# PDE-constrained optimization problems in the field of shallow geothermal energy

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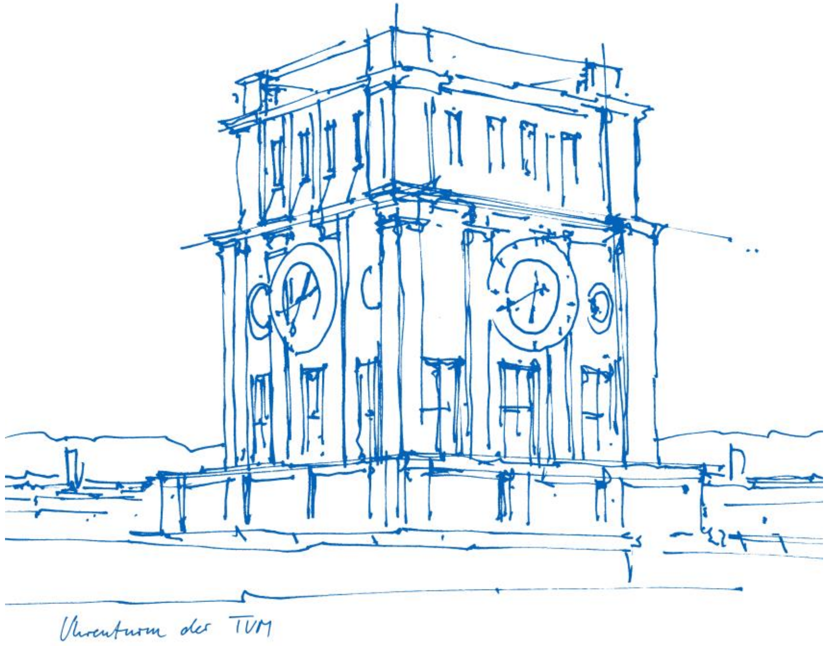
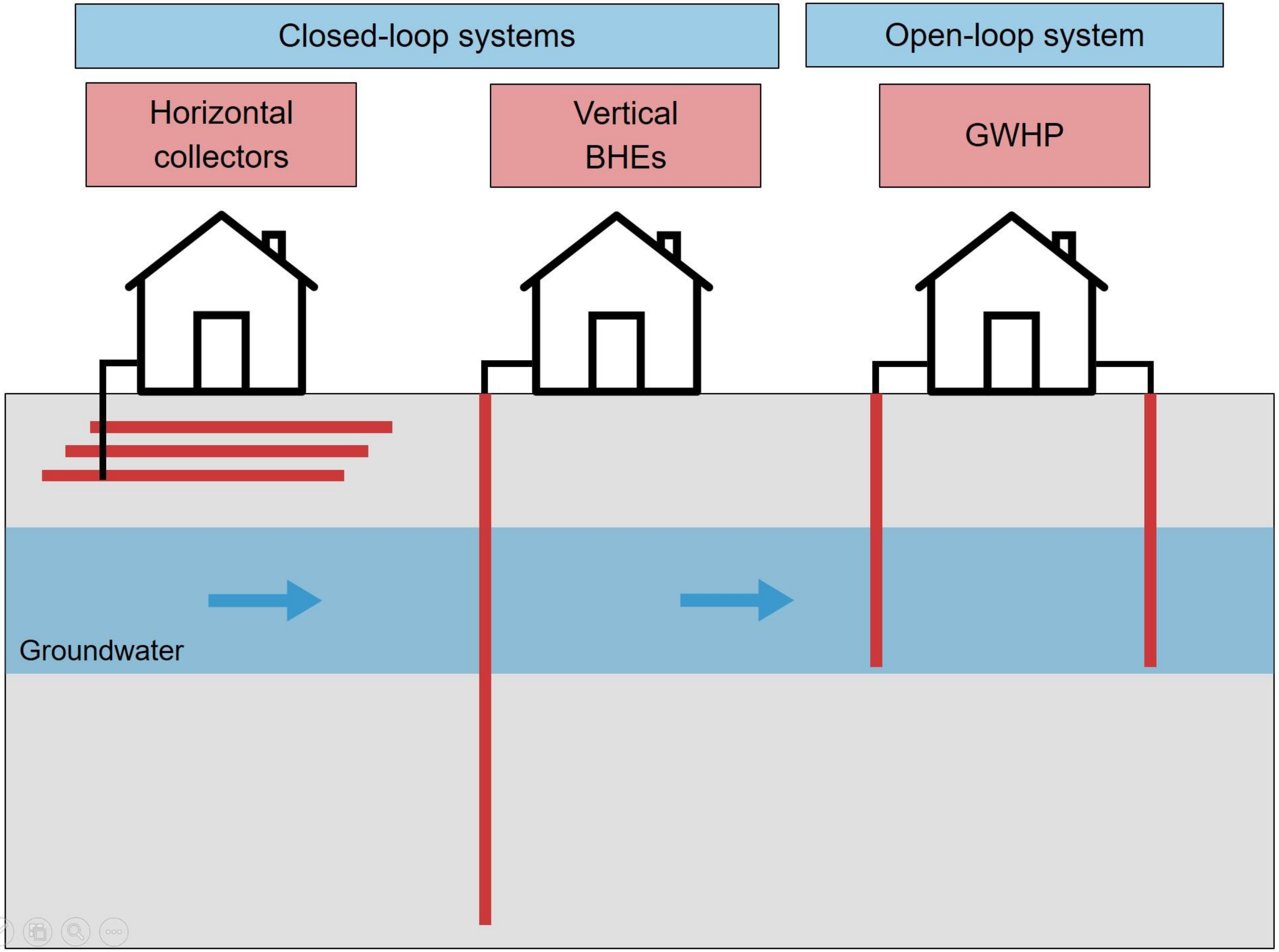
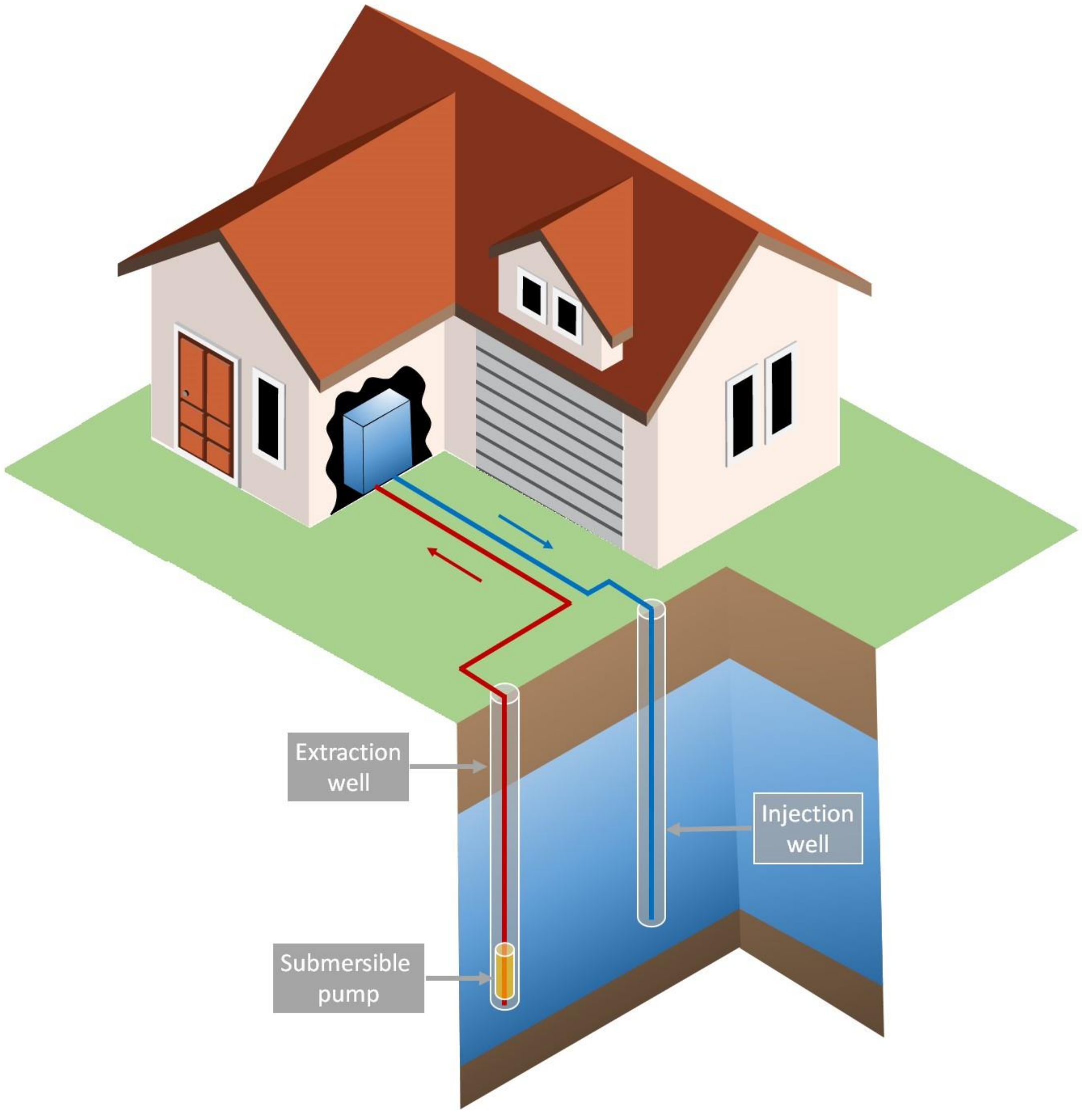
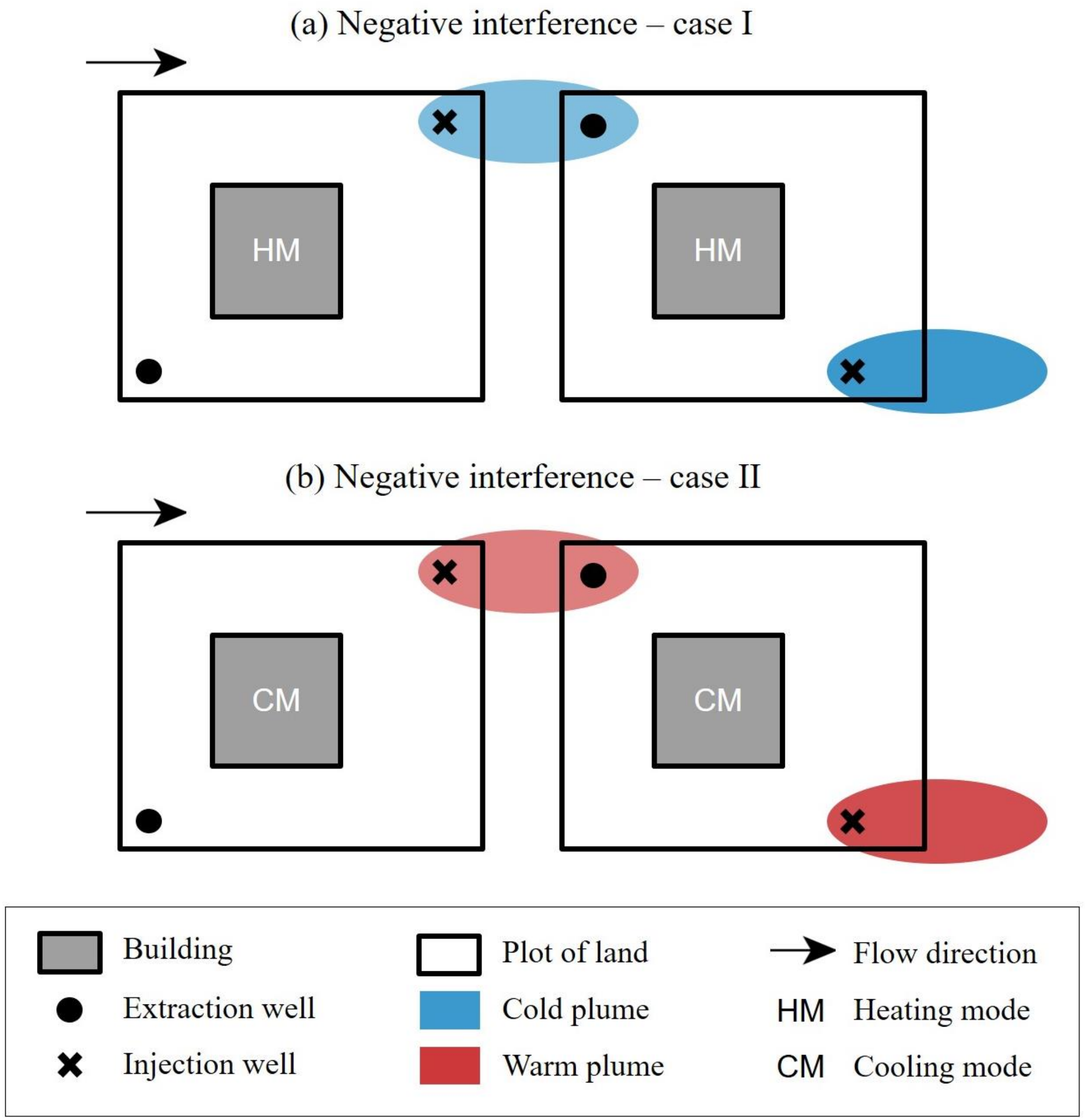


