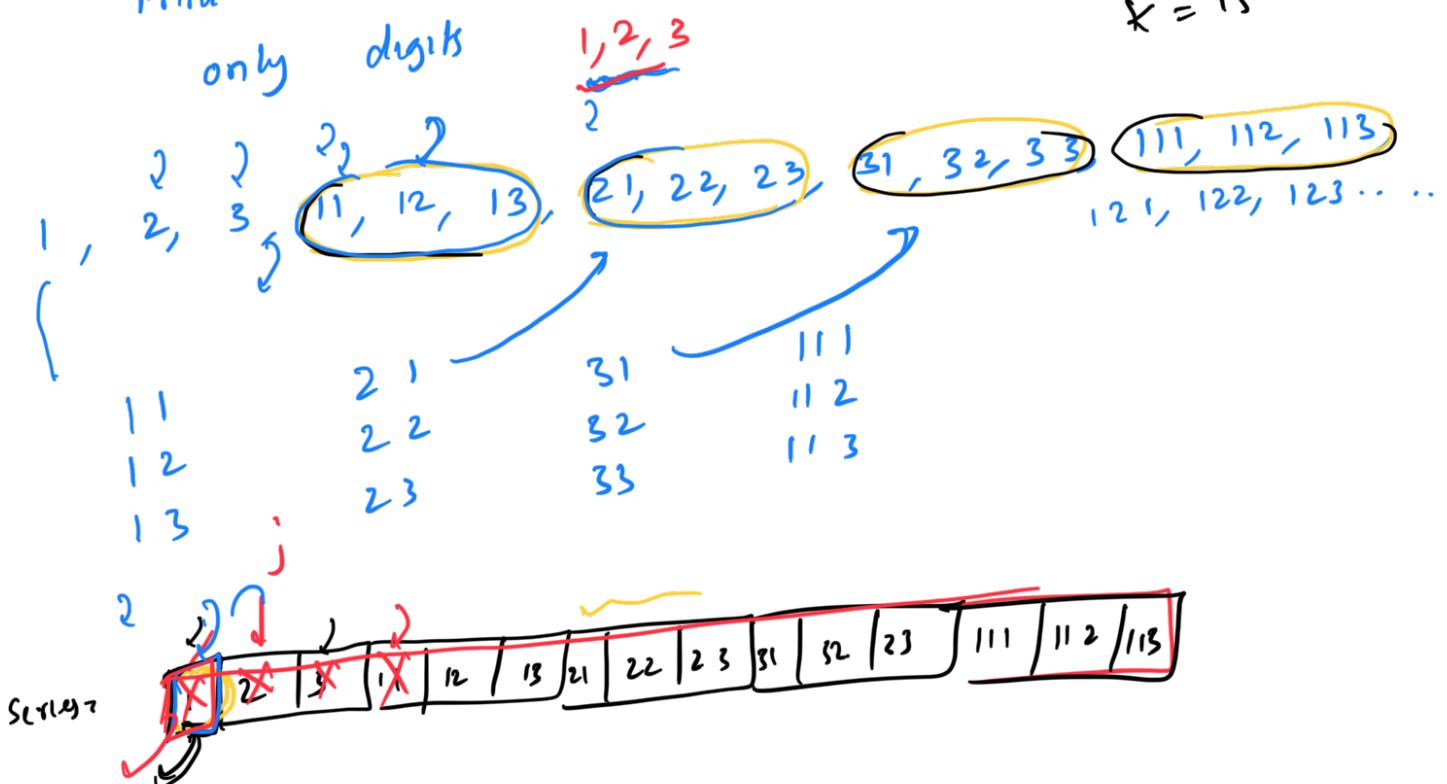


Queues

Question: Generate k numbers in sorted using
Print the first k numbers in sorted using



1 2 3 11

- Operations,
✓ 1) Deleting first element of the array $\Rightarrow O(n)$
✓ 2) Append numbers to the end $\Rightarrow O(1)$

