Matrix Computations

CPSC 5006-EL

Assignment4

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Q1

3.4.22

$$v_1 = \begin{bmatrix} 3 \\ -3 \\ 3 \\ -3 \end{bmatrix}, v2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

(a)

$$r_{11} = \|v_1\|_2 = \sqrt{4 * 9} = 6, q_1 = \frac{1}{r_{11}} v_1 = \frac{1}{6} \begin{bmatrix} 3 \\ -3 \\ 3 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$r_{12} = (v_2, q_1) = \frac{1}{2} + 2\left(-\frac{1}{2}\right) + 3\left(\frac{1}{2}\right) + 4\left(-\frac{1}{2}\right) = -1$$

$$\widetilde{q}_2 = v_2 - r_{12} q_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{7}{2} \\ \frac{7}{2} \end{bmatrix}$$

$$r_{22} = \| \widetilde{q}_2 \|_2 = \sqrt{\frac{9}{4} * 2 + \frac{49}{4} * 2} = \sqrt{29}, q_2 = \frac{1}{r_{22}} \widetilde{q}_2 = \frac{1}{\sqrt{29}} \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \\ \frac{7}{2} \\ \frac{7}{2} \end{bmatrix}$$

(b)

$$Q = \begin{bmatrix} q_1 & q_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & \frac{3}{\sqrt{29}} \\ -1 & \frac{3}{\sqrt{29}} \\ 1 & \frac{7}{\sqrt{29}} \\ -1 & \frac{7}{\sqrt{29}} \end{bmatrix}, R = \begin{bmatrix} r_{11} & r_{12} \\ & r_{22} \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 0 & \sqrt{29} \end{bmatrix}$$

Q2

3.4.27

(a)

$r_{1:k-1,k} \leftarrow (v_{1:k-1})^T v_k$, flop $= 2(k-1)n$

$v_k \leftarrow v_k - (v_{1:k-1}) r_{1:k-1,k}$, flop $= 2(k-1)n$

$s_{1:k-1} \leftarrow (v_{1:k-1})^T v_k$, flop $= 2(k-1)n$

$v_k \leftarrow v_k - (v_{1:k-1}) s_{1:k-1}$, flop $= 2(k-1)n$

$r_{1:k-1,k} \leftarrow r_{1:k-1,k} + s_{1:k-1}$, flop $= k-1$

$r_{\mathrm{kk}} \leftarrow \|v_k\|_2$, flop $= 2n$

$v_k \leftarrow \left(\frac{1}{r_{\mathrm{kk}}}\right) v_k$, flop $= n$

total flop for kth time $= 8(k-1)n + 3n + k - 1 \approx 8\mathrm{nk}$

(b)

$$\text{flops} = \sum_{k=1}^{m} 8\mathrm{nk} = 8n\frac{(1+m)m}{2} \approx 4\mathrm{nm}^2$$

Q3

3.4.28

(a)

```
for m=[3 4 5 6 7 8 9 10]
    V = hilb(m);
    [Q, R] = classical_gram_schmidt(V);
    fprintf("Dimension: %d", m);
    fprintf("I - Q'Q: %e", norm(eye(m) - Q'*Q));
    fprintf("V - QR: %e", norm(V - Q*R));
end
```

```
Dimension: 3
I - Q'Q: 6.866271e-14
V - QR: 0.000000e+00
Dimension: 4
I - Q'Q: 3.451110e-11
V - QR: 3.925231e-17
Dimension: 5
I - Q'Q: 1.032190e-07
V - QR: 2.775558e-17
Dimension: 6
I - Q'Q: 5.332758e-06
V - QR: 6.206335e-17
Dimension: 7
I - Q'Q: 3.133348e-01
V - QR: 6.850849e-17
Dimension: 8
I - Q'Q: 1.032494e+00
V - QR: 7.479507e-17
Dimension: 9
I - Q'Q: 2.007029e+00
V - QR: 4.734019e-17
Dimension: 10
I - Q'Q: 2.999394e+00
V - QR: 6.515183e-17
```