# Shallow geothermal energy (SGE) systems

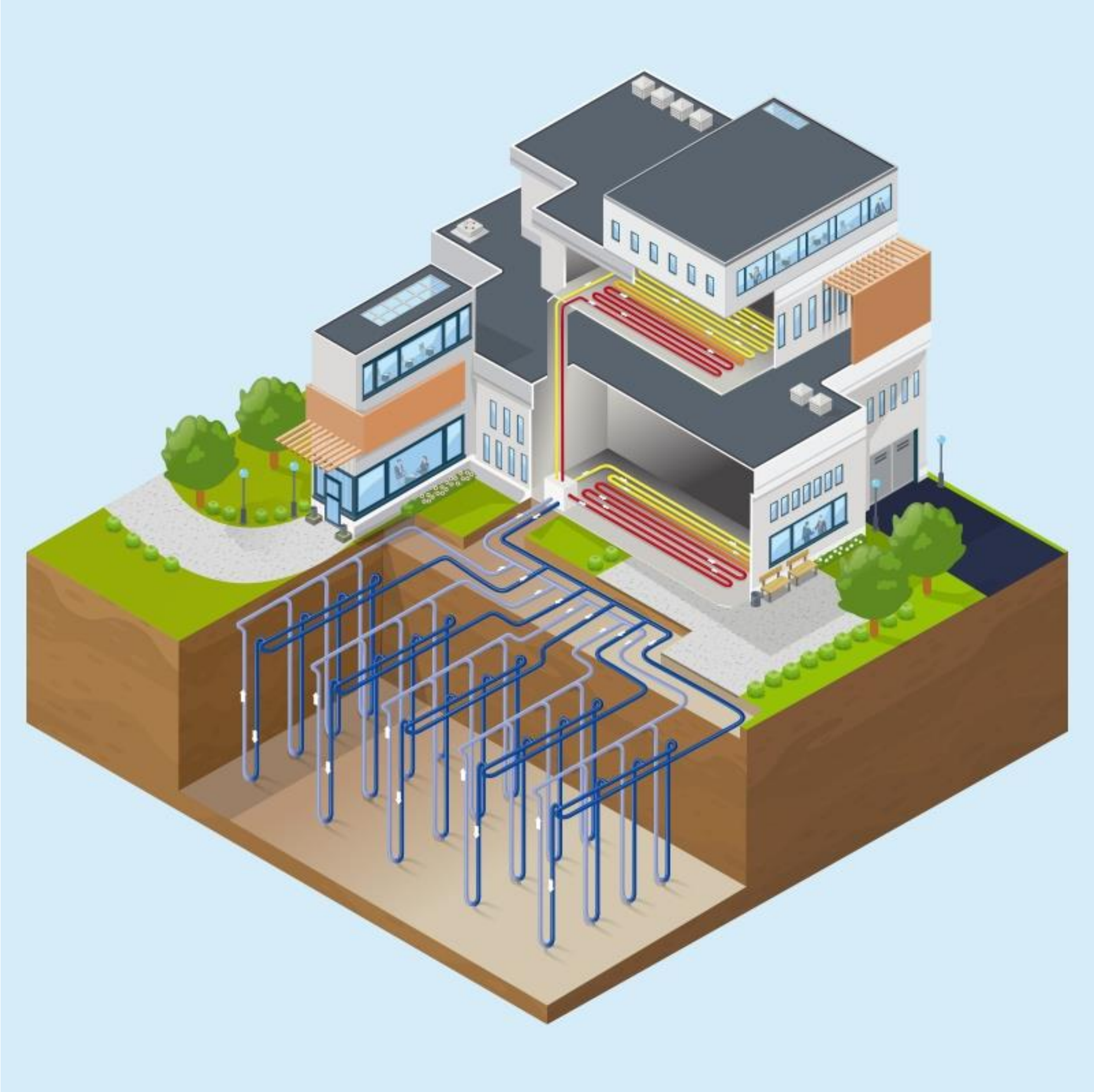


# Groundwater heat pumps (GWHPs)



# Ground source heat pumps (GSHPs) with v. BHEs



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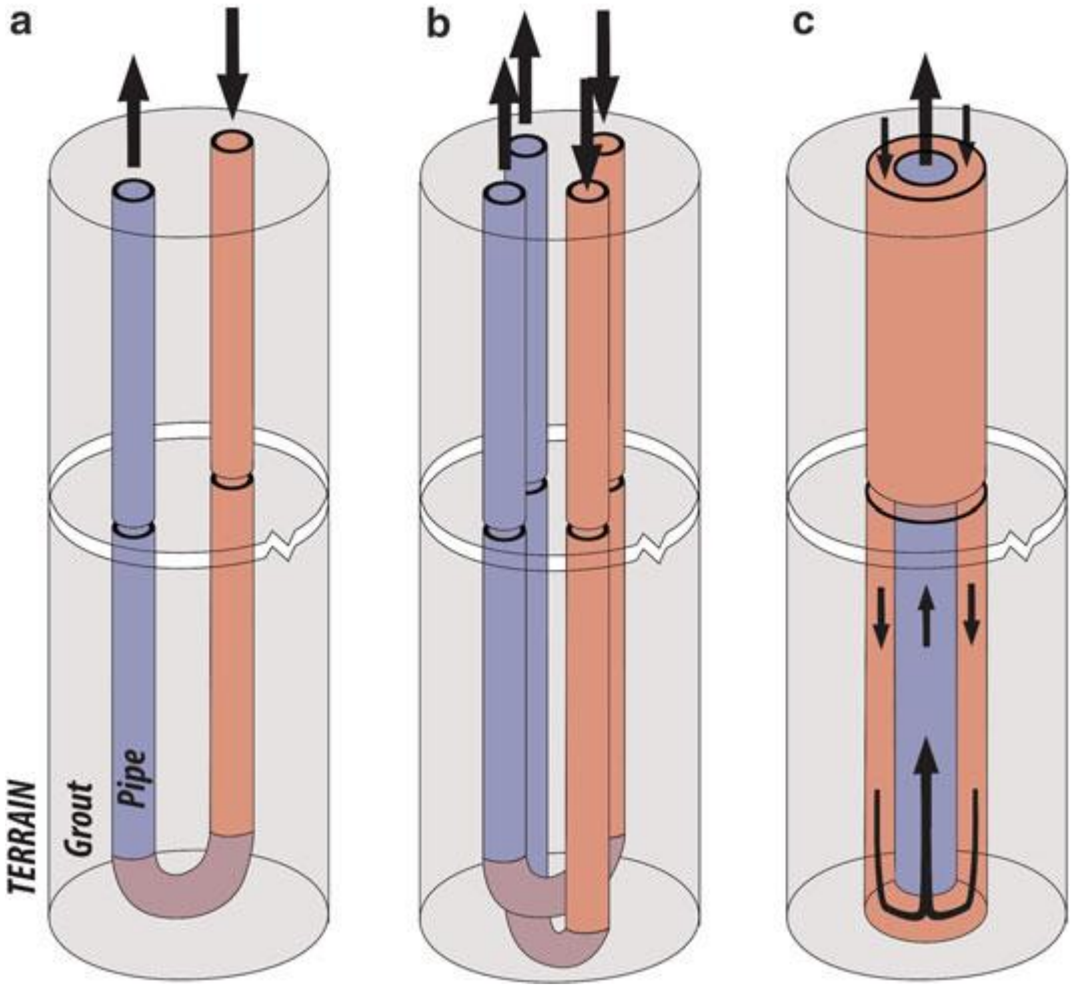