$O(n)$

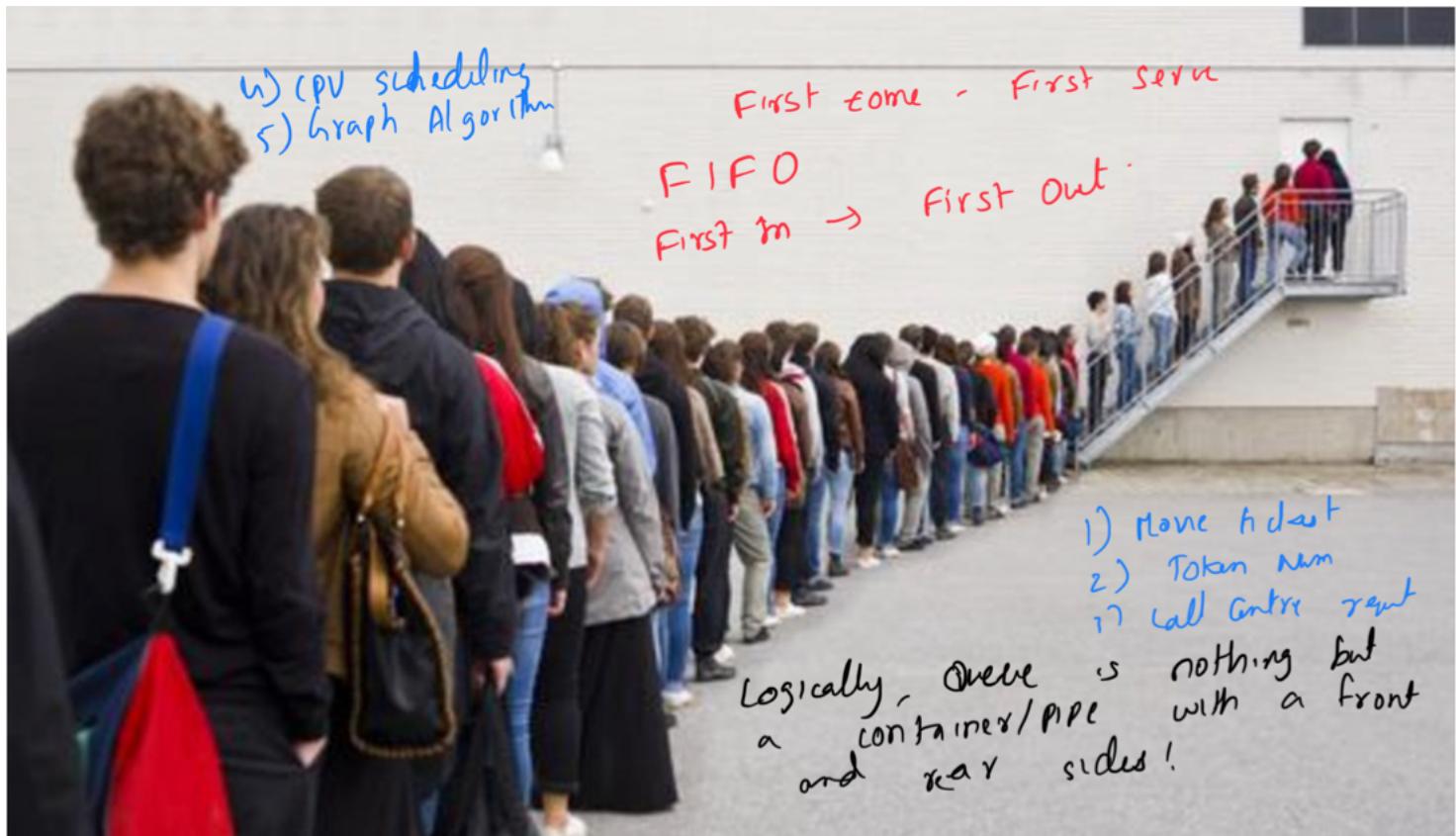
$O(1)$
 Queue DS

Does operations in just $O(1)$



... for more

Queue:
A sequence of items / people waiting to be processed.

