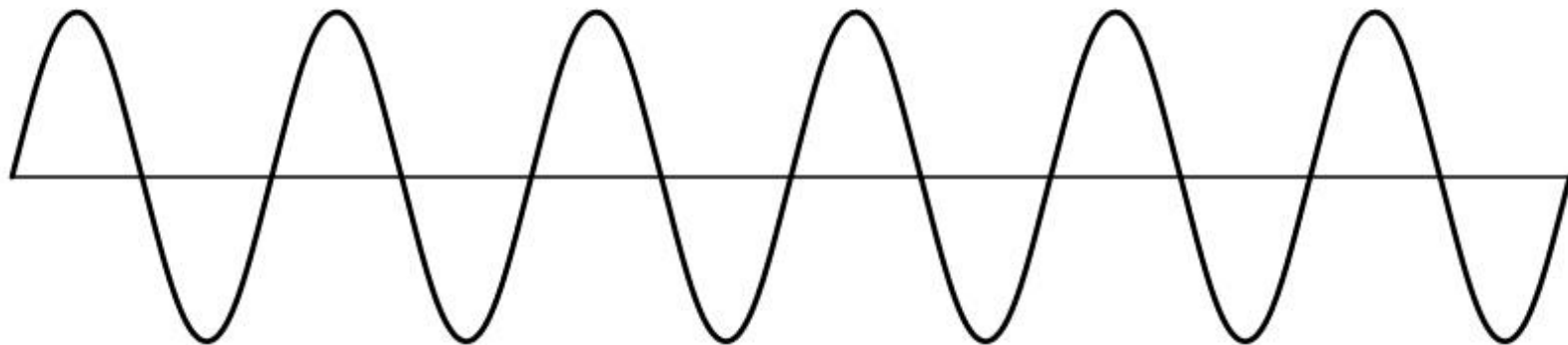
|    |   |
|----|---|
| 0  | 1 |
| -1 | 0 |

|   |    |
|---|----|
| 1 | 0  |
| 0 | -1 |

- How are the other filters derived and how do they relate to the Sobel filter?
- How would you derive a derivative filter that is larger than 3x3?

# Sampling

Very simple example: a sine wave

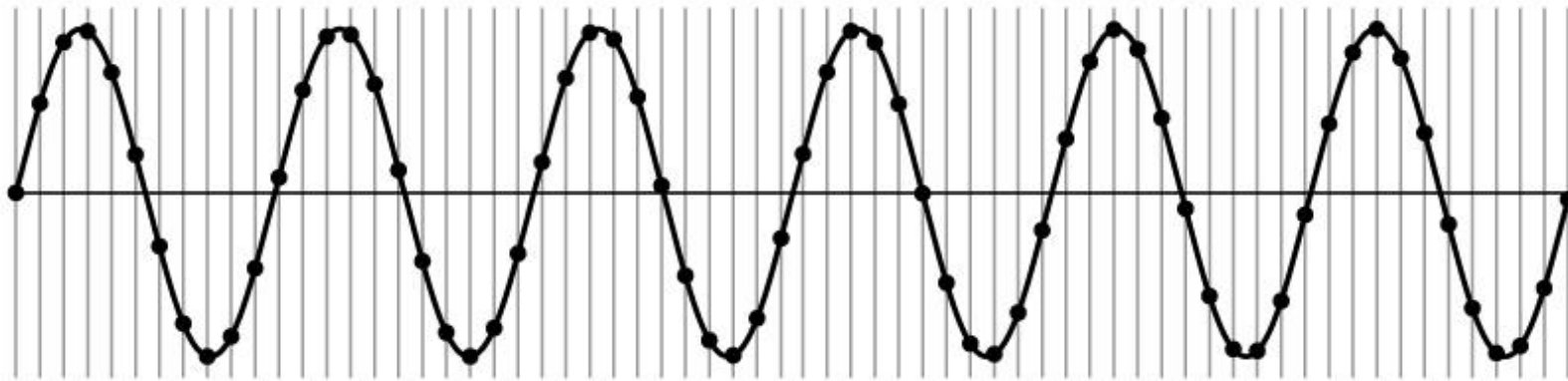


How would you discretize this signal?



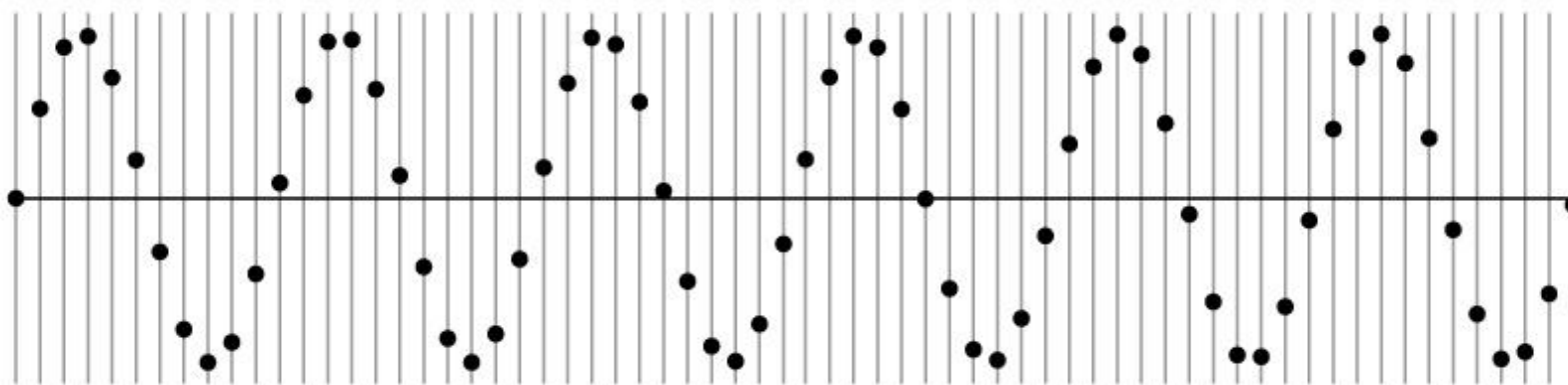
# Sampling

Very simple example: a sine wave



# Sampling

Very simple example: a sine wave



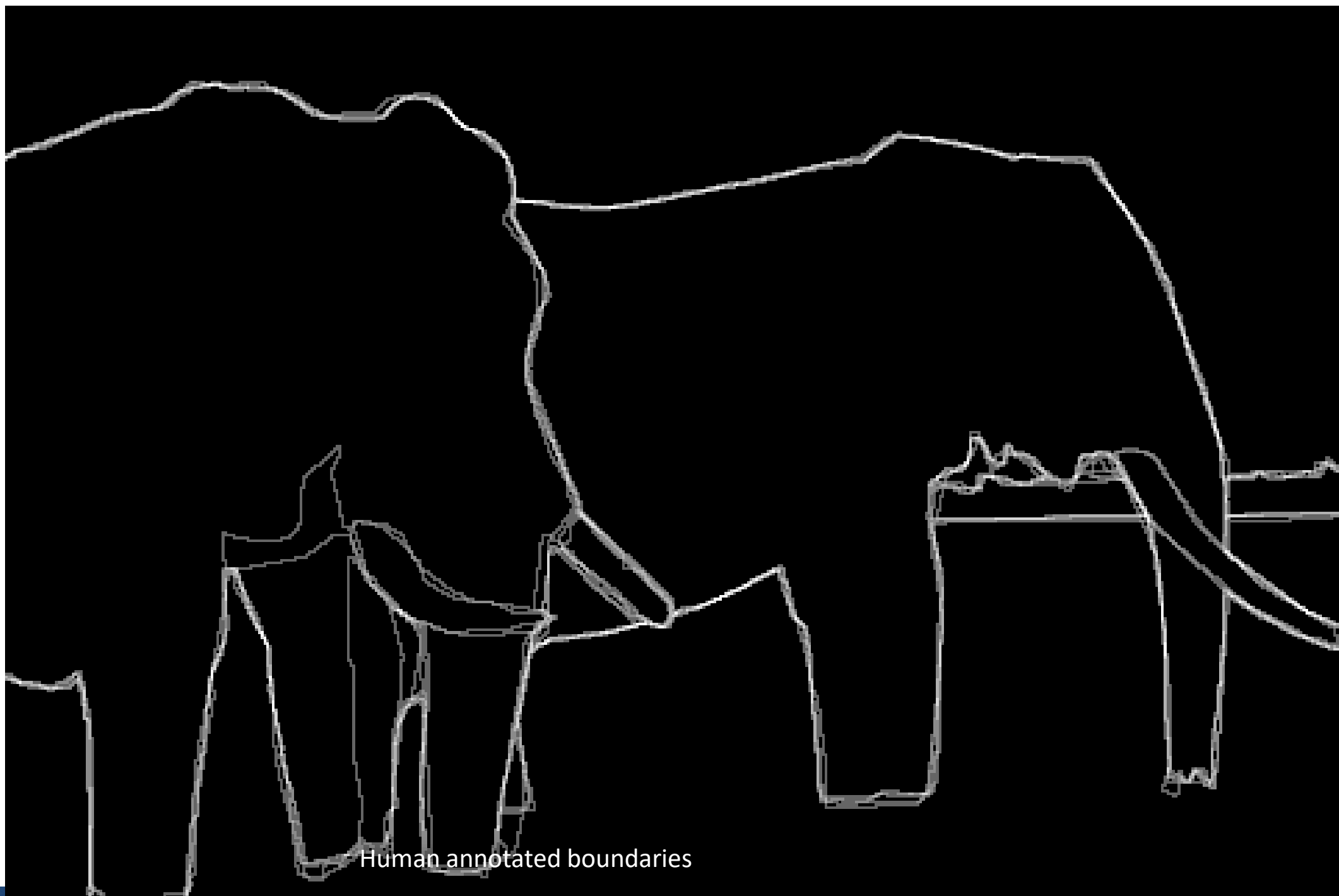
How many samples should I take?