(b)

```
for m=[3 4 5 6 7 8 9 10]
    V = hilb(m);
    [Q, R] = modified_gram_schmidt(V);
    fprintf("Dimension: %d", m);
    fprintf("I - Q'Q: %e", norm(eye(m) - Q'*Q));
    fprintf("V - QR: %e", norm(V - Q*R));
end
```

```
Dimension: 3
I - Q'Q: 8.612283e-15
V - QR: 0.000000e+00
Dimension: 4
I - Q'Q: 2.984511e-13
V - QR: 5.551115e-17
Dimension: 5
I - Q'Q: 8.492591e-12
V - QR: 5.551115e-17
Dimension: 6
I - Q'Q: 5.814916e-10
V - QR: 2.775558e-17
Dimension: 7
I - Q'Q: 3.829039e-09
V - QR: 5.915766e-17
Dimension: 8
I - Q'Q: 4.375355e-07
V - QR: 6.594918e-17
Dimension: 9
I - Q'Q: 1.700489e-06
V - QR: 6.457371e-17
Dimension: 10
I - Q'Q: 4.155022e-04
V - QR: 7.492457e-17
```

(c)

```
for m=[3 4 5 6 7 8 9 10]
    V = hilb(m);
    [Q, R] = reorthogonalization(V);
    fprintf("Dimension: %d", m);
    fprintf("I - Q'Q: %e", norm(eye(m) - Q'*Q));
    fprintf("V - QR: %e", norm(V - Q*R));
end
```

```
Dimension: 3
I - Q'Q: 2.022546e-16
V - QR: 5.551115e-17
Dimension: 4
I - Q'Q: 1.896146e-16
V - QR: 1.308499e-16
Dimension: 5
I - Q'Q: 3.302972e-16
V - QR: 8.591943e-17
Dimension: 6
I - Q'Q: 1.783775e-16
V - QR: 5.551115e-17
Dimension: 7
I - Q'Q: 2.869709e-16
V - QR: 8.219851e-17
Dimension: 8
I - Q'Q: 2.642104e-16
V - QR: 9.088681e-17
Dimension: 9
I - Q'Q: 3.899998e-16
V - QR: 7.889740e-17
Dimension: 10
I - Q'Q: 3.002226e-16
V - QR: 8.886774e-17
```

(d)

```
for m=[3 4 5 6 7 8 9 10]
    V = hilb(m);
    [Q, R] = qr(V, 0);
    fprintf("Dimension: %d", m);
    fprintf("I - Q'Q: %e", norm(eye(m) - Q'*Q));
    fprintf("V - QR: %e", norm(V - Q*R));
end
```

```
Dimension: 3
I - Q'Q: 2.501259e-16
V - QR: 2.580039e-16
Dimension: 4
I - Q'Q: 7.658024e-16
V - QR: 3.552512e-16
Dimension: 5
I - Q'Q: 9.583549e-16
V - QR: 6.463453e-16
Dimension: 6
I - Q'Q: 8.247379e-16
V - QR: 4.441084e-16
Dimension: 7
I - Q'Q: 6.341502e-16
V - QR: 6.219534e-16
Dimension: 8
I - Q'Q: 6.273268e-16
```

```
V - QR: 2.272497e-16
Dimension: 9
I - Q'Q: 1.126719e-15
V - QR: 3.223995e-16
Dimension: 10
I - Q'Q: 1.055787e-15
V - QR: 6.770664e-16
```

Q4

```
A = [3 6 9; 4 8 12];
```

4.1.14

$$\mathrm{AA}^T = \begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 6 & 8 \\ 9 & 12 \end{bmatrix} = \begin{bmatrix} 126 & 168 \\ 168 & 224 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 126-\lambda & 168 \\ 168 & 224-\lambda \end{bmatrix}\right) = 126*224 - 224\lambda - 126\lambda + \lambda^2 - 168*168 = (\lambda - 350)\lambda$$