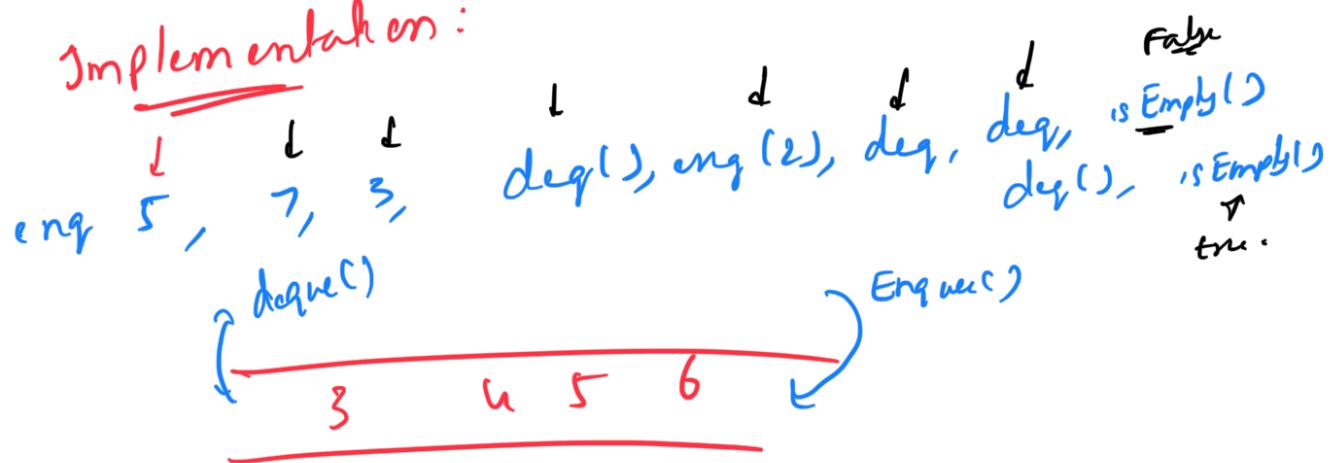


BFS: Queue

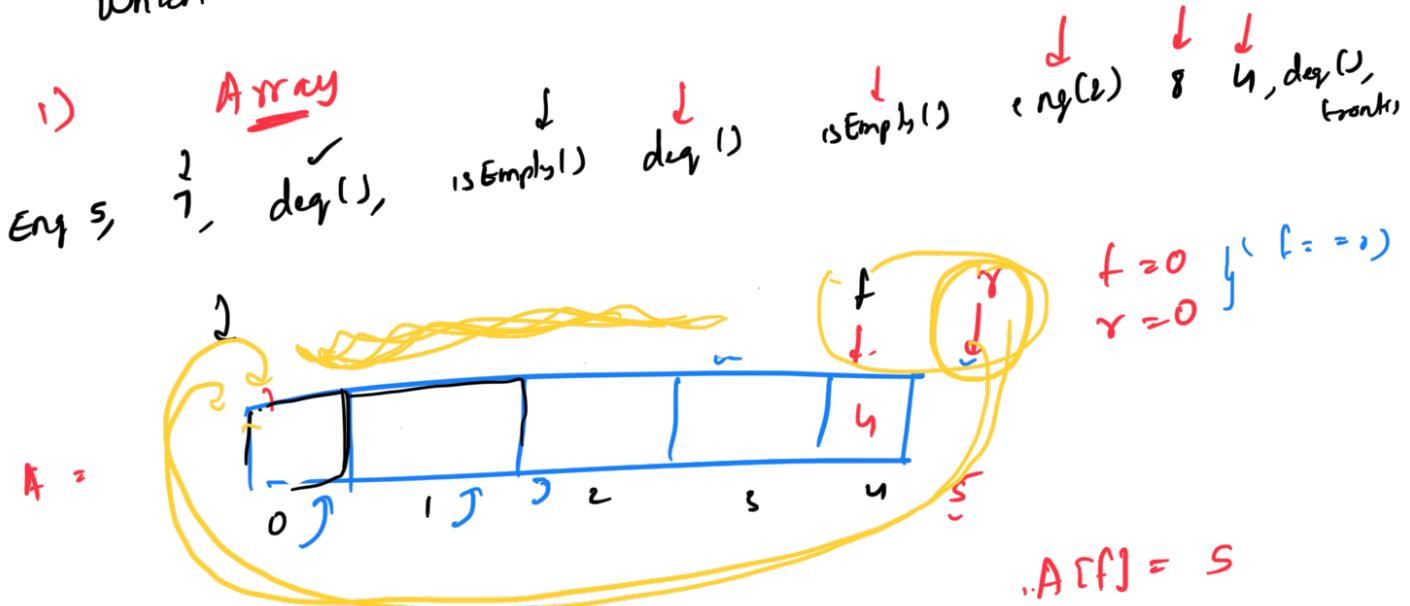
Operations

- 1) Enqueue(x) : Inserts an element at end of Queue
- 2) Dequeue() : Deletes the first element from queue
- 3) front() : Returns the first element
- 4) isEmpty() : Queue Empty or not

Implementation:



which D.S ?