Can I take as *many* samples as I want?

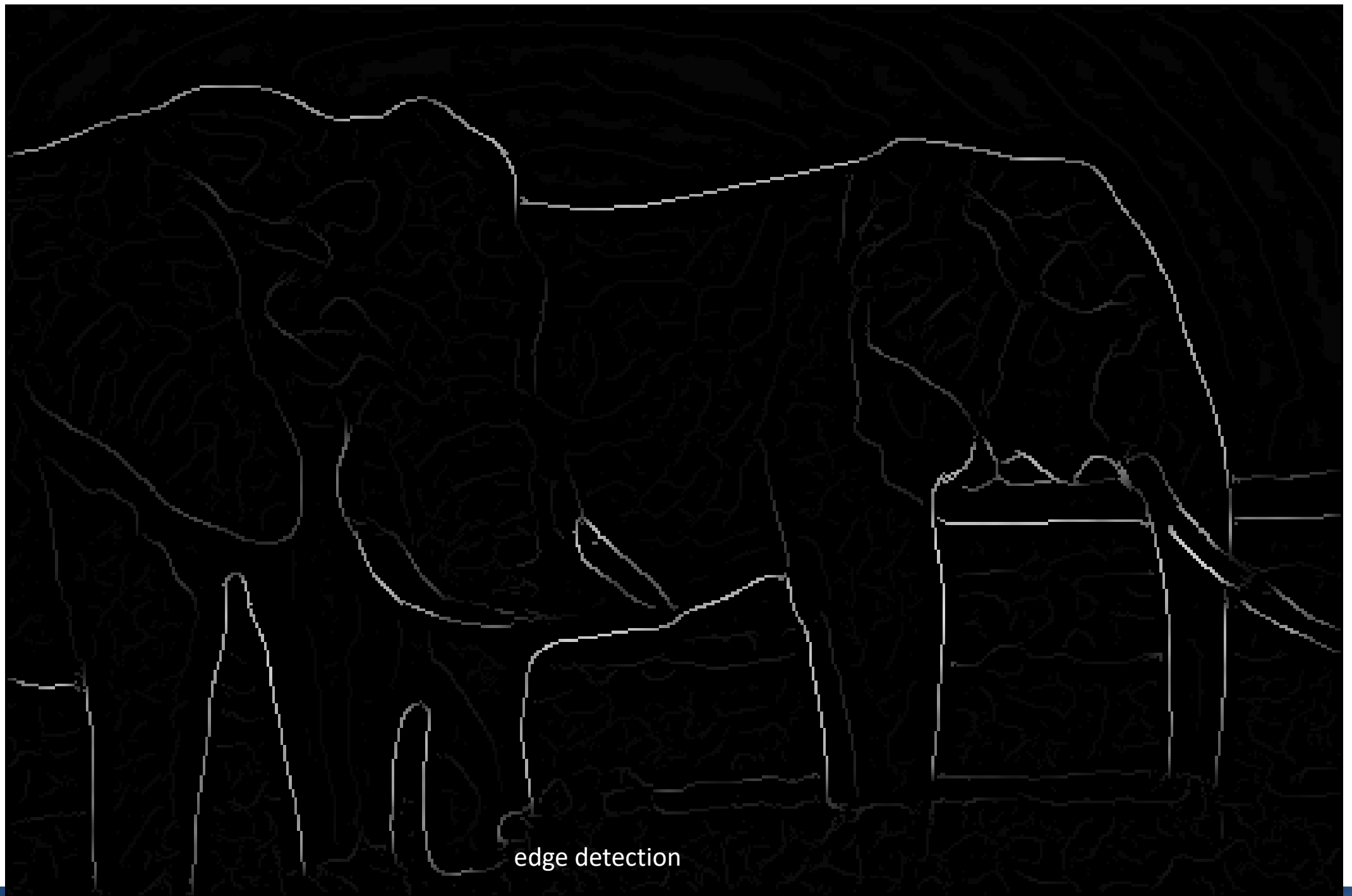
Where are the object boundaries?







Human annotated boundaries



edge detection

Where are the object boundaries?