$$\sigma_1 = \sqrt{350} = 5\sqrt{14}, \sigma_2 = 0, r = 1$$

$$\left(\mathrm{AA}^T - 350I\right)u_1 = 0, \begin{bmatrix} -224 & 168 \\ 168 & -126 \end{bmatrix} u_1 = 0 \Leftrightarrow u_1 = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$v_1 = \frac{1}{\sigma_1} A^T u_1 = \frac{1}{25\sqrt{14}} \begin{bmatrix} 3 & 4 \\ 6 & 8 \\ 9 & 12 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\hat{U} = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}, \hat{\Sigma} = [5\sqrt{14}], \hat{V} = \frac{1}{\sqrt{14}}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$A = \hat{U}\hat{\Sigma}\hat{V}^T = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix} [5\sqrt{14}] \frac{1}{\sqrt{14}} [1 \quad 2 \quad 3] = \begin{bmatrix} 3 \\ 4 \end{bmatrix} [1 \quad 2 \quad 3]$$

$$A = \sum_{j=1}^{r} \sigma_j u_j v_j^T = 5\sqrt{14}\left(\frac{1}{5}\right)\begin{bmatrix} 3 \\ 4 \end{bmatrix}\left(\frac{1}{\sqrt{14}}\right)[1 \quad 2 \quad 3] = \begin{bmatrix} 3 \\ 4 \end{bmatrix}[1 \quad 2 \quad 3]$$

$A = \hat{U}\hat{\Sigma}\hat{V}^T$ and $A = \sum_{j=1}^{r} \sigma_j u_j v_j^T$ are the same.

4.1.15

```
[U, S, V] = svd(A, "econ")
```

```
U = 2×2
  -6.0000e-01   8.0000e-01
  -8.0000e-01  -6.0000e-01
S = 2×2
```

```
   1.8708e+01            0
            0   1.1038e-15
V = 3×2
  -2.6726e-01   9.5141e-01
  -5.3452e-01  -2.7844e-01
  -8.0178e-01  -1.3151e-01
```

Q5

4.2.20

```
A = randn(8, 4);
A(:, 5:6) = A(:, 1:2) + A(:, 3:4);
[Q, R] = qr(randn(6));
A = A*Q;
```

(a)

```
A
```

```
A = 8×6
   1.8006e+00  -1.3827e+00   7.5407e-01   1.1916e+00  -9.2131e-01   9.7806e-01
  -7.1190e-01  -8.2721e-01  -7.6783e-01   1.3887e+00   3.4733e-01   3.6096e-01
  -1.5849e+00  -5.1495e-01   3.6949e-01   1.2369e+00  -8.0920e-01   4.4398e-01
  -9.7515e-01   1.2749e+00  -4.2530e-01  -1.4033e+00  -2.8775e-01  -2.7467e-01
  -3.7988e+00   3.1514e-01  -7.6148e-01   8.4647e-01  -1.6337e+00   8.3545e-01
  -3.3808e-01   4.8169e-01  -1.0965e+00  -4.8740e-01   1.1813e+00  -5.2930e-01
   1.2723e+00  -5.3754e-01   1.5786e+00   2.8530e-01  -2.1017e-01  -1.2903e-01
   1.6145e+00  -3.4943e-01  -1.1445e+00  -3.1882e-03   2.3296e+00  -7.0652e-01
```

I can't tell A has rank 4 by looking at it.

(b)

```
format short e
svd(A)
```

```
ans = 6×1
   5.6870e+00
   4.2503e+00
   2.9333e+00
   1.1130e+00
   8.5976e-16
   1.7753e-16
```

4 of the singular values are "large", 2 of the singular values "tiny".

(c)

```
rank(A)
```

```
ans = 
     4
```

(d)

```
rank(A, 1e-17)
```

```
ans = 
     6
```

Q6

4.3.4

$$\Sigma = \begin{bmatrix} \widehat{\Sigma} & 0 \\ 0 & 0 \end{bmatrix}, \widehat{\Sigma} = \operatorname{diag}\{\sigma_1, \cdots, \sigma_r\}$$

$$\widehat{\Sigma}^{-1} = \operatorname{diag}\left\{\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \cdots, \frac{1}{\sigma_r}\right\}$$

