School of Electrical Engineering & Telecommunications

ELEC3115 - Electromagnetic Engineering PART A

Workshop 1 – week 1

1. A field vector measured in the free space is given as $\vec{E} = (0.5x + 5)\vec{a}_x$. It is suspected to be an electric field caused by a stationary charge density of a uniformly charged body of unit volume (1 m^3). How will you confirm this? If it is a static electric field, find the charge density that produces this electrostatic field?

2. The radius of a storm cloud is 1m. What should be the charge density of the cloud to produce lightening? You must state the assumptions that you make for this calculation.

3. The parallel-plate arrangement of figure 1 has mica as the dielectric materials between them. The relative permittivity of mica $\varepsilon_r = 6$.

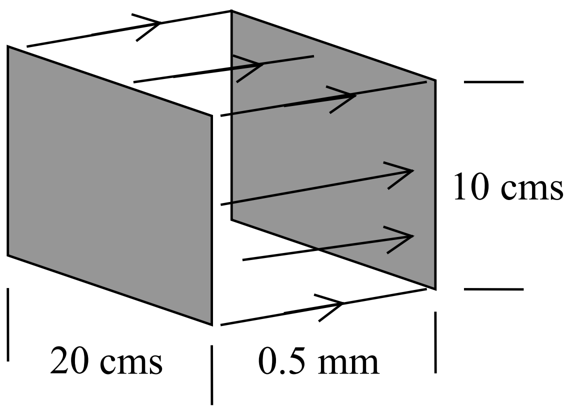

Figure 1

The left and the right plates have charges of +0.4 and –0.4 micro-Coulombs respectively.

(a) What assumptions must you have to use Gauss's law to find the electric field in the space between the plates?

(b) find the electric field ***E*** and the electric flux density ***D*** vectors between the plates using Gauss's law. How would this field vary with separation between the plates?

(c) Find the potential difference *V* between the plates. How would the potential difference vary with separation between the plates?

(d) If the dielectric is air (relative permittivity $\varepsilon_r = 1$), what will be E, D and V. Compare them with the values obtained with mica and comment.

[Ans: $\boldsymbol{E}_{mica}$ = 0.3765 MV/m, V_{mica} = 188 V, $\boldsymbol{E}_{air}$ = 2.259 MV/m, V_{air} = 1130 V]

4. The two parallel plates of figure 1 are connected to a dc voltage source of 800 volts. The left side plate is at the higher potential. For the two dielectric materials of question 3,

(a) Find the electric field ***E*** and the electric flux density ***D*** vectors between the plates using Gauss's law. How would this field vary with separation between the plates?

[$\vec{E} = \vec{a}_x 1.6\times 10^6\ V/m, \vec{D}_{air} = \vec{a}_x 14.167\times 10^{-6}\ C/m^2, \vec{D}_{mica} = \vec{a}_x 85.2\times 10^{-6}\ C/m^2$]

(b) Find the charge Q on each plate for both materials and compare. How would the Q vary with separation between the plates?

[$Q_{air} = 0.283 \times 10^{-6}\ C, Q_{mica} = 1.7 \times 10^{-6}\ C$]

5. The space between two large parallel plates, separated by a distance d =0.1mm is filled with a dielectric material of relative permittivity ε_r = 4. The plates are connected to 12 V battery. What is the polarization vector in the dielectric? [Ans: $\vec{P}_x = 3.187\vec{a}_x$ μC/m^2]

6. In the internal combustion engines of cars, the spark plug is designed to ignite the gasoline mixture. For this, a large electric field causes a breakdown of gasoline mixture to create the spark. Let's consider, in a car, due to malfunctioning, ignition voltage reduced to 10kV. The separation between the electrodes of the spark plug is 1mm and the dielectric strength of the gasoline is 15kV/mm.

   (i) When the spark plug was tested outside in air, a spark was produced but when put back into the car, engine did not run. Why did this happen?

   (ii)What must be the minimum voltage for the engine to operate?

   (iii)As an emergency measure, the separation between the electrodes (spark plug gap) could be reduced. What must be the separation gap to produce a spark with 10kV?

   [Ans: 15kV, 0.667mm.]

   (Check how a spark plug works - https://www.youtube.com/watch?v=TqQE0xkCJ8c)

7. Integrated circuit (IC) miniaturization is possible by separating closely spaced conducting layers by dielectric materials. One of the well-known limits of miniaturization is the breakdown of the dielectric layers due to applied voltage or accumulation of electrostatic charges from the surroundings. Consider this problem: two conducting strips in an IC are separated by distance d. The dielectric strength of the separator is 30kV/mm.

   (a) Calculate the smallest separation possible at an operating voltage of 5V.

   (b) If the smallest separation is 0.2 micrometer, what is the maximum potential difference (voltage) the IC can withstand without damage?

   (c) This IC got damaged when a person took it out from the package and touched the pins accidently with his woolen glove. Why did this happen?

   [Ans: 0.167micrometer, 6V.]

8. Consider two parallel identical transmission lines. The first line is located d = 10 m above ground and charged with line charge density ρ_l = 10^{-7} C/m. The second line is a = 2 m below the first line and charged negatively with ρ_l = -10^{-7} C/m.

   (a)Find an expression of **E** at a general point in space P(x, y).

$$\vec{E}_x = \frac{\rho_l}{2\pi\varepsilon_0} x \left[\frac{1}{x^2 + (d-y)^2} - \frac{1}{x^2 + (d-a-y)^2} \right] \vec{a}_x, \quad \vec{E}_y = \frac{\rho_l}{2\pi\varepsilon_0} \left[\frac{y-d}{x^2 + (d-y)^2} - \frac{y-(d-a)}{x^2 + (d-a-y)^2} \right] \vec{a}_y$$

   (b) Use the expression of (a) to calculate the magnitude of **E** at the ground level immediately below the two lines? Ans: 44.94V/m.

Useful formula and constants:

$$\varepsilon_o = 8.854 \times 10^{-12}\ C/m$$

$$\vec{\nabla} \bullet \vec{\mathrm{D}} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \vec{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \vec{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \vec{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\oint_s \vec{E} \bullet d\vec{s} = \frac{Q_{enclosed}}{\varepsilon} \quad \text{or} \quad \oint_s \vec{D} \bullet d\vec{s} = Q_{enclosed}$$

$$Q_{enclosed} = \int_v \rho dv \text{ or } \int_s \rho_s ds \text{ or } \int_l \rho_l dl$$

$$V_{ba} = -\int_a^b \vec{E} \bullet d\vec{l}$$

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E}, \quad \vec{D} = \varepsilon \vec{E}, \quad \varepsilon = \varepsilon_0 \varepsilon_r$$

For long cylindrical conductor with line charge density:

$$\vec{E} = \left| \frac{\rho_l}{2\pi\varepsilon r} \right| \vec{a}_r \ \text{V/m}, \quad V_{21} = \frac{\rho_l}{2\pi\varepsilon} \ln\left(\frac{r_1}{r_2} \right)$$