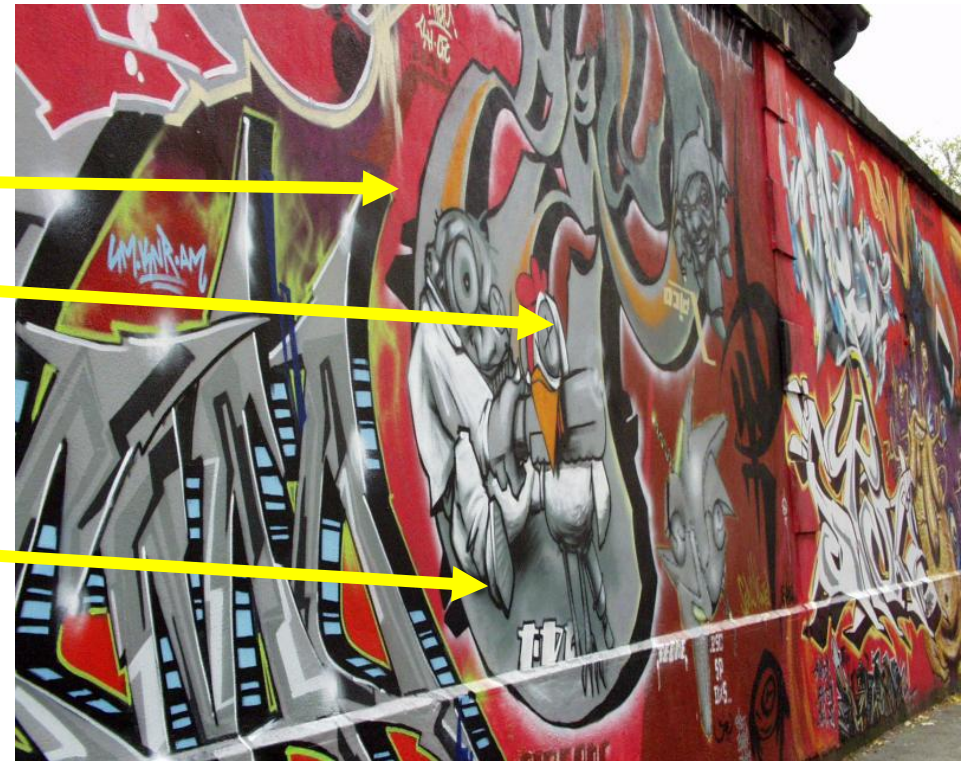
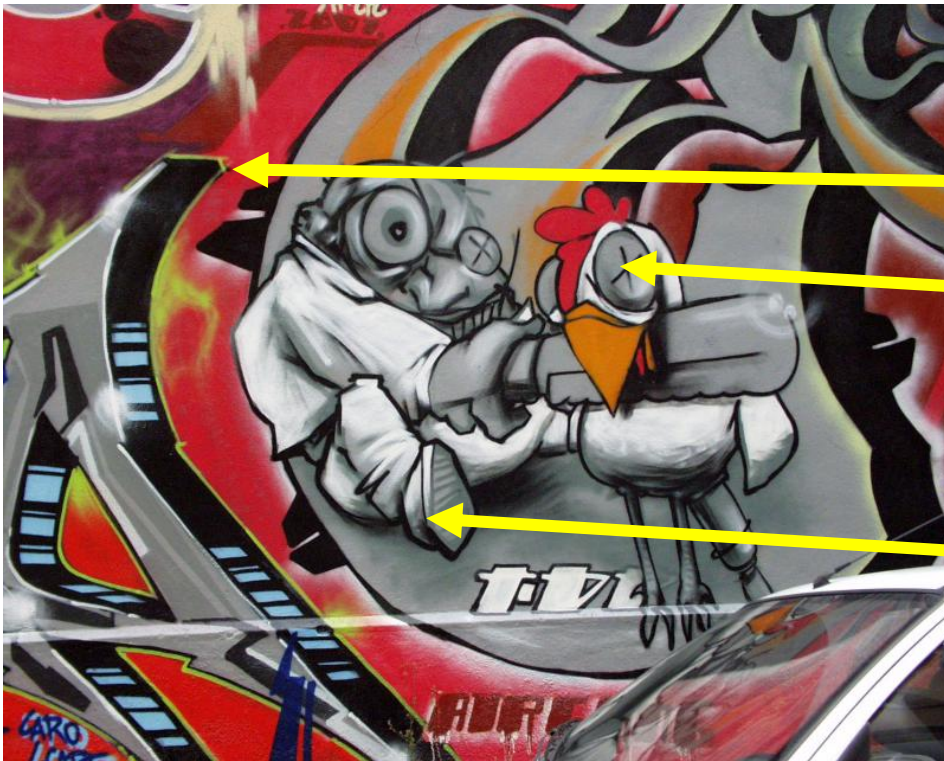


edge detection

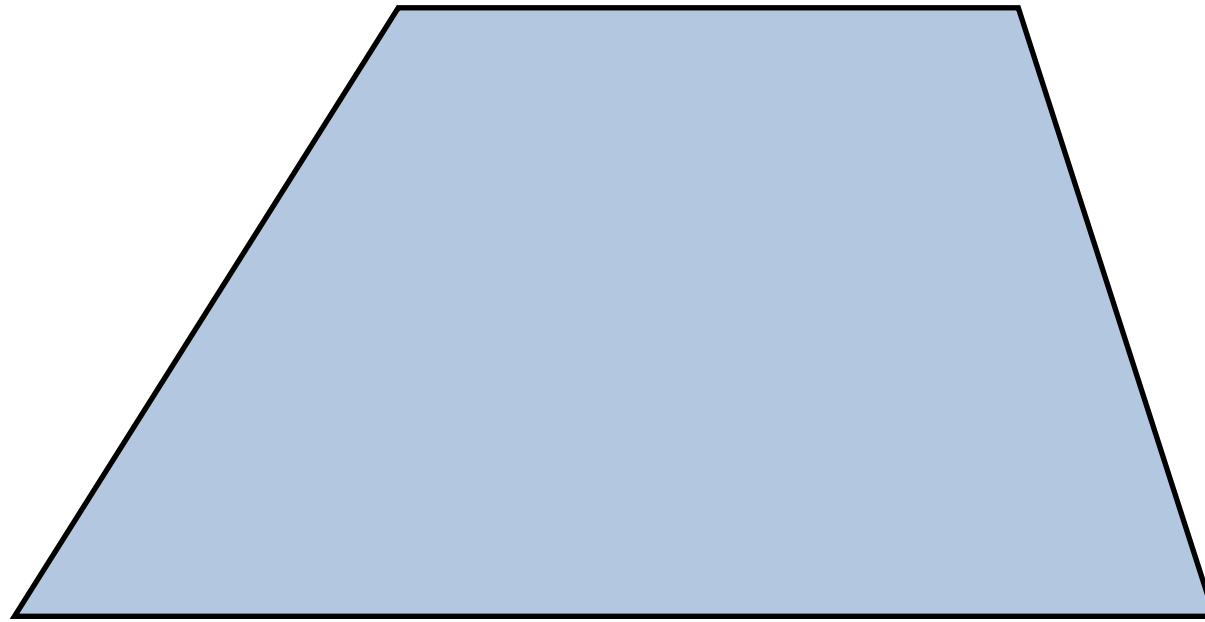


Human annotated boundaries

# Image matching

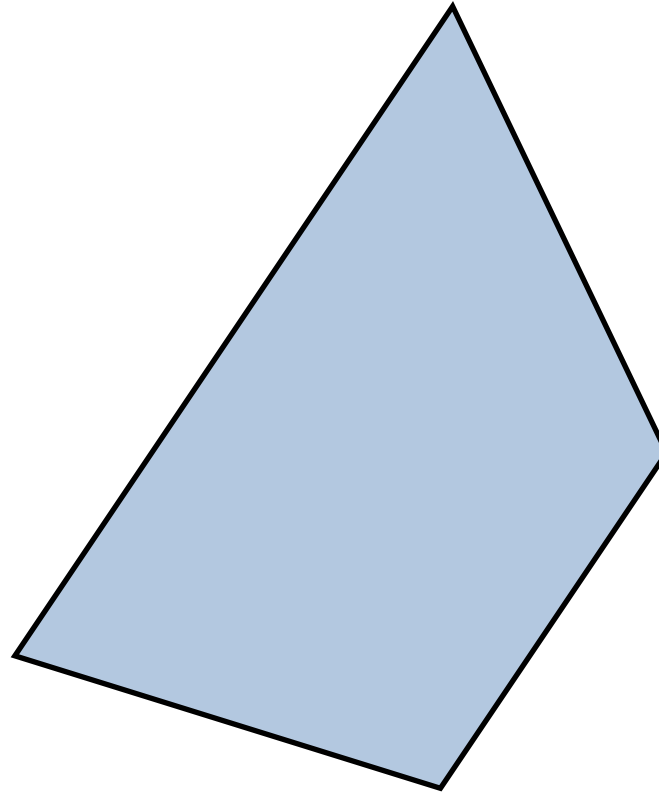






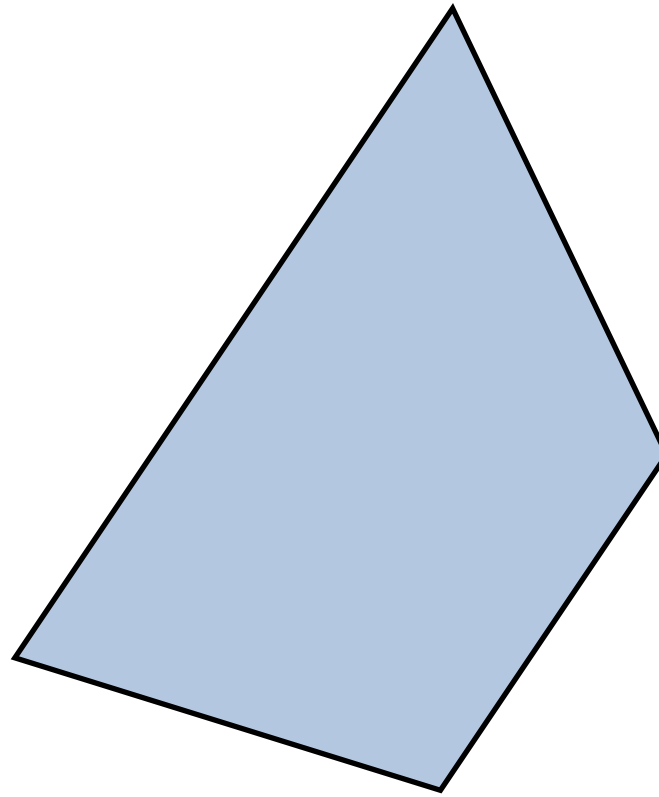
Pick a point in the image.  
Find it again in the next image.