Since $\begin{bmatrix} \widehat{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \sigma_1 & & & & & 0 \\ & \ddots & & & & \\ & & \sigma_n & & & \\ & & & 0 & & \\ & & & & \ddots & \\ 0 & & & & & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1} & & & & & 0 \\ & \ddots & & & & \\ & & \frac{1}{\sigma_r} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ 0 & & & & & 0 \end{bmatrix} = \begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ 0 & & & & & 0 \end{bmatrix}$

Let $\Sigma v_i = \sigma_i u_i$, if $i = r+1 \cdots m$, $\Sigma v_i = 0$, then $\begin{bmatrix} \widehat{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} v_i = \sigma_i u_i,\ u_i = \frac{1}{\sigma_i} \begin{bmatrix} \widehat{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} v_i$

then $\begin{bmatrix} \widehat{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix} u_i = \begin{bmatrix} \widehat{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{\sigma_i} \begin{bmatrix} \widehat{\Sigma} & 0 \\ 0 & 0 \end{bmatrix} v_i = \frac{1}{\sigma_i} \begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ 0 & & & & & 0 \end{bmatrix} v_i = \frac{1}{\sigma_i} v_i$, if $i = r+1 \cdots n$, $\begin{bmatrix} \widehat{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix} u_i = 0$

So if $\Sigma v_i = \sigma_i u_i$, then $\begin{bmatrix} \widehat{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix} u_i = \frac{1}{\sigma_i} v_i$

So $\Sigma^{\dagger} = \begin{bmatrix} \widehat{\Sigma}^{-1} & 0 \\ 0 & 0 \end{bmatrix}$

4.3.8

$$A = \sum_{i=1}^{r} \sigma_i u_i v_i^T$$

$$A^\dagger = \left(U\Sigma V^T\right)^\dagger = \left(V^T\right)^\dagger \Sigma^\dagger U^\dagger = V\Sigma^\dagger U^T$$

$$= \begin{bmatrix} u_1 & \cdots & u_r & u_{r+1} & \cdots & u_n \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1} & & & & & 0 \\ & \ddots & & & & \\ & & \frac{1}{\sigma_r} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ 0 & & & & & 0 \end{bmatrix} \begin{bmatrix} u_1^T \\ \vdots \\ u_r^T \\ u_{r+1}^T \\ \vdots \\ u_n^T \end{bmatrix} = \sum_{i=1}^{r} \sigma_i^{-1} v_i u_i^T + \sum_{r+1}^{n} 0 * v_i u_i^T = \sum_{i=1}^{r} \sigma_i^{-1} v_i u_i^T$$

Q7

4.3.9

```
A = [1 2; 2 4; 3 6];
b = [1;1;1];
```

(a)

$$A^T A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 14 & 28 \\ 28 & 56 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 14-\lambda & 28 \\ 28 & 56-\lambda \end{bmatrix}\right) = (14-\lambda)(56-\lambda) - 28^2 = (\lambda - 70)\lambda$$

$$\sigma_1 = \sqrt{70}, \sigma_2 = 0, r = 1$$

$$\left(A^T A - 70I\right) v_1 = 0, \begin{bmatrix} -56 & 28 \\ 28 & -14 \end{bmatrix} v_1 = 0 \Leftrightarrow v_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$u_1 = \frac{1}{\sigma_1} \mathrm{Av}_1 = \frac{1}{\sqrt{70}} \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{\sqrt{14}}{70} \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}, \text{ normalize } u_1 = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\hat{U} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \hat{\Sigma} = \left[\sqrt{70}\right], \hat{V} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$A = \hat{U}\hat{\Sigma}\hat{V}^T = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \left[\sqrt{70}\right] \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$A = \sum_{j=1}^{r} \sigma_j u_j v_j^T = \sqrt{70} \left(\frac{1}{\sqrt{14}}\right) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \left(\frac{1}{\sqrt{5}}\right) \begin{bmatrix} 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix}$$

```
[U, S, V] = svd(A, "econ")
```

```
U = 3×2
  -2.6726e-01   9.5141e-01
  -5.3452e-01  -2.7844e-01
  -8.0178e-01  -1.3151e-01
V = 2×2
  -4.4721e-01   8.9443e-01
  -8.9443e-01  -4.4721e-01
```

