Data structure & Algorithm

Chapter I

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1)

$\log_2 2048 = \log_{10} 2048 / \log_{10} 2$ #

2)

$3+5+7+9+\cdots+(2k+1)$

$= 2(1+2+3+k) + (1+1+1+\cdots+1)$

$= 2[k(k+1)/2] + k$ #

3) counterexamples

$n^3 > 2^n$ (for any $n \geq 1$)

if $n=1 \rightarrow 2^3 = 1 < 2^1 = 2 \rightarrow$ prove $n^3 > 2^n$ not true #

4) contradiction.

Q: Assume k is an even number

$\rightarrow k^2$ is also even.

Ans: ① Assume it is false, then k^2 is odd

$\rightarrow k^2 = 2c+1$

② But $k = 2a$, mean $(2a)^2 = 2c+1$

$\rightarrow 2(2a^2) = 2c+1$

③ But an even number equals an odd number is impossible

$\Rightarrow k$ is even number, k^2 is also even #

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5) Induction

a. $\sum_{i=1}^{n} i^3 = [n^2][(n+1)^2]/4$

Base case: $i=1 \rightarrow 1 = (1)(4/4) = 1$

Inductive step:

Assume true for k

$$\sum_{i=1}^{k} i^3 = (k^2)(k+1)^2/4$$

Show true for $k+1$

$$\sum_{i=1}^{k+1} i^3 = \sum_{i=1}^{k} i^3 + (k+1)^3$$

$$= (k^2)(k+1)^2/4 + 4(k+1)(k+1)^2/4$$

$$= [k^2 + 4(k+1)](k+1)^2/4$$

$$= (k+2)^2(k+1)^2/4$$

$$= (k+1)^2[(k+1)+1]^2/4$$

Conclusion:

$\sum_{i=1}^{n} i^3 = n^2(n+1)^2/4$ is true #

b. $n^2 - n$ is even for any $n \geq 1$

Base case: $n=1 \rightarrow 1^2 - 1 = 0$ is even

Inductive step:

Assume true for k

$k^2 - k = 2a$.

Show $k+1$

$$(k+1)^2 - (k+1) = k^2 + 2k + \cancel{1} - k - \cancel{1}$$

$$= (k^2 - k) + 2k = 2(a+k) \rightarrow \text{even}$$

Conclusion: $n^2 - n$ is even for any $n \geq 1$ #

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