Home Work - 1

MLE & MAP

(1)(a) * Performance Function for given function i.e geometric distribution

$$P_q(k) = (1-q)^{k-1} q$$

$$P(D \mid P_q(k)) = (1-q)^{k_1-1} q \cdot (1-q)^{k_2-1} q \cdot (1-q)^{k_3-1} q \cdot \ldots \cdot (1-q)^{k_n-1} q$$

$$= (1-q)^{\sum_{i=1}^{n} k_i - n} \cdot q^n$$

* Optimization Criteria (MLE)

~~TODO~~ = Taking Log

$$\ln P(D \mid P_q(k)) = n \log q + \left(\sum_{i=1}^{n} k_i - n\right) \log(1-q)$$

Now finding parameter (q) putting derivative as zero

$$\frac{d\left[\ln P(D \mid P_q(k))\right]}{dq} = \frac{n}{q} - \frac{\sum_{i=1}^{n} k_i - n}{(1-q)} = 0$$

$$\therefore \frac{n}{q} = \frac{\sum_{i=1}^{n} k_i - n}{1-q} \qquad \therefore q = \frac{n}{\sum_{i=1}^{n} k_i}$$

$$n - \cancel{nq} = q\sum_{i=1}^{n} k_i - \cancel{nq}$$

/

(c) Now we know from database that

$\alpha = 1.65$, $\beta = 1.65$

Now the Beta Distribution optimization would be carried as

$$\hat{q} = \cancel{\arg} \operatorname{argmax}_q \left((1-q)^{\sum k_i - n + \beta - 1} \cdot q^{n+\alpha-1} \right)$$

$$\hat{q} = \operatorname{argmax}_q \left((1-q)^{\sum k_i - n + 1.65 - 1} \cdot q^{n+1.65-1} \right)$$

$$\hat{q} = \operatorname{argmax}_q \left((1-q)^{\sum k_i - n + 0.65} \cdot q^{n+0.65} \right)$$

$$\log \hat{q} = \sum k_i - n + 0.65 \log(1-q) + n + 0.65 \log q$$

$$\frac{d}{d\hat{q}} \log \hat{q} = -\left(\frac{\sum k_i - n + 0.65}{1-q} \right) + \frac{n+0.65}{q}$$

Now putting L.H.S $= 0$ for maximizing

$$0 = -\left(\frac{\sum k_i - n + 0.65}{1-q} \right) + \frac{n+0.65}{q}$$

$$(\sum k_i - n + 0.65)(q) = (n + 0.65)(1-q)$$

$$q \sum k_i - \cancel{nq} + 0.65q = -\cancel{nq} - 0.65q + n + 0.65$$

$$q \sum k_i + 1.3q = n + 0.65$$

$$\therefore q = \frac{n + 0.65}{\sum k_i + 1.3}$$