(b)

$$A^{\dagger} = V\Sigma^{\dagger}U^{T} = \frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\end{bmatrix}\left[\frac{1}{\sqrt{70}}\right]\left(\frac{1}{\sqrt{14}}\right)\begin{bmatrix}1 & 2 & 3\end{bmatrix} = \frac{1}{70}\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\end{bmatrix}$$

$$A^{\dagger} = \sum_{j=1}^{r}\frac{1}{\sigma_j}v_j u_j^T = \frac{1}{\sqrt{70}}\left(\frac{1}{\sqrt{5}}\right)\begin{bmatrix}1\\2\end{bmatrix}\left(\frac{1}{\sqrt{14}}\right)\begin{bmatrix}1 & 2 & 3\end{bmatrix} = \frac{1}{70}\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\end{bmatrix}$$

```
pinv(A)
```

```
ans = 2×3
   1.4286e-02   2.8571e-02   4.2857e-02
   2.8571e-02   5.7143e-02   8.5714e-02
```

(c)

$$\min\|b - \mathrm{Ax}\|_2 = \|b - \mathrm{AA}^{\dagger}b\|_2 = \|(I - \mathrm{AA}^{\dagger})b\|_2 = \left\|\left(\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\2 & 4\\3 & 6\end{bmatrix}\frac{1}{70}\begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\end{bmatrix}\right)\begin{bmatrix}1\\1\\1\end{bmatrix}\right\|_2$$

$$= \left\|\left(\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} - \frac{1}{70}\begin{bmatrix}5 & 10 & 15\\10 & 20 & 30\\15 & 30 & 45\end{bmatrix}\right)\begin{bmatrix}1\\1\\1\end{bmatrix}\right\|_2 = \left\|\begin{bmatrix}\frac{13}{14} & -\frac{1}{7} & -\frac{3}{14}\\-\frac{1}{7} & \frac{5}{7} & -\frac{3}{7}\\-\frac{3}{14} & -\frac{3}{7} & \frac{5}{14}\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}\right\|_2 = \left\|\begin{bmatrix}\frac{4}{7}\\\frac{1}{7}\\-\frac{2}{7}\end{bmatrix}\right\|_2$$

$$= \sqrt{\left(\frac{4}{7}\right)^2 + \left(\frac{1}{7}\right)^2 + \left(-\frac{2}{7}\right)^2} = \sqrt{\frac{3}{7}}$$

```
norm(b-A*pinv(A)*b)
```

```
ans =
   6.5465e-01
```

(d)

$$\text{Let } \mathrm{N}(A) = \begin{bmatrix}n_1\\n_2\end{bmatrix}$$

$$\begin{bmatrix}1 & 2\\2 & 4\\3 & 6\end{bmatrix}\begin{bmatrix}n_1\\n_2\end{bmatrix} = 0 \Leftrightarrow n_1 + 2n_2 = 0 \Leftrightarrow n_1 = -2n_2$$

$$\mathrm{N}(A) = a\begin{bmatrix} 2 \\ -1 \end{bmatrix}, a \in R - \{0\}$$

(e)

$$x = A^{\dagger}b = \frac{1}{70}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{70}\begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

```
pinv(A)*b
```

```
ans = 2×1
   8.5714e-02
   1.7143e-01
```

Q8

4.3.10

```
A = [1 2 3; 2 4 6];
pinv(A)
```

```
ans = 3×2
   1.4286e-02   2.8571e-02
   2.8571e-02   5.7143e-02
   4.2857e-02   8.5714e-02
```

```
svd(A*pinv(A))
```

```
ans = 2×1
   1.0000e+00
   4.9252e-17
```

```
svd(pinv(A)*A)
```

```
ans = 3×1
   1.0000e+00
   9.2332e-17
   2.4978e-17
```