*What type of feature would you select?*



Pick a point in the image.  
Find it again in the next image.

*What type of feature would you select?*



corner

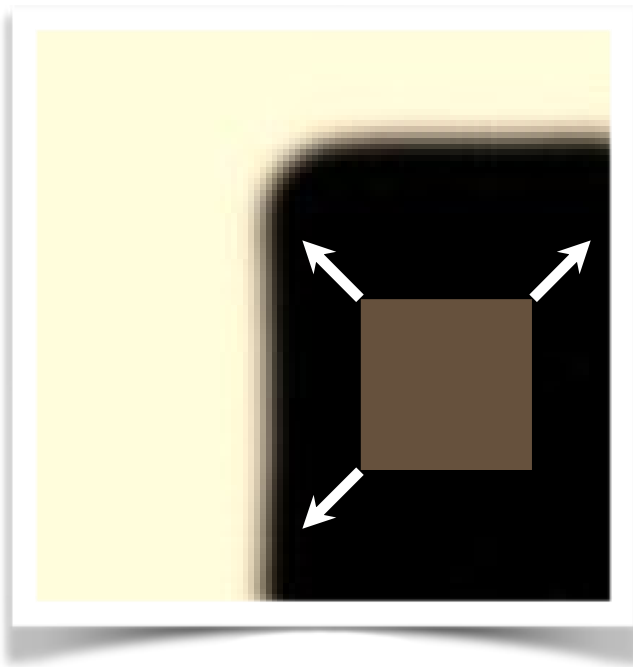
Pick a point in the image.  
Find it again in the next image.

*What type of feature would you select?*

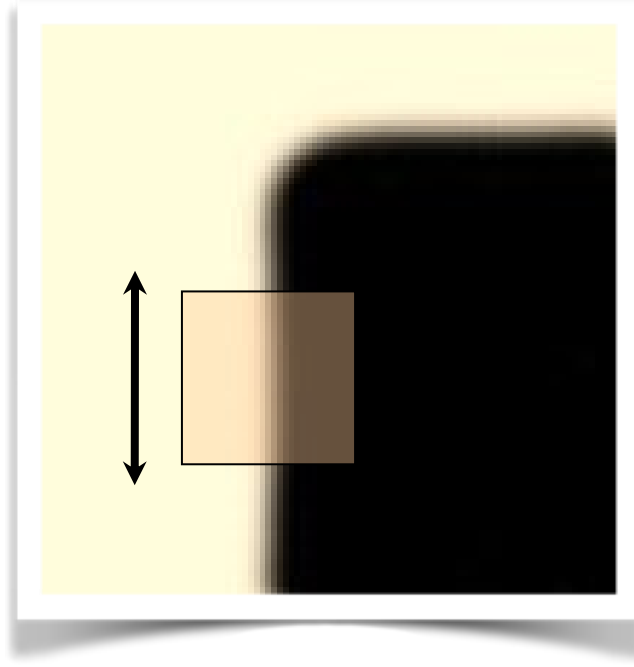


Easily recognized by looking through a small window

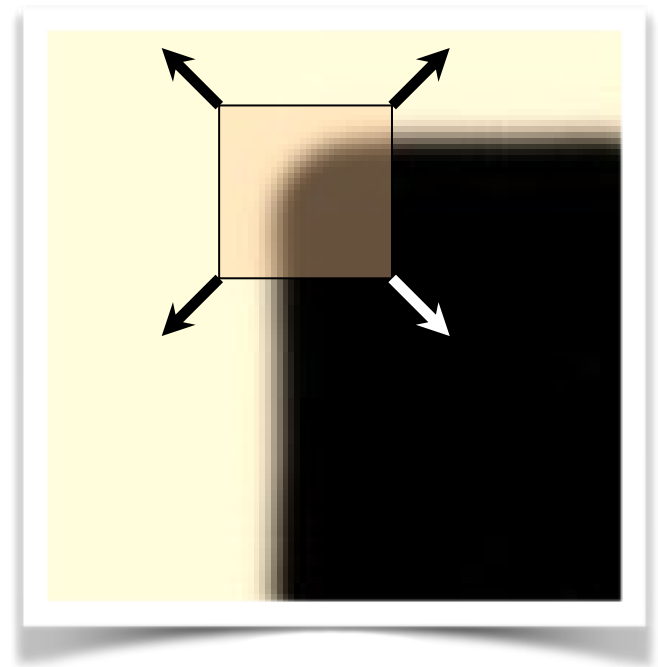
Shifting the window should give large change in intensity



“flat” region:  
no change in all  
directions

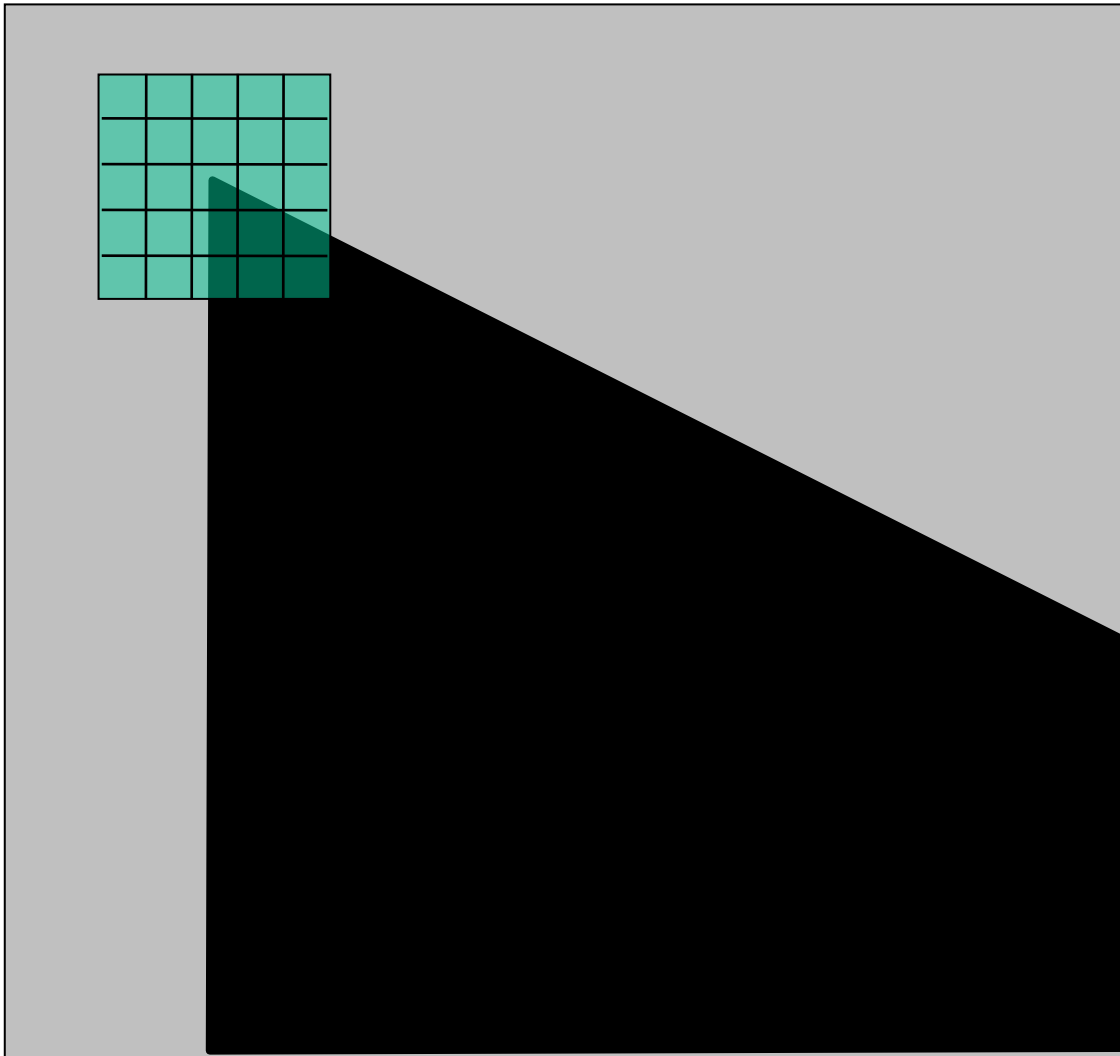


“edge”:  
no change along the edge  
direction



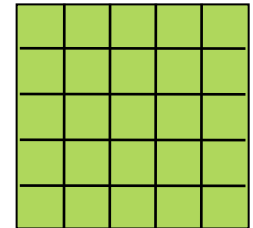
“corner”:  
significant change in all  
directions