Different designs of vertical BHEs [2]

# Optimization of SGE systems

Maximizing the efficiency of SGE systems while meeting regulatory and technical conditions

| GWHP optimization [3] | Adjoint approach | GSHP optimization [4] |
|---|---|---|
| Optimal well placement for multiple GWHPs | FEM (Firedrake) | Optimal control of individual BHE loads |

Collaborators:

- Fabian Böttcher, Stefan C. Kramer, Prof. Matthew D. Piggott, Kai Zosseder, Prof. Thomas Hamacher
- Bodo Lipp, Prof. Michael Ulbrich

[3] Halilovic, S., Böttcher, F., Kramer, S. C., Piggott, M. D., Zosseder, K., & Hamacher, T. (2022). *Well layout optimization for groundwater heat pump systems using the adjoint approach*. Energy Conversion and Management, 268, 116033. https://doi.org/10.1016/j.enconman.2022.116033

# GWHP optimization – Forward model

## 2D vertically averaged steady state groundwater flow and heat transport in aquifer

- Flow PDEs:

$$(BS_0+\varepsilon_e)\frac{\partial h}{\partial t}+\nabla\cdot(B\mathbf{q})=\overline{Q}$$

$$\mathbf{q}=-\mathbf{K}\cdot\nabla h$$

$$B=h-f^{\mathrm{B}}$$

$$\boxed{\overline{Q}=\sum_{i=1}^{N}q_i(t)\delta(\mathbf{x}-\mathbf{x}_i^{\mathrm{inj}})-q_i(t)\delta(\mathbf{x}-\mathbf{x}_i^{\mathrm{ext}})}$$

- Heat transport PDE:

$$BC_{\mathrm{m}}\frac{\partial T}{\partial t}+BC_{\mathrm{w}}\mathbf{q}\cdot\nabla T-\nabla\cdot(B\mathbf{\Lambda}\cdot\nabla T)=\overline{P}_{\mathrm{t}}$$

$$\boxed{\overline{P}_{\mathrm{t}}=\sum_{i=1}^{N}q_i(t)C_{\mathrm{w}}[T(\mathbf{x}_i^{\mathrm{inj}},t)-T]\delta(\mathbf{x}-\mathbf{x}_i^{\mathrm{inj}})}$$

- Steady state PDE system:

$$\nabla\cdot(B\mathbf{q})=\overline{Q}$$

$$\mathbf{q}=-\mathbf{K}\cdot\nabla h$$

$$BC_{\mathrm{w}}\mathbf{q}\cdot\nabla T-\nabla\cdot(B\mathbf{\Lambda}\cdot\nabla T)=\overline{P}_{\mathrm{t}}$$

- Tensor of hydrodynamic thermal dispersion:

$$\mathbf{\Lambda}=\mathbf{\Lambda}_0+\mathbf{\Lambda}_0^s+\rho c\mathbf{D}_{\mathrm{mech}}$$

$$\mathbf{\Lambda}_0=\varepsilon s\Lambda\mathbf{I}$$

$$\mathbf{\Lambda}_0^s=(1-\varepsilon)\Lambda^s\mathbf{I}$$

$$\mathbf{D}_{\mathrm{mech}}=\beta_T\|\mathbf{q}\|\mathbf{I}+(\beta_L-\beta_T)\frac{\mathbf{q}\otimes\mathbf{q}}{\|\mathbf{q}\|}$$

# Smooth approximation of Dirac functions

Dirac delta functions problematic for gradient based optimization
→ Replace them with smooth ‘bump’ functions

- Using exponential terms

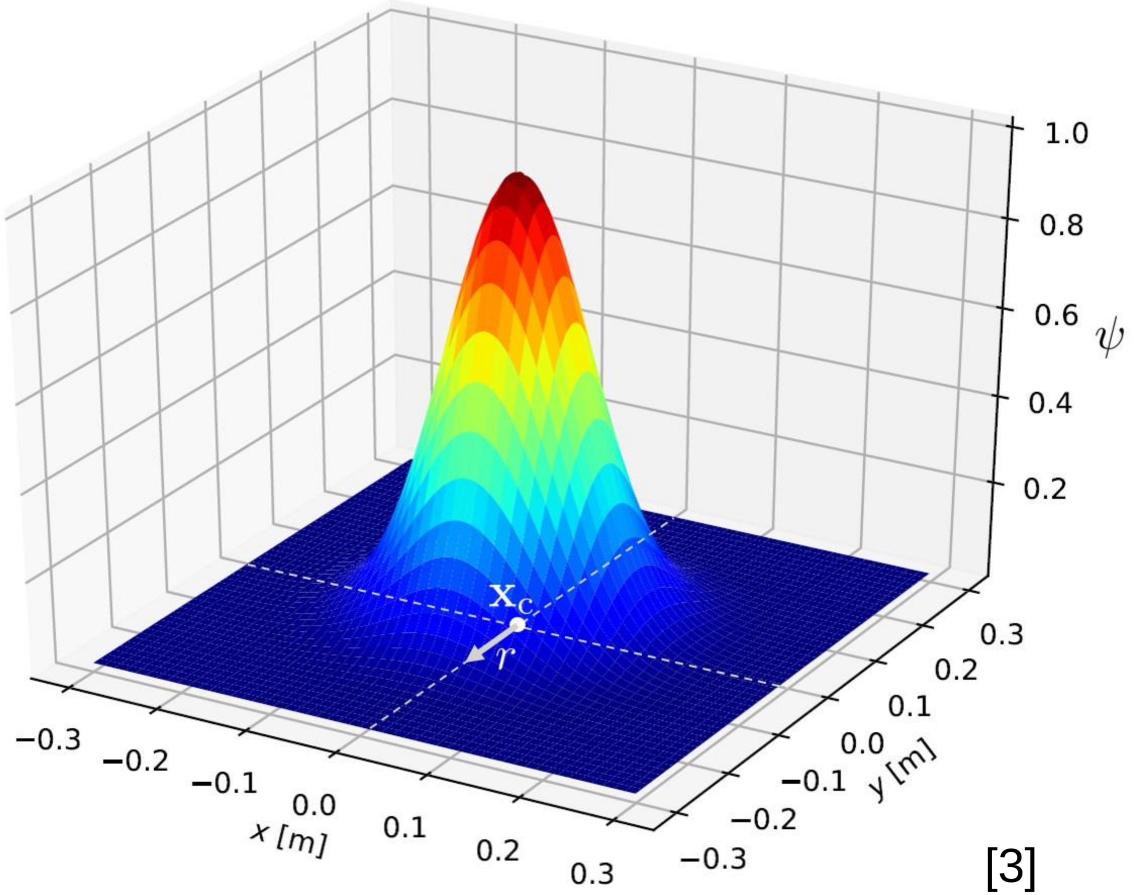
$$\delta(\mathbf{x}-\mathbf{x}_i)=\delta(x-x_i)\delta(y-y_i)$$

$$\delta(\mathbf{x}-\mathbf{x}_i)\rightarrow \mathrm{e}^{-\frac{(x-x_i)^2+(y-y_i)^2}{r^2}}=\psi(x,y,x_i,y_i)$$

- Normalize the values with the volume:

$$V=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(x,y,x_i,y_i)dxdy=r^2\pi$$

$r$: Radius of wells




[3]

# Functional of interest and constraints

- Controls:

$$\boldsymbol{m} = [x_1^{\text{ext}}, \dots, x_N^{\text{ext}}, y_1^{\text{ext}}, \dots, y_N^{\text{ext}}, x_1^{\text{inj}}, \dots, x_N^{\text{inj}}, y_1^{\text{inj}}, \dots, y_N^{\text{inj}}]^T$$

- Functional of interest:

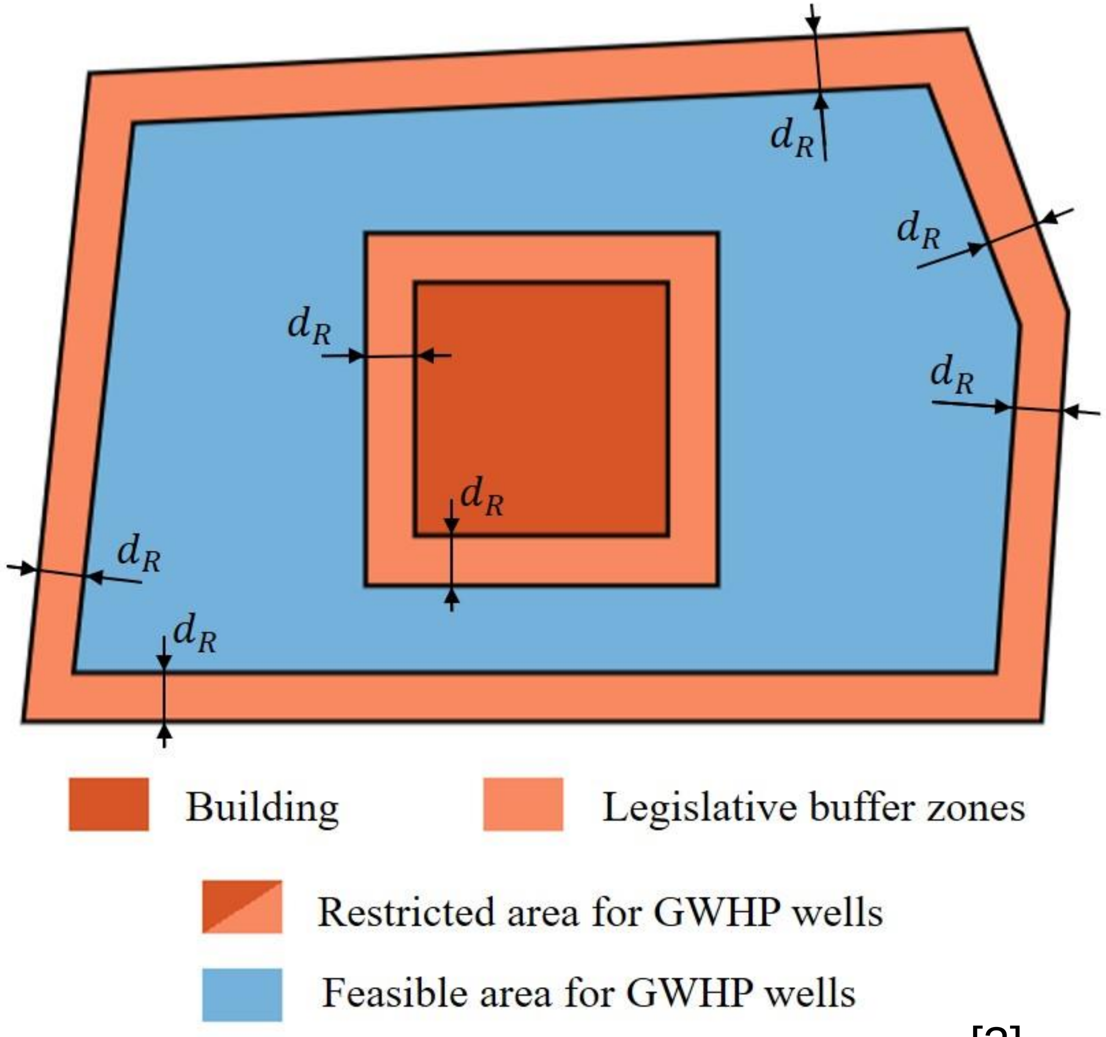
$$J_0(\boldsymbol{u}, \boldsymbol{m}) = \sum_{i=1}^{N} T(\mathbf{x}_i^{\text{ext}}) = \sum_{i=1}^{N} \int_{\Omega} T(\mathbf{x}) \cdot \delta(\mathbf{x} - \mathbf{x}_i^{\text{ext}})\, \mathrm{d}\Omega$$

- Constraints:

$$\mathbf{x}_i^{\text{ext}}, \mathbf{x}_i^{\text{inj}} \in D_i, \quad \forall i \in \{1, \dots, N\} \quad \Rightarrow \quad \mathbf{A}\boldsymbol{m} \leq \mathbf{b}$$

$$d(\mathbf{x}_i^{\text{ext}}, \mathbf{x}_i^{\text{inj}})^2 = (x_i^{\text{ext}} - x_i^{\text{inj}})^2 + (y_i^{\text{ext}} - y_i^{\text{inj}})^2 \geq d_{\min}^2,$$

$$\mathbf{x}_i^{\text{ext}}, \mathbf{x}_i^{\text{inj}} \notin R_i^j, \quad \forall i \in \{1, \dots, N\}, \forall j \in \{1, \dots, r_i\}$$



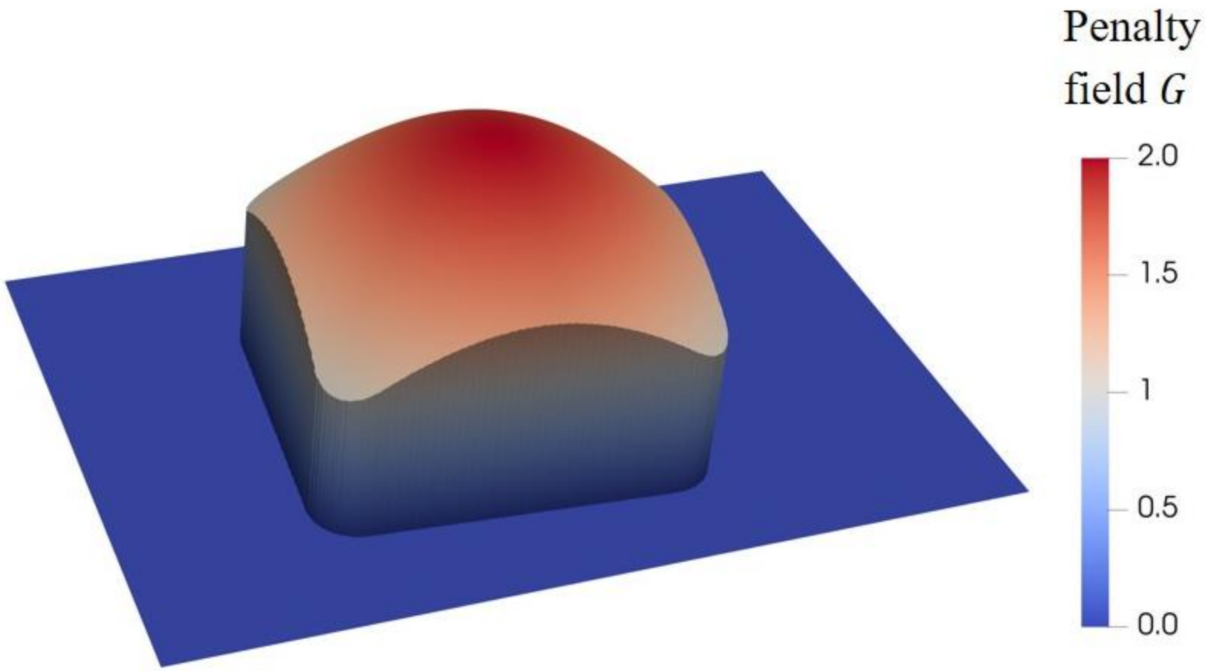

[3]

# Soft constraints – penalty approach

- Penalty function:

$$P_R = \alpha \cdot \sum_{i=1}^{N} \left( p(\mathbf{x}_i^{\text{ext}}) + p(\mathbf{x}_i^{\text{inj}}) \right)$$

$$p(\mathbf{x}_i) = \int_{\Omega} G(\mathbf{x}) \cdot \delta(\mathbf{x} - \mathbf{x}_i)\, \mathrm{d}\Omega$$




[3]

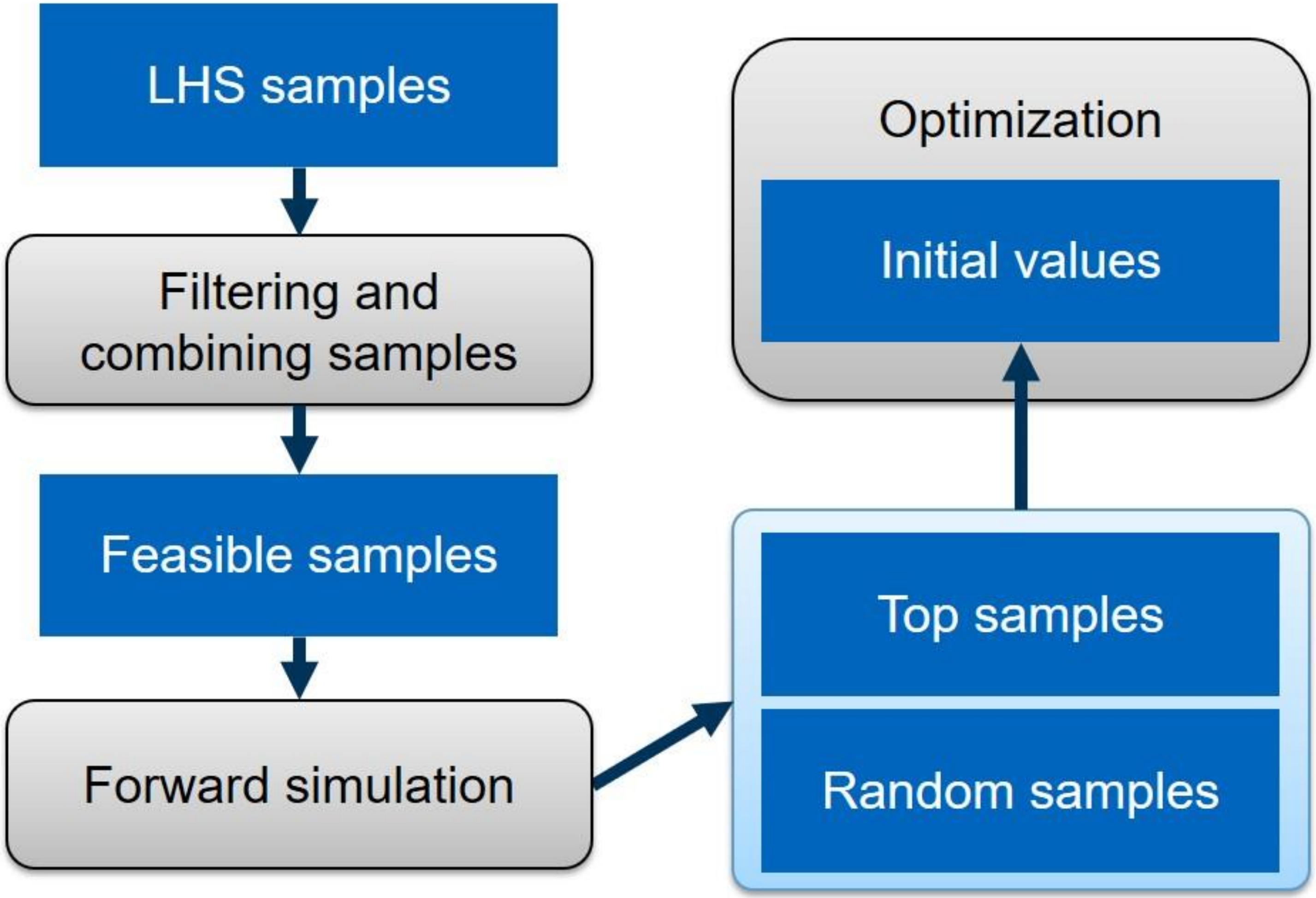
- Penalty field:

$$G(\mathbf{x}) = \begin{cases} 0 & \text{if} \quad \mathbf{x} \notin R_i^j, \\ & \quad \forall i \in \{1, \ldots, N\}, \forall j \in \{1, \ldots, r_i\} \\ 1 + \cos\left[ \frac{d(\mathbf{x}, \mathbf{x}_{i,j}^{\mathrm{p}})}{r_{i,j}^{\mathrm{p}}} \cdot \frac{\pi}{2} \right] & \text{if} \quad \mathbf{x} \in R_i^j, \end{cases}$$