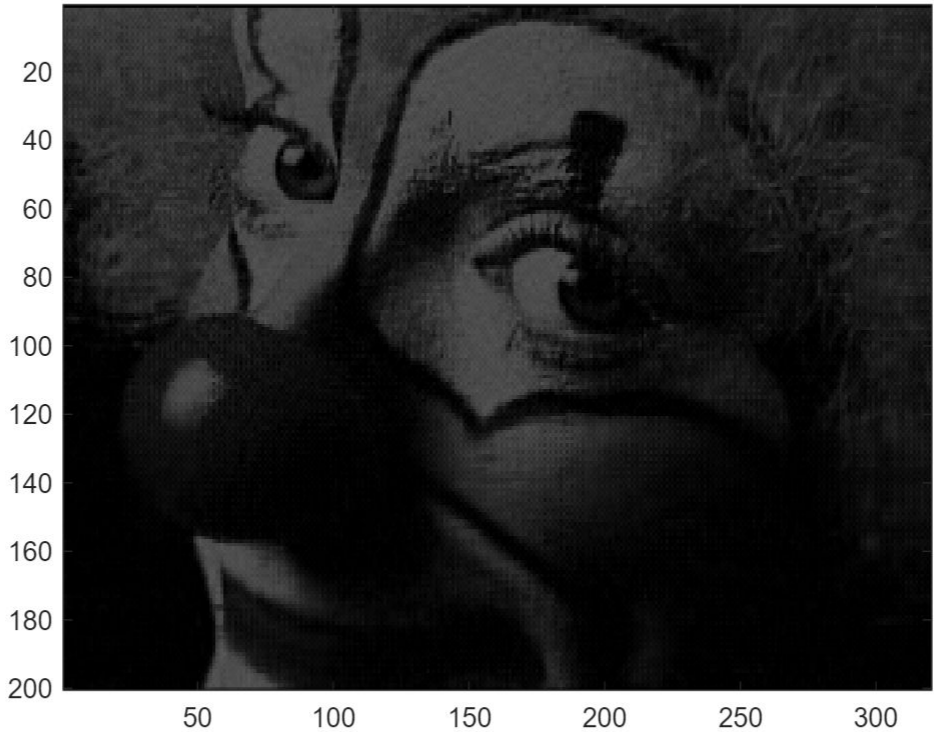
Since A has a rank of 1, $\mathrm{AA}^{\dagger}$ or $A^{\dagger}A$ has a rank of 1.

So each of them has only one "large" singular value, which is 1, other singular values are very small.

Q9

```
load('clown.mat');
image(X);
colormap("gray"); axis off;
```

```
[U, S, V] = svd(X);
n = 50;
Xk = U(:, 1:n) * S(1:n, 1:n) * V(:, 1:n)';
image(Xk);
```



If we take rank of 15, it still have a decent looking.

If we take a very low rank like 3, the image will blurred.

1)

$A = \hat{U}\hat{\Sigma}\hat{V}^T$

A has has dimension nxm.

$\hat{U}$ has dimension nxr, needs to store nr numbers.

$\hat{\Sigma}$ has dimension rxr, and it is diagnal, needs to store r numbers.

$\hat{V}^T$ has dimension rxm, needs to store mr numbers.

$\mathrm{nr} + r + \mathrm{rm} = r(m + n + 1)$.

2)

Original image needs to store $n \times m = 200 \times 320 = 64000$ numbers.