# 1. Compute image gradients over a small region (not just a single pixel)



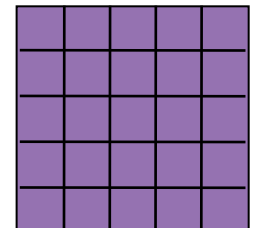
array of x gradients

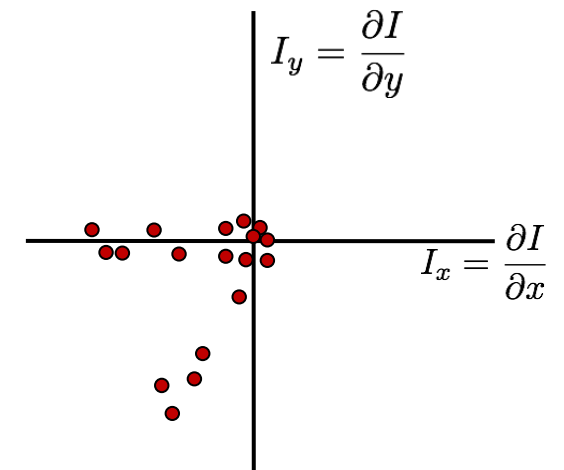
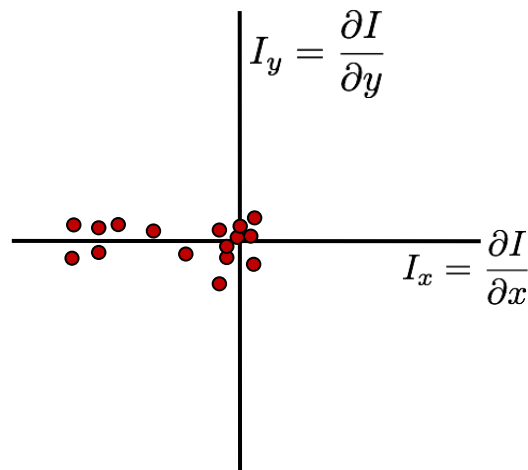
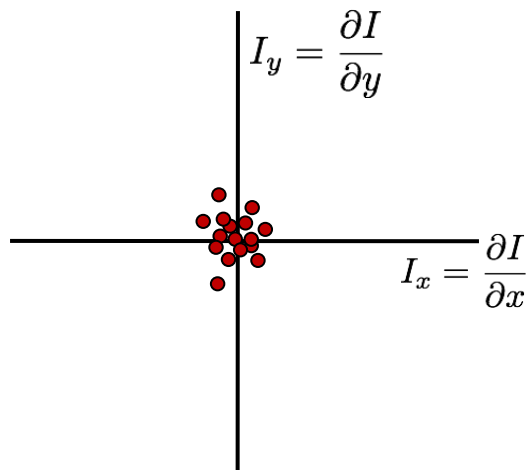
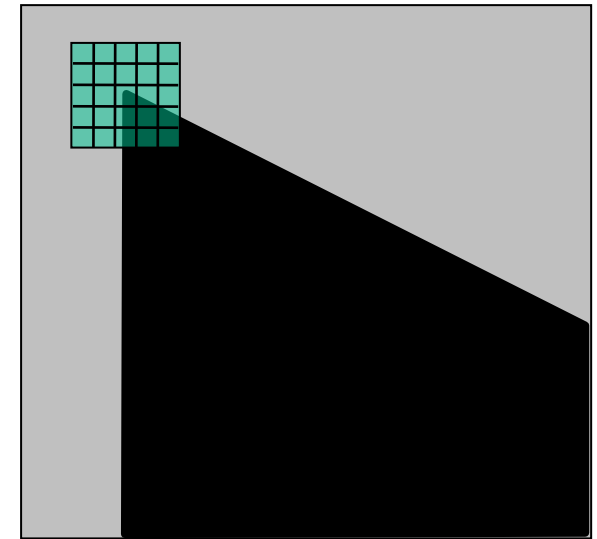
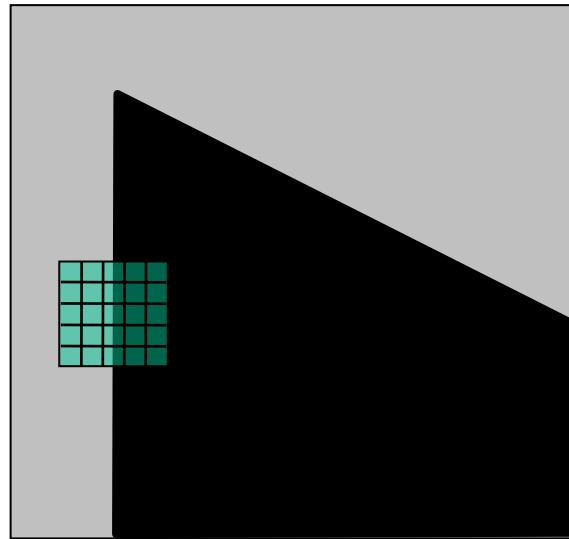
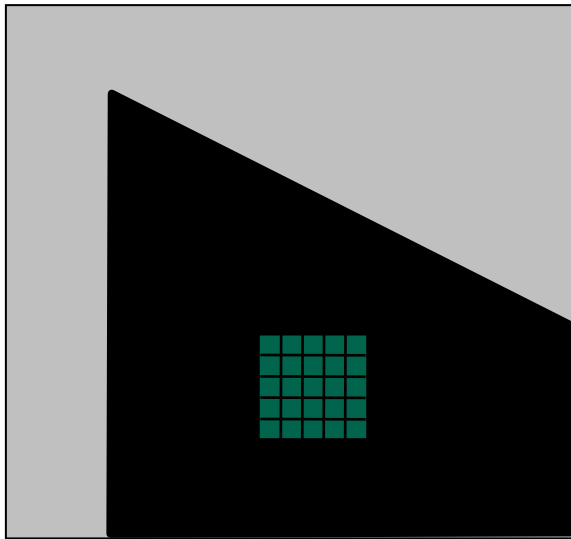
$$I_x = \frac{\partial I}{\partial x}$$