# Problem formulation and initialization

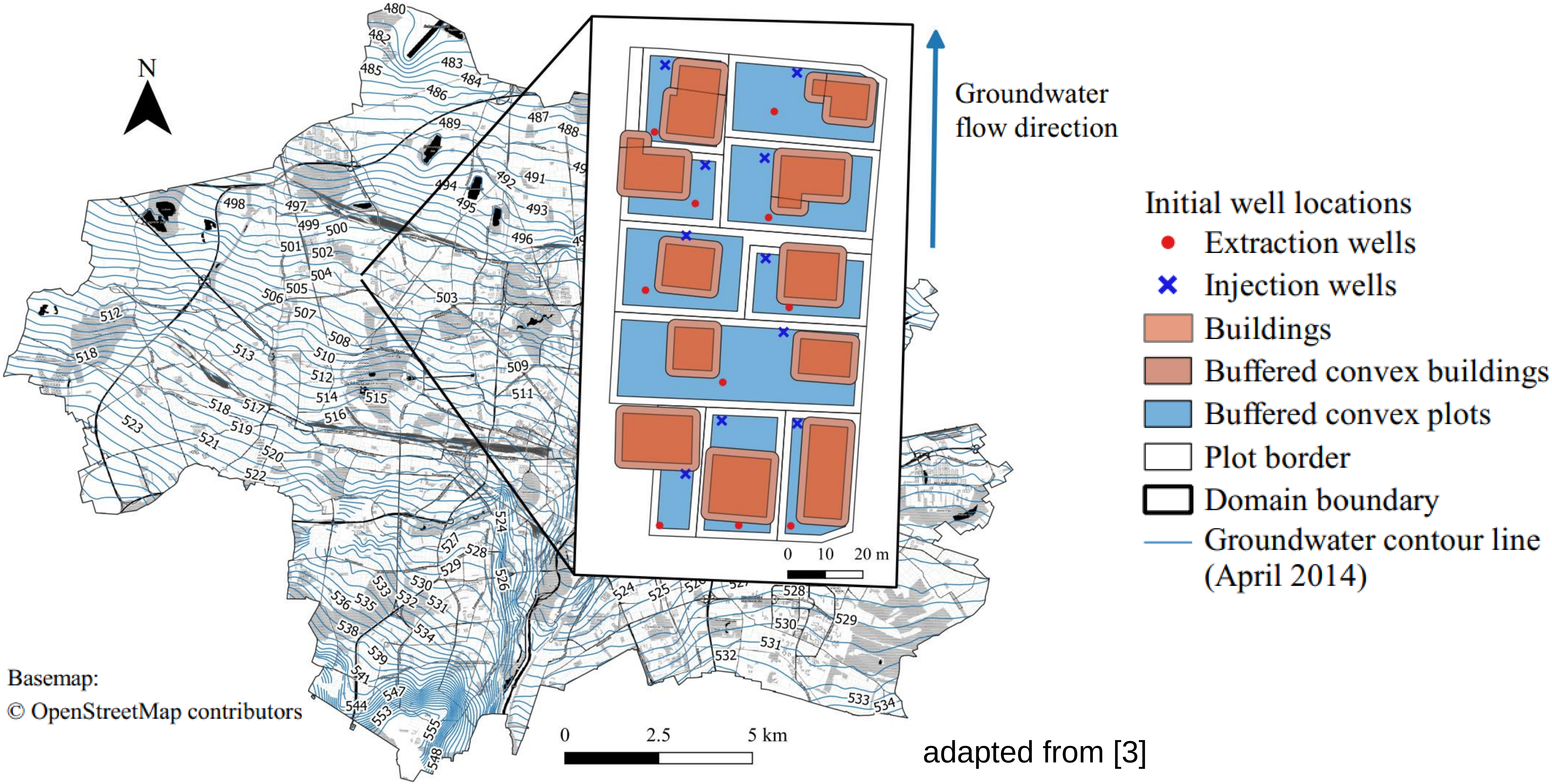
Reduced-space problem:

$$\min_{\boldsymbol{m}} \quad -\hat{J}_0(\boldsymbol{m}) + P_R(\boldsymbol{m}),$$
$$\text{subject to} \quad g_1(\boldsymbol{m}) \leq 0,$$
$$g_2(\boldsymbol{m}) \leq 0,$$



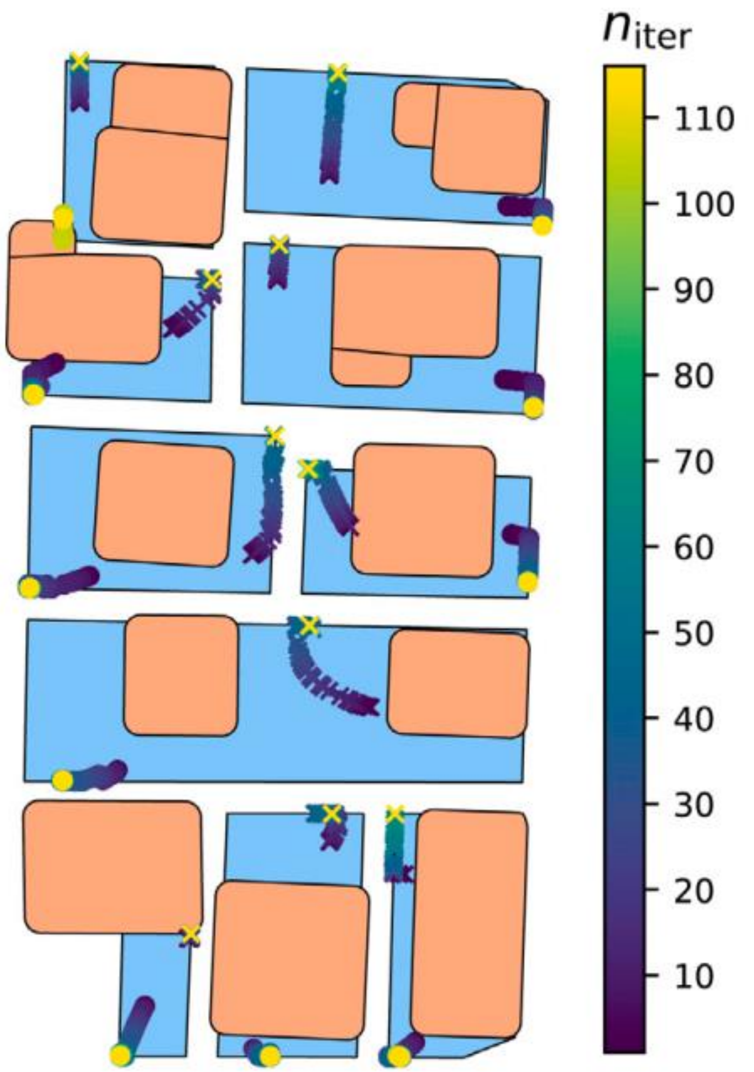

Multi-start initialization strategy [3]

