Nuclear Astrophysics: Neutron Star Physics

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June 2025

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Introduction to Particle Physics

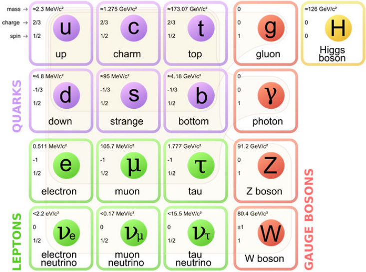
Particle physics forms the foundation of nuclear astrophysics by describing the fundamental particles and interactions that govern matter at the smallest scales. It helps us understand phenomena in extreme environments, such as the early universe and neutron stars.

Key Elements:

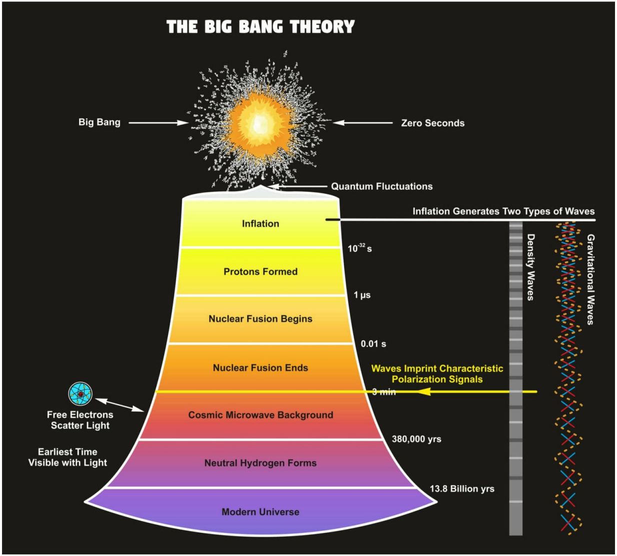
- **Particles**: Quarks, Leptons, and Bosons
- **Forces**: Strong (QCD), Weak, Electromagnetic, Gravitational

Standard Model Overview

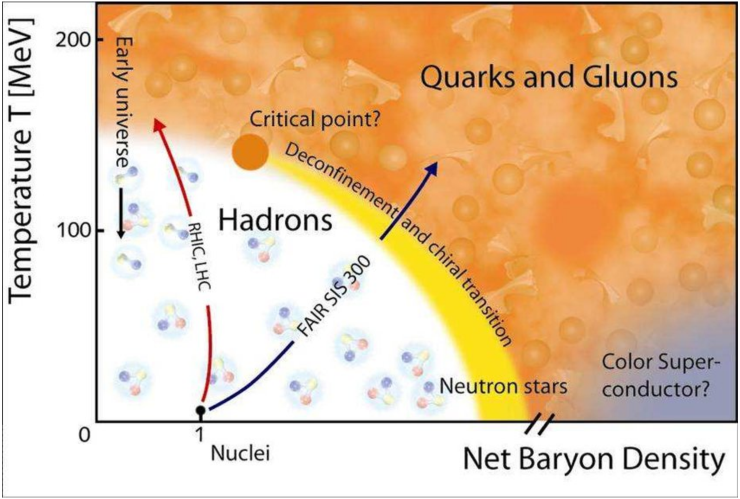
The Standard Model describes the fundamental particles of matter and the interactions mediated by the electromagnetic, weak, and strong forces. It groups these particles into **quarks**, **leptons**, and **gauge bosons**, providing a basis for studies in nuclear and particle physics.



Big Bang Theory and Early Universe

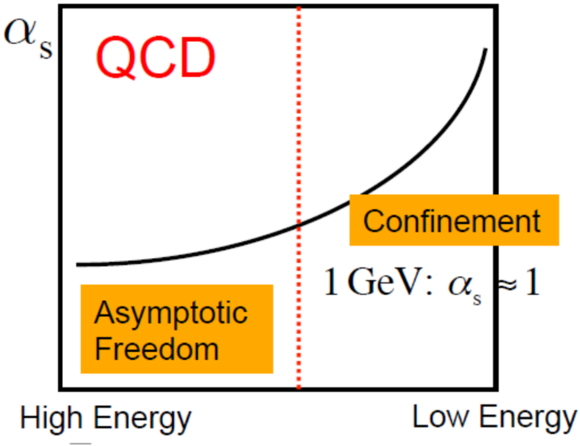


QCD Phase Diagram



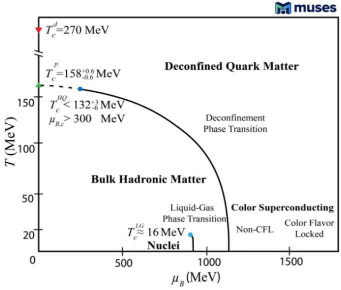
Temperature vs baryon chemical potential

QCD Concepts: Confinement and Asymptotic Freedom

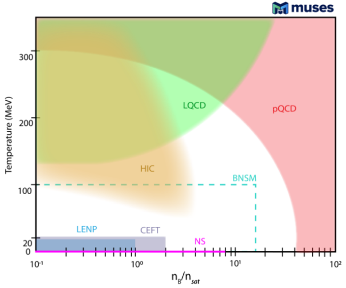


Confinement and Asymptotic Freedom

Models of the QCD Phase Diagram – MUSES



R.K., et al., Phys. Rev. D 109, 074008 (2024)



R.K., et al., Living Rev. Rel. 27, 3 (2024)

Models representing QCD matter under varying conditions

Introduction to the Walecka Model

The Walecka model is a relativistic mean-field approach used to describe dense nuclear matter. It models nucleons interacting via the exchange of scalar (σ) and vector (ω^μ) mesons.

