

LAURENTIAN UNIVERSITY

CPSC 5006 EL

Matrix Computations

Assignment 4

DUE Monday, April 8th, 2024

**Question 9**

- Enter the following into Matlab:  
     $\gg \text{load('clown.mat')};$   
     $\gg \text{image}(X);$   
     $\gg \text{colormap(gray)}; \text{axis off};$
  - The image  $X$  is a  $200 \times 320$  matrix (type  $\text{size}(X)$  into Matlab's command line). The numbers range from 1 to 81 and correspond to different shades of gray.
  - We compute the SVD of our image  $X$  by executing  
     $\gg [U, S, V] = \text{svd}(X);$
  - Note that the rank of  $X$  is 200. We can reduce our clown image to a rank of 60 by executing the following:  
     $\gg n = 60;$   
     $\gg X_k = U(:, 1:n) * S(1:n, 1:n) * V(:, 1:n)';$   
     $\gg \text{image}(X_k);$
  - Note that the clown's left cheek is a little blurry, but it otherwise looks OK.  
How low can you take the rank and still have a decent looking image?  
What happens when you take the rank too low?
1. Show that the reduced form of the SVD requires knowledge of only  $r(m + n + 1)$  numbers, where an  $n \times m$  matrix  $A$  is of rank  $r$ .
  2. Explore the clown picture for several different values of rank. Conduct the experiments described above. Note that the original image takes 64,000 integers to store. Compare this with the storage needs for various lower-rank SVD approximations. What conclusions can you draw?