# GWHP optimization – Case study

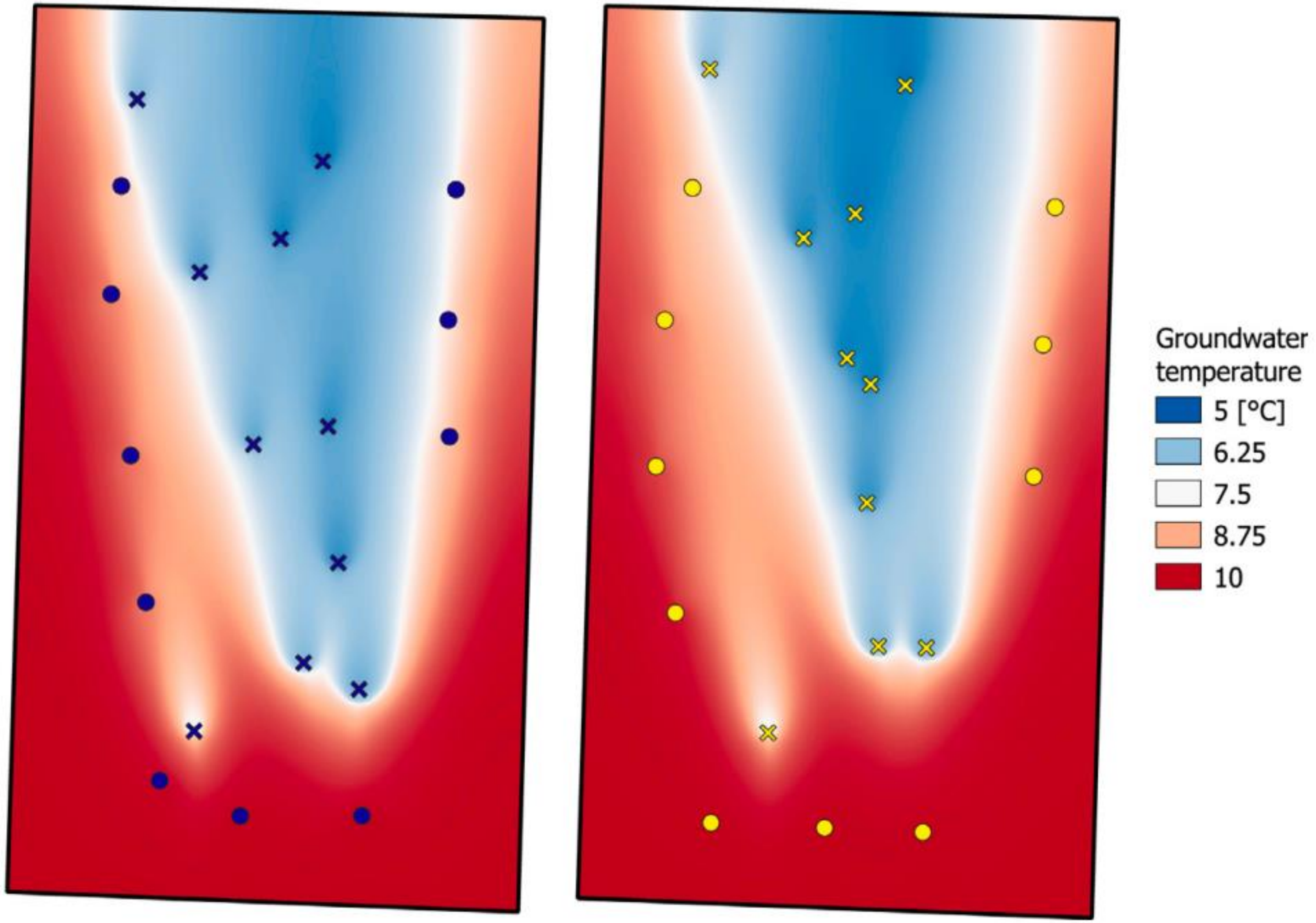



adapted from [3]

# GWHP optimization – Results

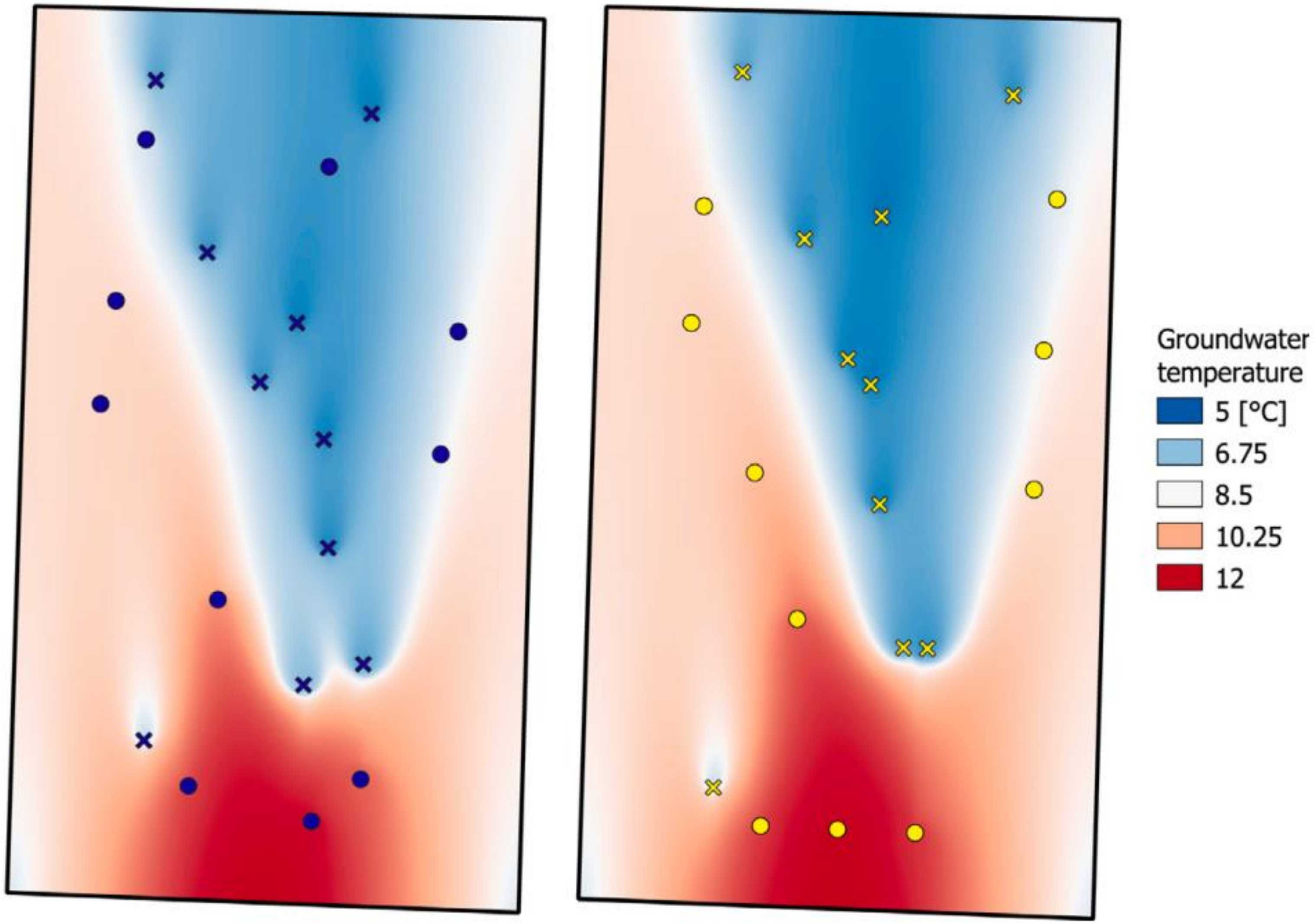



Well positions during optimization [3]

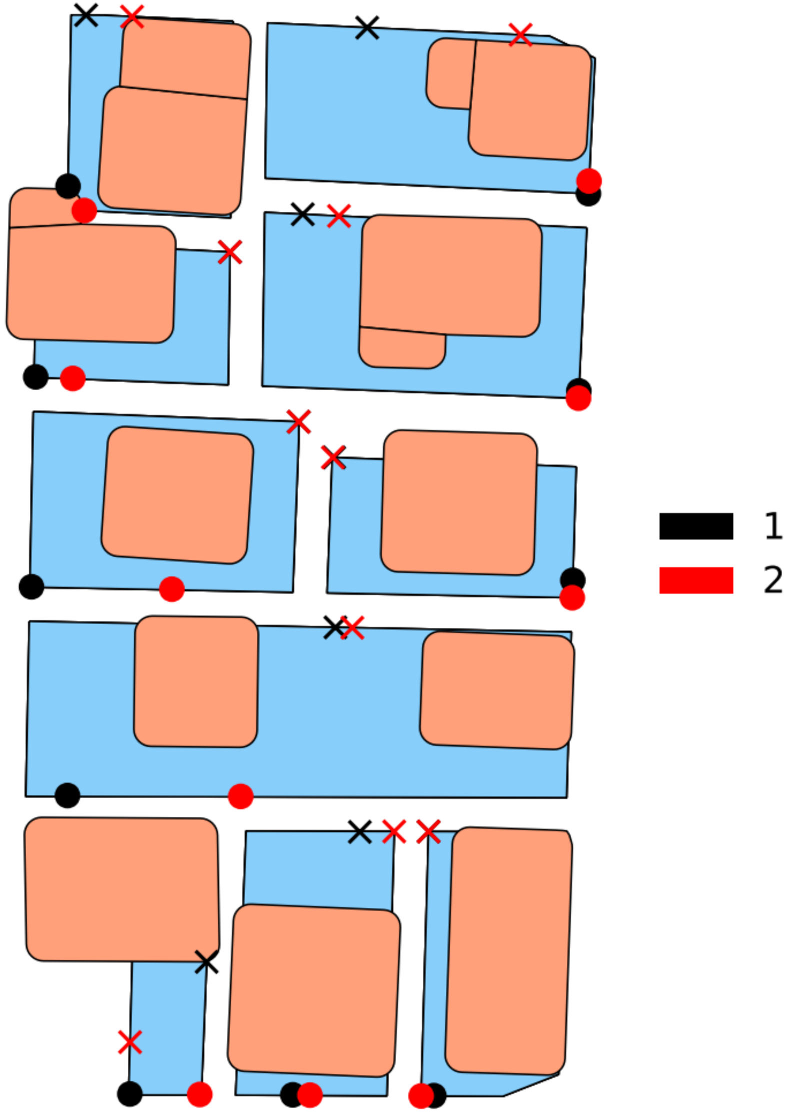



Initial and final GW temperature fields and well layouts [3]

# GWHP optimization – Results 2




Initial and final GW temperature fields and well layouts [3]




Optimal well layouts – Scenario 1 & 2 [3]

# GSHP optimization – Forward model

2D vertically averaged transient heat transport in porous media

<u>Scenario 1</u>

without groundwater flow

$$
\begin{aligned}
C^s \frac{\partial T}{\partial t} - \nabla \cdot (\mathbf{\Lambda} \nabla T) &= 0 && I \times \Omega \\
\mathbf{n} \cdot (\mathbf{\Lambda} \nabla T) &= -q_i && I \times \Gamma_i \\
\mathbf{n} \cdot (\mathbf{\Lambda} \nabla T) &= 0 && I \times \Gamma_0 \\
T(0) &= T_0 && \{0\} \times \Omega
\end{aligned}
$$