array of y gradients

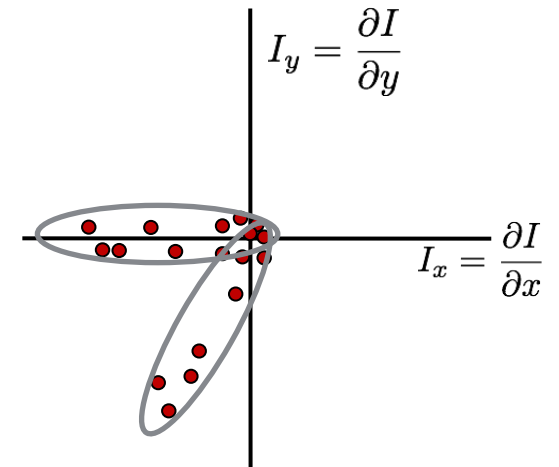
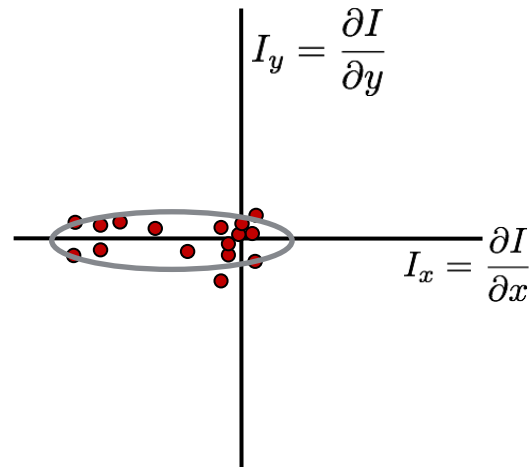
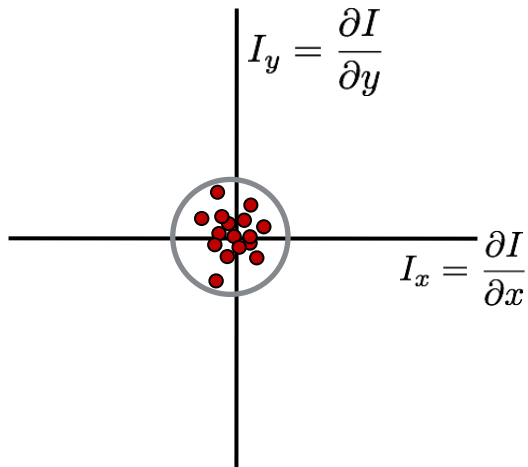
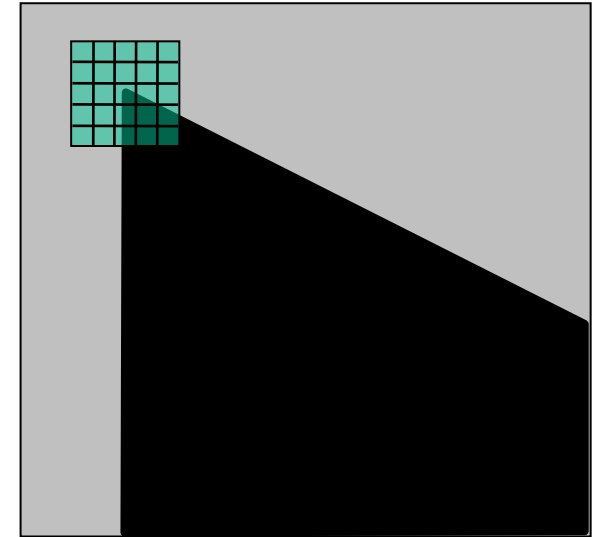
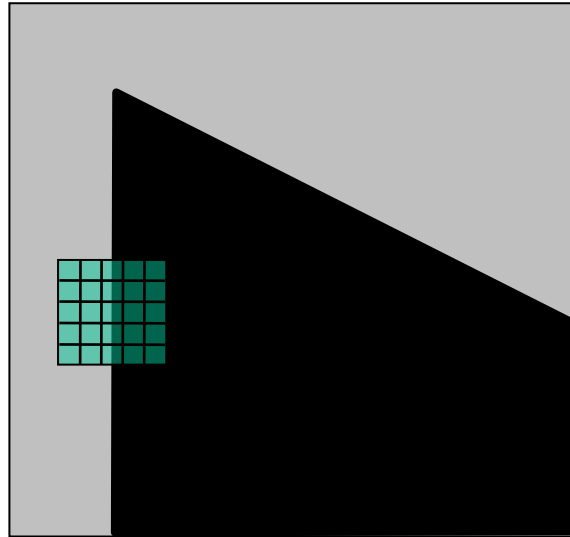
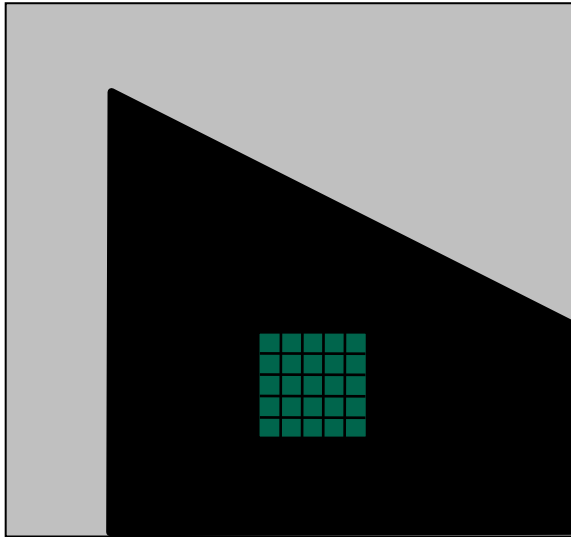
$$I_y = \frac{\partial I}{\partial y}$$





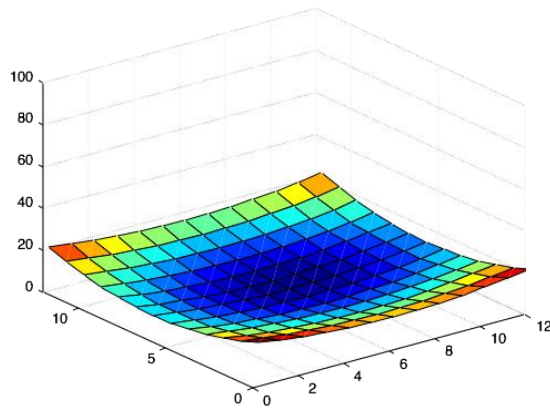
*What does the distribution tell you about the region?*



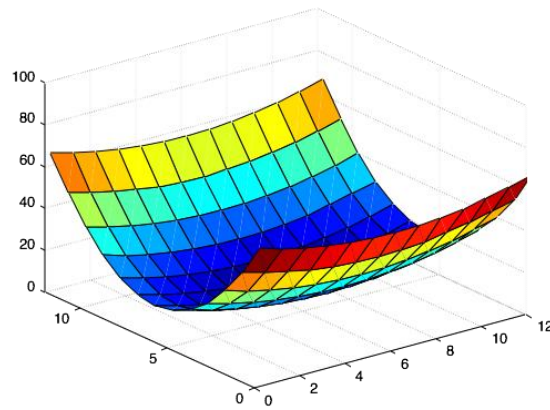


*distribution reveals edge orientation and magnitude*

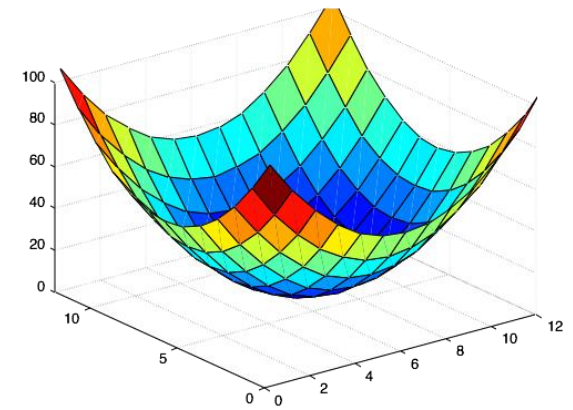
*Which error surface indicates a good image feature?*



flat



edge  
'line'



corner  
'dot'

# Compute eigenvalues and eigenvectors

eigenvalue



$$Me = \lambda e$$



eigenvector

$$(M - \lambda I)e = 0$$

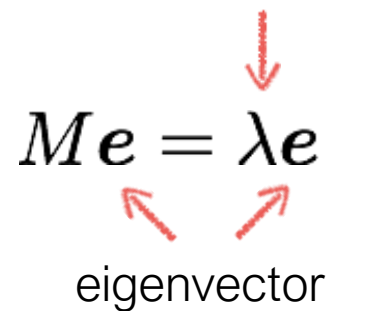


# Compute eigenvalues and eigenvectors

eigenvalue

$$M\mathbf{e} = \lambda\mathbf{e}$$

eigenvector



$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of  
(returns a polynomial)