Scalar Meson (σ) Term:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\left(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2\right)$$

Vector Meson (ω^μ) Term:

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2 \omega^\mu \omega_\mu$$

where $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$

Saturation Properties of Nuclear Matter

Empirical nuclear matter exhibits a saturation point at a density $\rho_0 \approx 0.16\,\mathrm{fm}^{-3}$, where the energy per nucleon is minimized (approximately $-16\,\mathrm{MeV}$). This reflects a balance between attractive and repulsive interactions in nuclear forces.

A reliable nuclear model should reproduce the following properties:

- Saturation density ρ_0
- Binding energy per nucleon at saturation
- Compressibility modulus K
- Symmetry energy S
- Slope parameter L

Lagrangian of the Model

The total Lagrangian of the Walecka model combines contributions from the fermionic, scalar, and vector fields, along with interaction terms. It forms the foundation for deriving the equations of motion in mean-field theory.

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{int}} \\
&= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_N)\psi_i + g_\sigma \bar{\psi}_i \sigma \psi_i - g_\omega \bar{\psi}_i \gamma^\mu \omega_\mu \psi_i \\
&\quad + \frac{1}{2}(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu
\end{aligned}
$$

Mean-Field Approximation

In uniform nuclear matter, the mean-field approximation simplifies the equations of motion by treating meson fields as classical constants. Only time-like components of the vector field are retained.

Assumptions:

$$\sigma(x) \to \bar{\sigma}, \quad \omega^{\mu}(x) \to \delta^{\mu 0}\bar{\omega}_0$$

Nucleon Field Equation:

$$\left[i\gamma^{\mu}\partial_{\mu} - m_N + g_{\sigma}^{i}\bar{\sigma} - g_{\omega}^{i}\gamma^{0}\bar{\omega}_0\right]\psi_i = 0$$

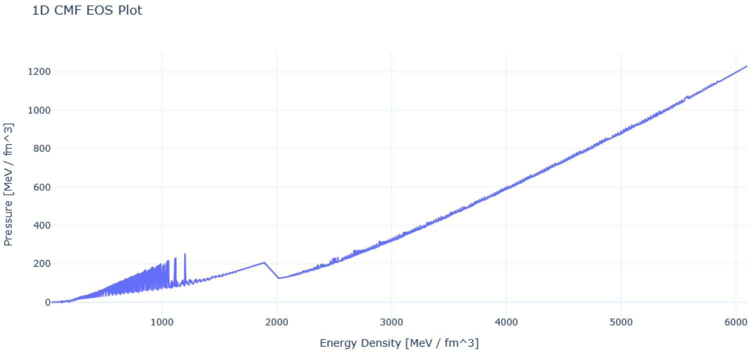
$$M_i^{*} = m_N - g_{\sigma}^{i}\bar{\sigma}$$

$$\left[i\gamma^{\mu}\partial_{\mu} - M_i^{*} - g_{\omega}^{i}\gamma^{0}\bar{\omega}_0\right]\psi_i = 0$$

Meson Field Equations:

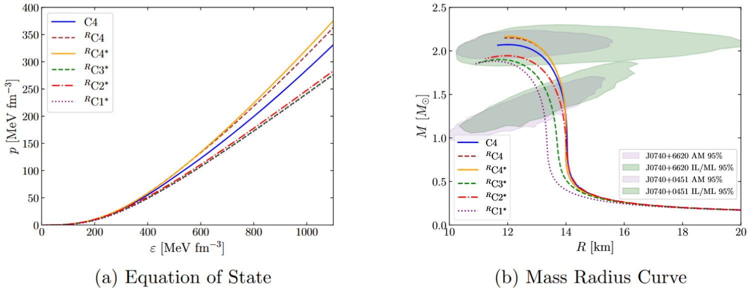
$$\bar{\sigma} = \frac{g_{\sigma}^{i}}{m_{\sigma}^{2}} n_{s_i}, \quad \bar{\omega}_0 = \frac{g_{\omega}^{i}}{m_{\omega}^{2}} n_{B_i}$$

EoS in Neutron Star Matter from the CMF Model



Equation of State curve generated using the CMF model

EoS and Mass-Radius Curve of Neutron Stars



EoS and MR of neutron stars

Conclusion

This study explored the structure of nuclear matter through the QCD phase diagram and the Walecka model. The mean-field approximation provided tractable equations for modeling interactions in dense systems. Using the CMF model, the equation of state was computed and applied to neutron star matter, yielding physically consistent mass-radius relationships. These results highlight the importance of combining field-theoretical models with observational constraints in high-density astrophysics.