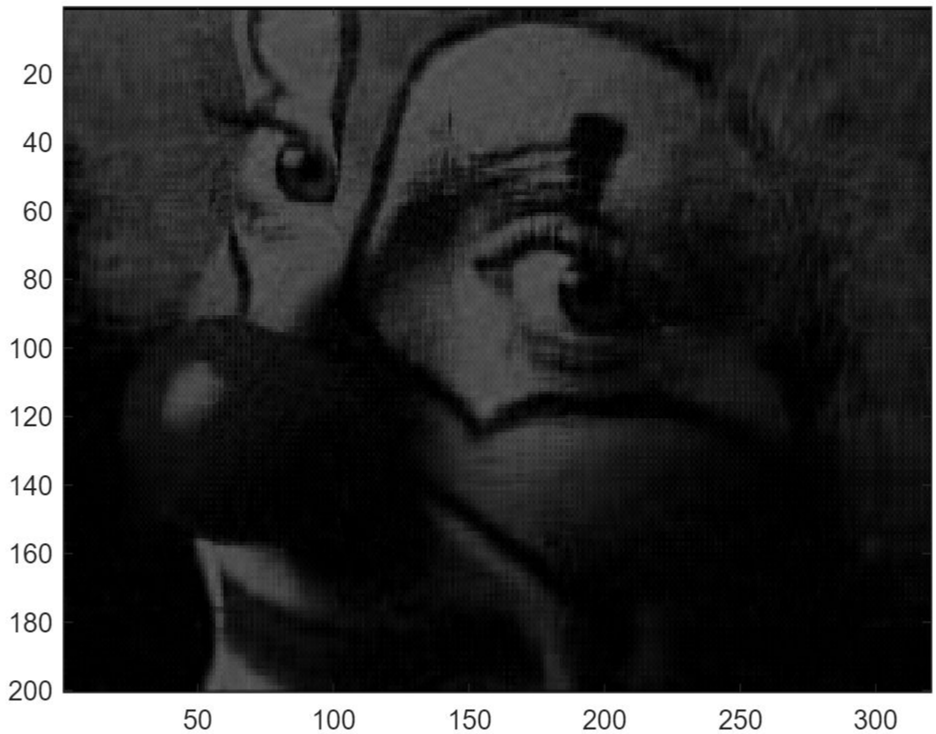
When r = 30, image needs to store $n \times m = 30(200 + 320 + 1) = 15630$ numbers.

When r = 15, image needs to store $n \times m = 15(200 + 320 + 1) = 7815$ numbers.

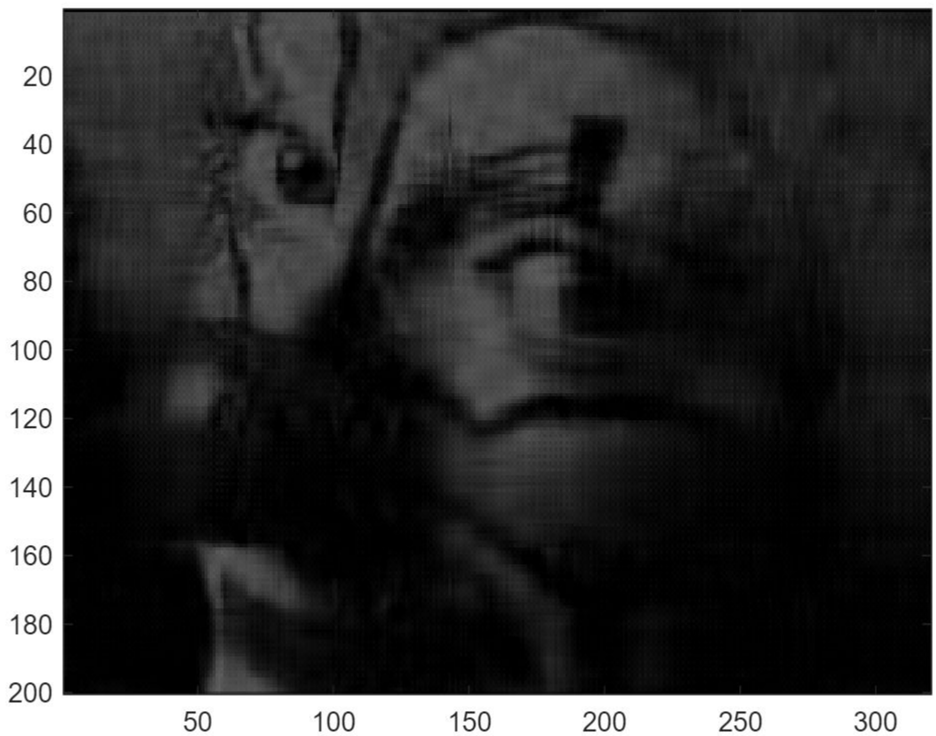
The lower-rank we choose, the lower storage we need, but the image will be more blurry.

```
n = 30;
```

```
Xk = U(:, 1:n) * S(1:n, 1:n) * V(:, 1:n)';
image(Xk);
```



```
n = 15;
Xk = U(:, 1:n) * S(1:n, 1:n) * V(:, 1:n)';
image(Xk);
```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