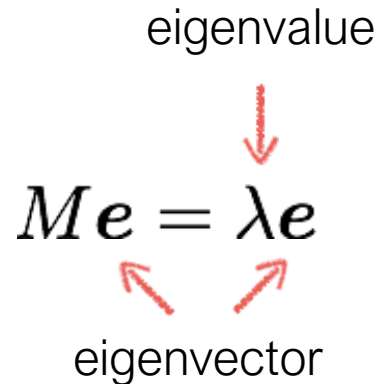
$$M - \lambda I$$

# Compute eigenvalues and eigenvectors

eigenvalue

$$M\mathbf{e} = \lambda\mathbf{e}$$

eigenvector



$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of  
(returns a polynomial)

$$M - \lambda I$$

2. Find the roots of polynomial  
(returns eigenvalues)

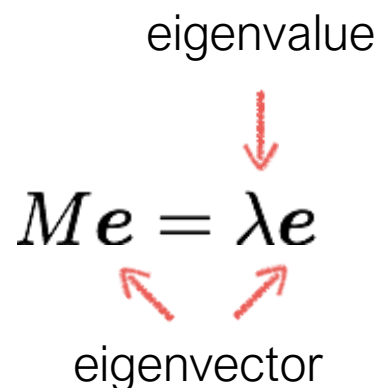
$$\det(M - \lambda I) = 0$$

# Compute eigenvalues and eigenvectors

eigenvalue

$$M\mathbf{e} = \lambda\mathbf{e}$$

eigenvector



$$(M - \lambda I)\mathbf{e} = 0$$

1. Compute the determinant of  
(returns a polynomial)