<u>Scenario 2</u>

with groundwater flow (confined aquifer):

$$
\begin{aligned}
C^g \frac{\partial T}{\partial t} + C^f \mathbf{u} \cdot \nabla T - \nabla \cdot (\mathbf{\Lambda} \nabla T) &= 0 && I \times \Omega \\
\mathbf{n} \cdot (\mathbf{\Lambda} \nabla T) &= -q_i && I \times \Gamma_i \\
\mathbf{n} \cdot (\mathbf{\Lambda} \nabla T) &= 0 && I \times \Gamma_0 \\
T(0) &= T_0 && \{0\} \times \Omega
\end{aligned}
$$

- $T$ ground temperature
- $q_i$ load at BHE i
- $C$ volumetric heat capacity
- $\mathbf{u}$ velocity of groundwater
- $\mathbf{\Lambda}$ tensor of hydrodynamic thermodispersion

# Functional of interest

- Controls:

$$\mathbf{q} = (q_1, \dots, q_{n_b})^T \in Q = L^2(I; \mathbb{R}^{n_b})$$

- Borehole wall temperature as the mean over the respective boundary:

$$T_{b_i} = \frac{\int_{\Gamma_i} T\, ds}{\int_{\Gamma_i} ds} = \frac{1}{2\pi r_b} \int_{\Gamma_i} T\, ds \quad \Rightarrow$$

$$\frac{\sum_{i=1}^{n_b} T_{b_i}}{n_b} = \frac{1}{2\pi r_b n_b} \sum_{i=1}^{n_b} \int_{\Gamma_i} T\, ds$$ Average temperature on all boreholes

- Functional of interest:

$$\frac{1}{t_{end}} \int_I \frac{\sum_{i=1}^{n_b} T_{b_i}}{n_b} dt = \frac{1}{2\pi r_b n_b t_{end}} \int_I \sum_{i=1}^{n_b} \int_{\Gamma_i} T ds\, dt \quad \Rightarrow$$ **Problematic**

# Functional of interest and constraints

- Functional of interest:

$$J_1(T,\mathbf{q}) := \frac{1}{t_{end}} \int_I \sum_{i=1}^{n_b} T_{b_i} q_i dt = \frac{1}{2\pi r_b n_b t_{end}} \int_I \sum_{i=1}^{n_b} \int_{\Gamma_i} T ds\, q_i\, dt$$

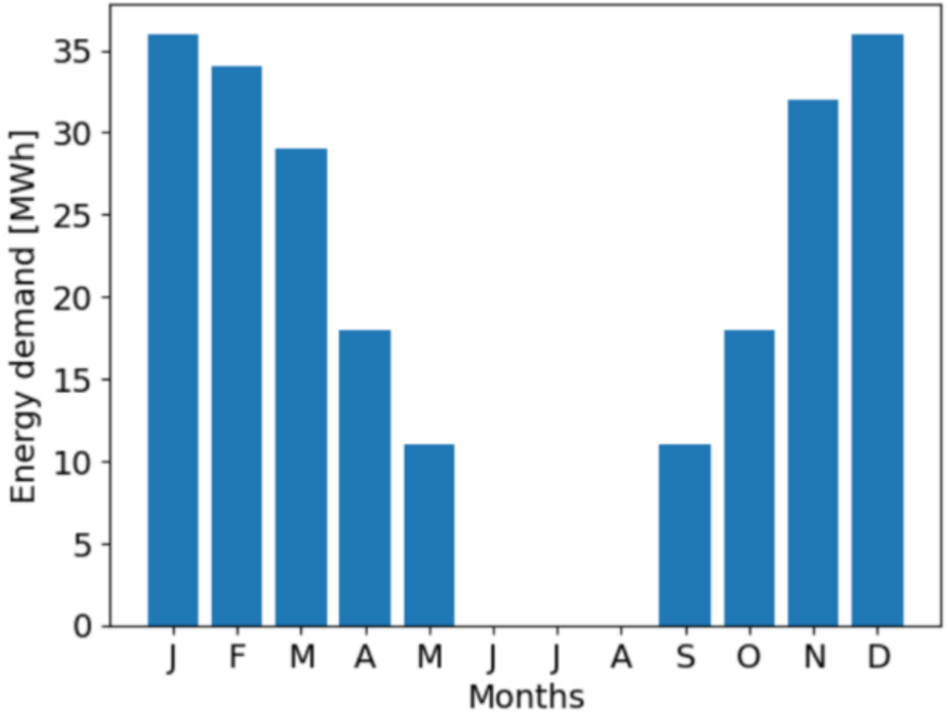
Constraints:

- The energy demand must be satisfied at each time step:

$$\int_{t_k}^{t_{k+1}} \sum_{i=1}^{n_b} 2\pi r_b l_b q_i(t)\, dt = Q_k$$

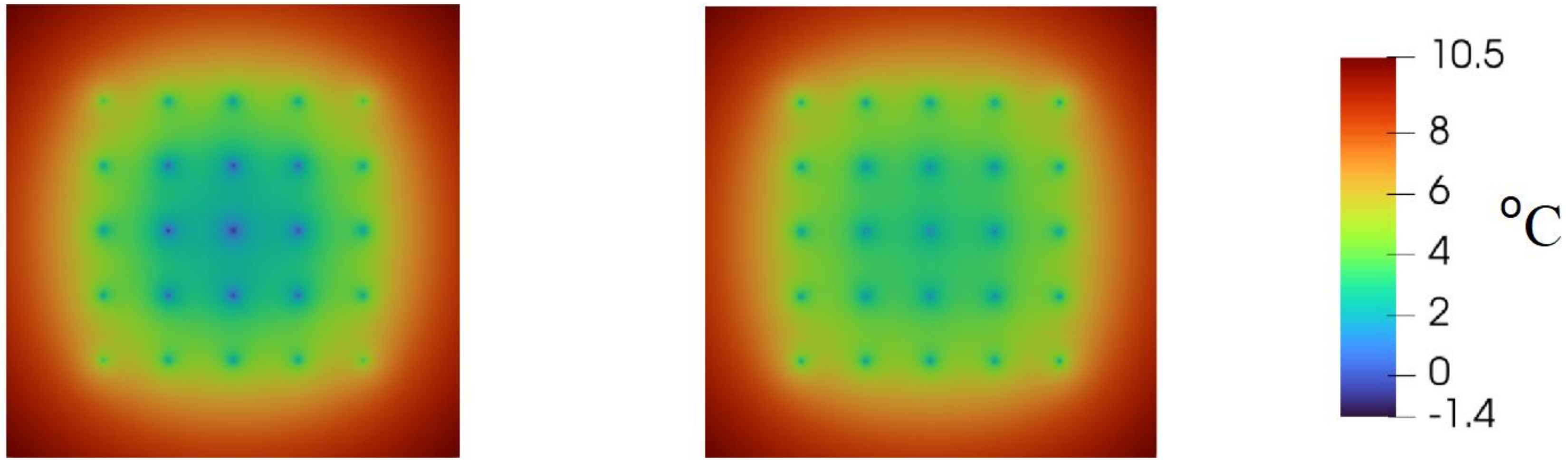
- Only heating is considered:

$$0 \leq q_i \leq q_{up}$$

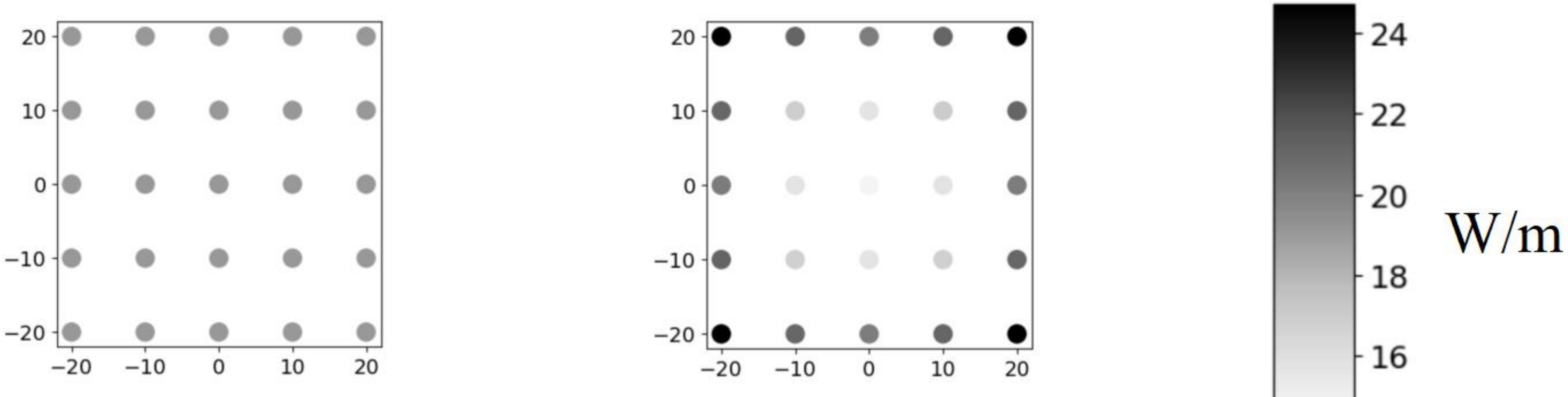



Monthly energy demand [4]