$$M - \lambda I$$

2. Find the roots of polynomial  
(returns eigenvalues)

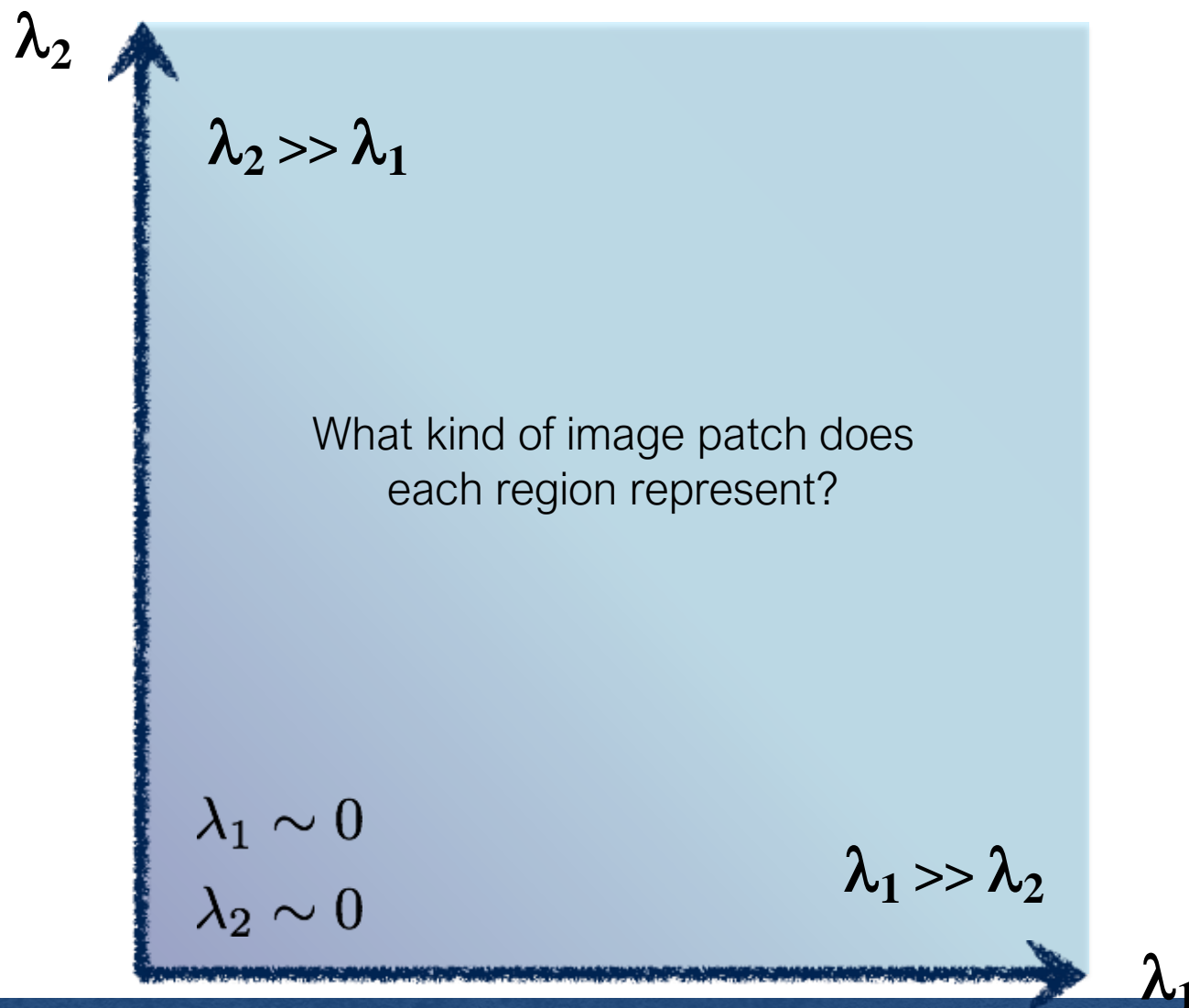
$$\det(M - \lambda I) = 0$$

3. For each eigenvalue, solve  
(returns eigenvectors)

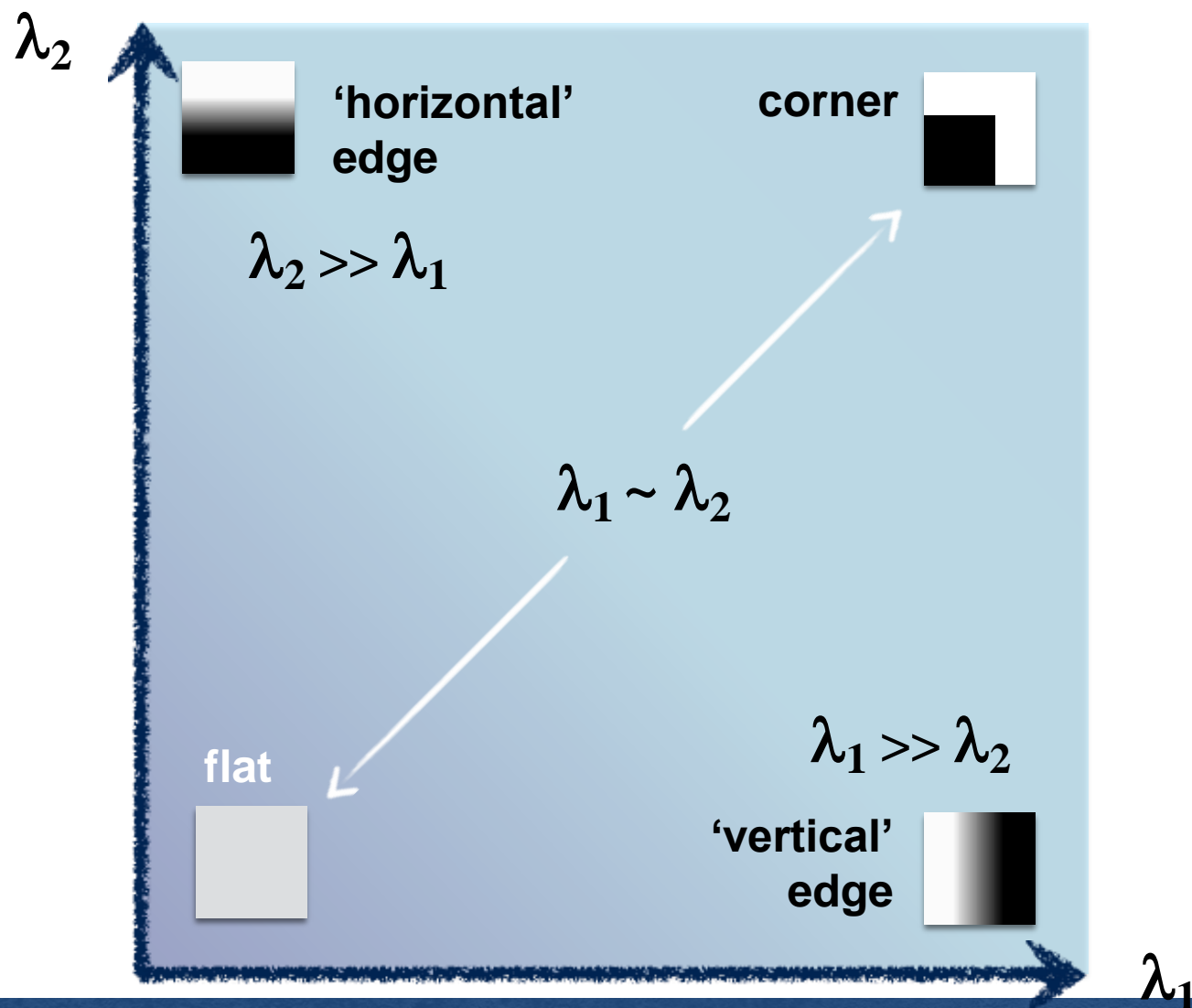
$$(M - \lambda I)\mathbf{e} = 0$$



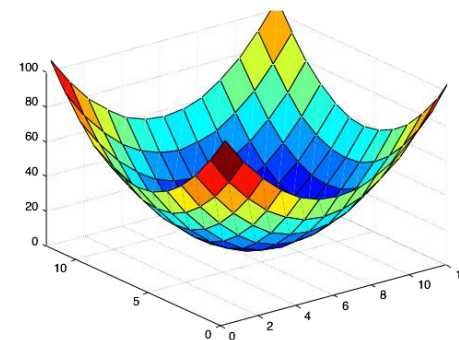
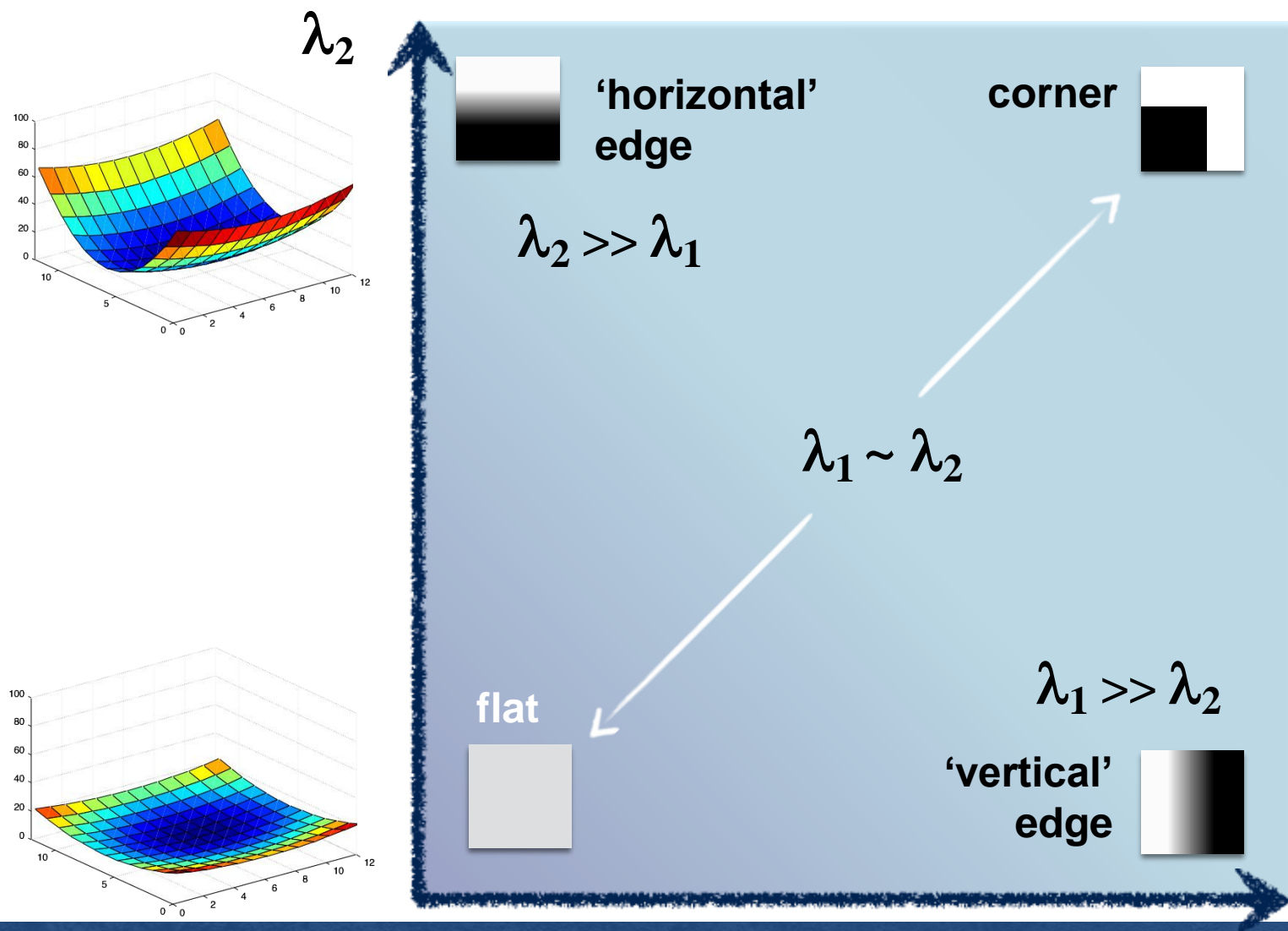
# interpreting eigenvalues



# interpreting eigenvalues

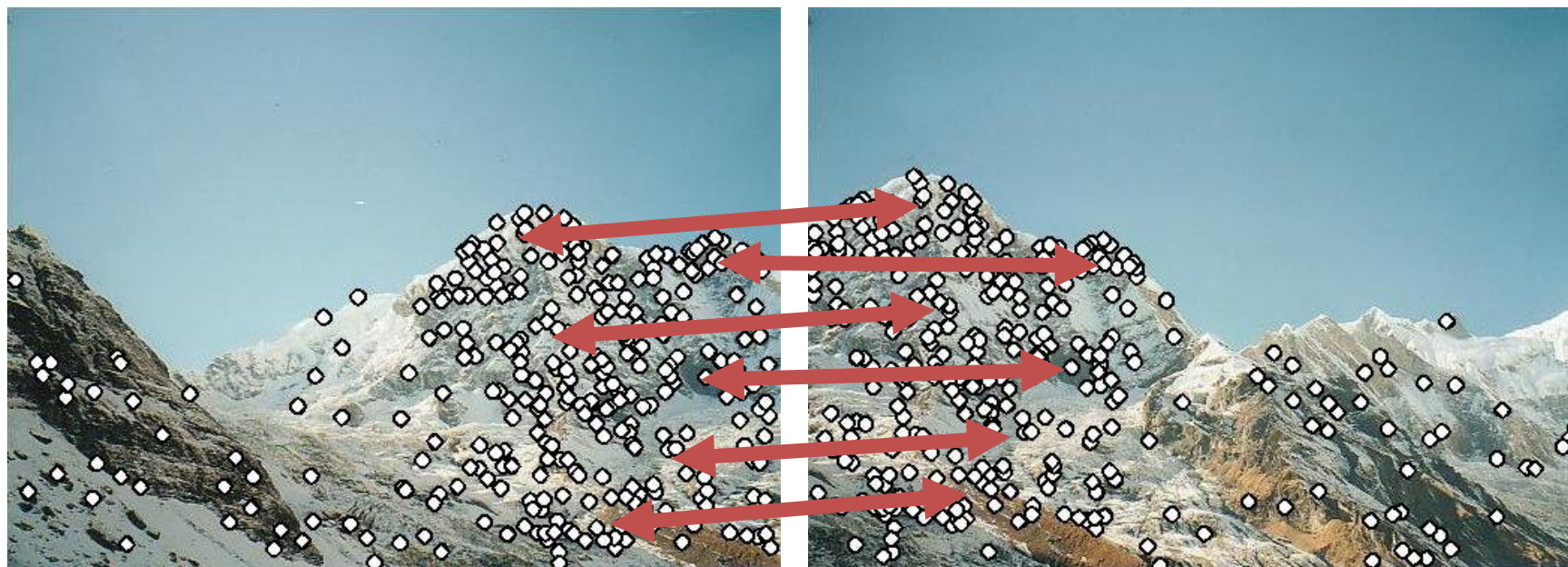


# interpreting eigenvalues



# Finding + Matching

## Finding and Matching



1: find corners+features

2: match based on local image data