# GSHP optimization results (Scenario 1)



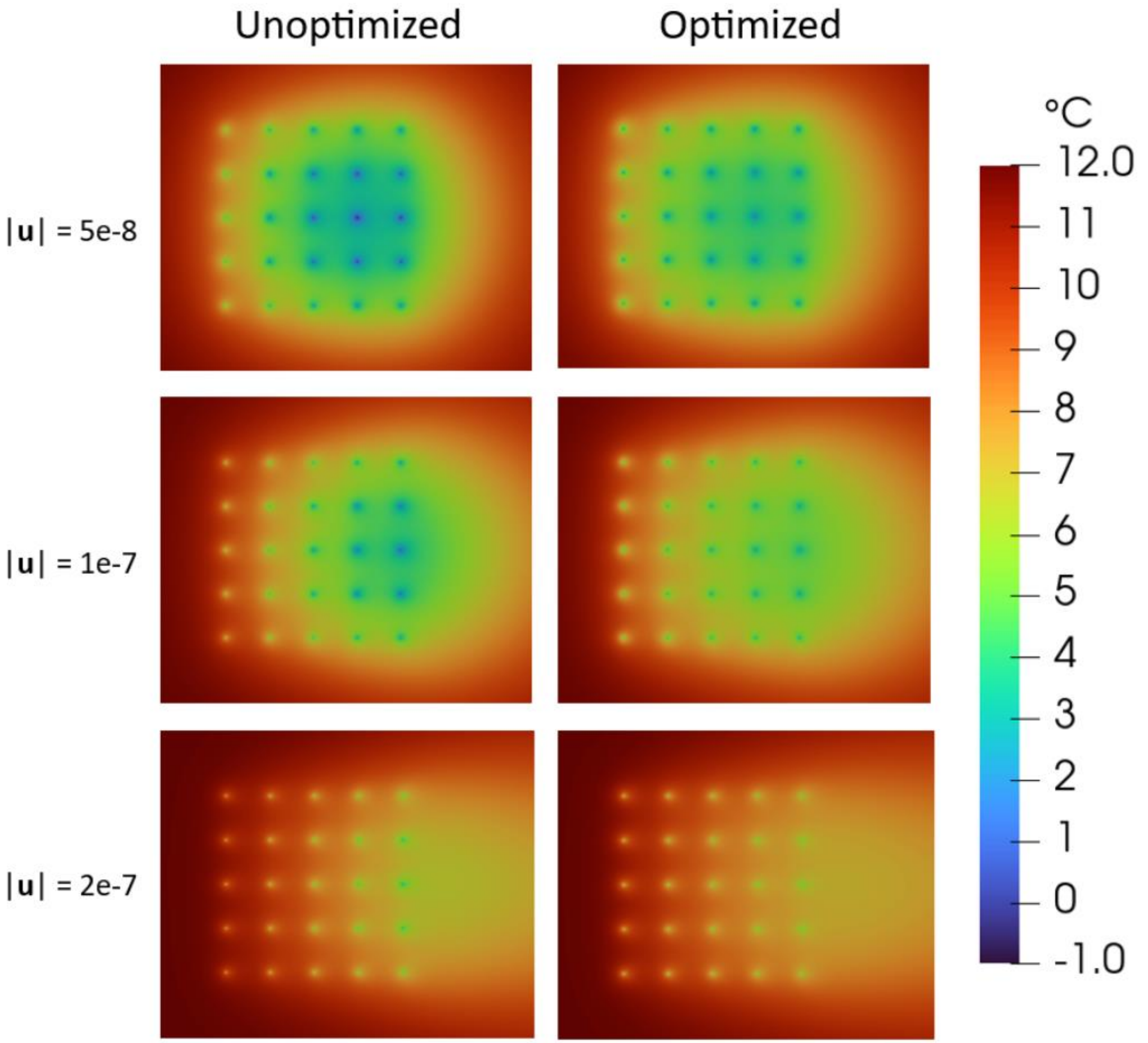

(a) Unopt. temperature after 10a (b) Opt. temperature after 10a




(c) Equal loads in January (d) Optimized loads in January

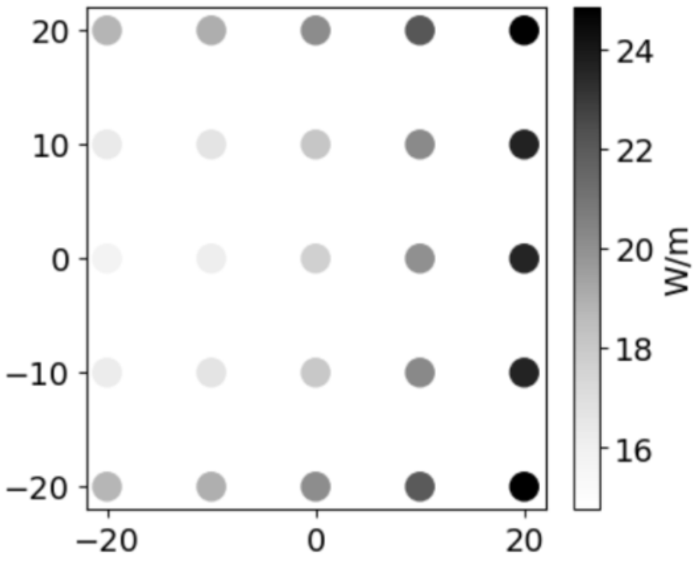
[4]

# GSHP optimization results (Scenario 2)

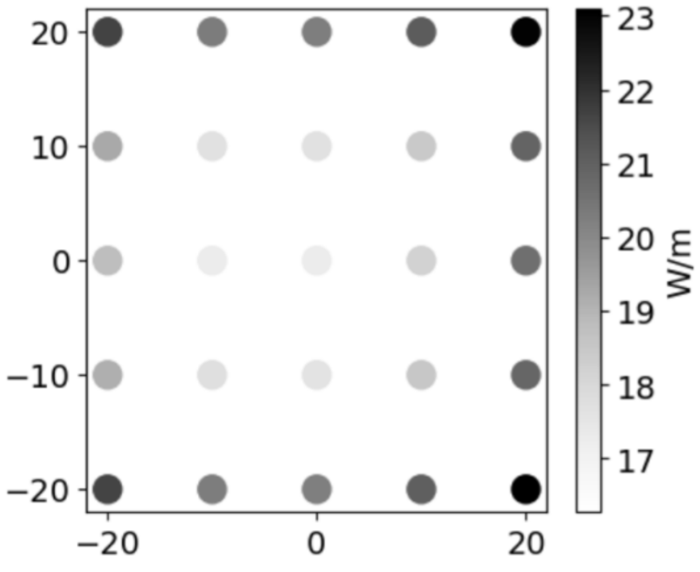



Temperature fields for different groundwater velocities [4]

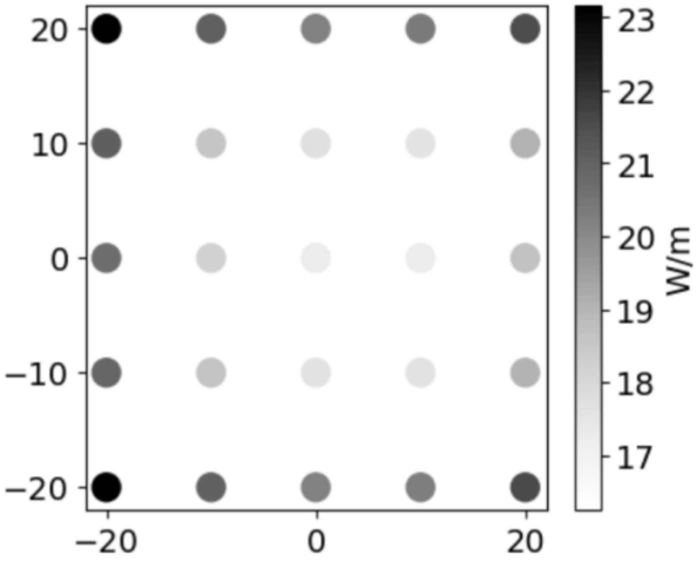
# GSHP optimization results (Scenario 2)



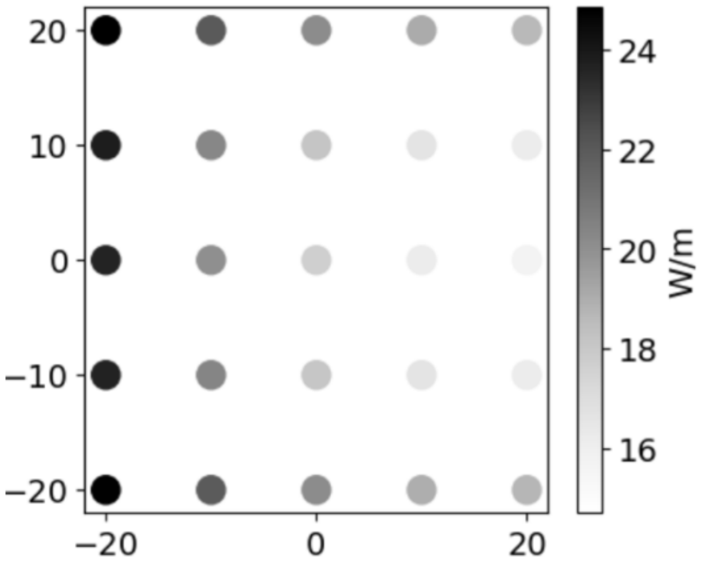

(a) 2nd year




(b) 5th year




(c) 7th year




(d) 10th year

Optimized loads for |u| = $10^{-7}$ m/s in January [4]

# Conclusion and outlook

| | Optimization application | |
|---|---|---|
| | **GWHP** | **GSHP** |
| Forward model | 2D | 2D |
| Time dependency | no | yes |
| Convexity | no | yes |
| Type of problem | Design / Distributed control | Boundary control |

Comparison of the application examples

## Outlook

- 3D models, design + control
- Rigorous mathematical proofs
- Other SGE applications: ATES, BTES

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https://www.epe.ed.tum.de/en/ens

# Literature

[1] © GRETA - INTERREG ALPINE SPACE FEDER https://rapport-activite.brgm.fr/en/complementing-energy-mix-with-properties-resources-subsurface

[2] A. García Gil, E. A. Garrido Schneider, M. Mejías Moreno, and J. C. Santamarta Cerezal. Shallow Geothermal Energy. Springer, 2022.

[3] Halilovic, S., Böttcher, F., Kramer, S. C., Piggott, M. D., Zosseder, K., & Hamacher, T. (2022). *Well layout optimization for groundwater heat pump systems using the adjoint approach*. Energy Conversion and Management, 268, 116033. https://doi.org/10.1016/j.enconman.2022.116033

[4] B. Lipp. Optimization of closed-loop shallow geothermal systems using the adjoint approach. Master's Thesis, 2023.