Enqueue:

```

if(r == N)
    overflow;
A[r] = x;
r++;

```

$$A[4] = 5 \checkmark$$

front():

```

if(isEmpty())
    UNDERFLOW
else
    return A[f];

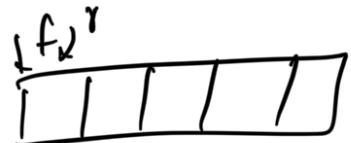
```

Dequeue:

```

ifisEmpty()
    UNDERFLOW
else
    f++;

```



Issues

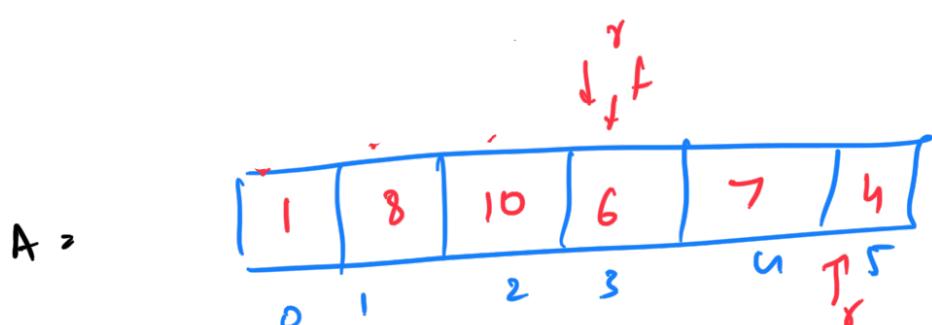
1) Unused space

Circular Queue

enq	3	2	5	6	7	
deq	deq	deq()	enq	4		
~			enq	10		
enq	1	enq 8	enq	10		

$$r = 6$$

$$r = (r + 1) \% N$$



$$\text{flag} = 0$$

->

If $f == r$ and $\text{flag} == 0$
 Queue is Empty

If $f == r$ so $\text{flag} == 1$
 Queue is FULL

```
int flag = 0;
f = 0, r = 0;
```

```
isEmply() {
    if (f == r so flag == 0)
        return true;
    else false O(1)
```

↳

```
enqueue(x) {
    if (isFull())
        overflow; O(1)
    A[r] = x;
    r = (r + 1) % N;
    if (f == r) flag = 1;
```

↳

T.C:

Implement Queue Using Stack

enq	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
4	5	3	deg	deg	enq	in	9	deg			

Visualization



Approach 1:

| | | |

```
isFull() {
    if (f == r so flag == 1)
        return true;
    return false;
O(1)
```

```
Dequeue() {
    if (isEmpty())
        return "UNDERFLOW";
    f = (f + 1) % n;
    if (f == r) flag = 0;
```

O(1)

push()
 pop()
 top()

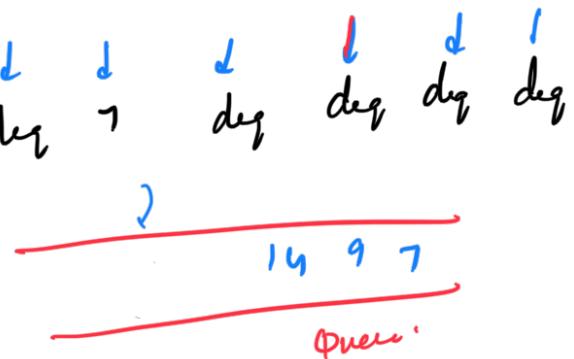


Enqueue();
push to stk1 : $O(1)$

Dequeue(); $\Rightarrow O(n)$

Approach 2:

enq h s 3 deg 14 9 deg deg deg deg
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓



Enqueue(x):
push to stack 1

Dequeue():
if (stack2 is not empty)
stack2.pop();

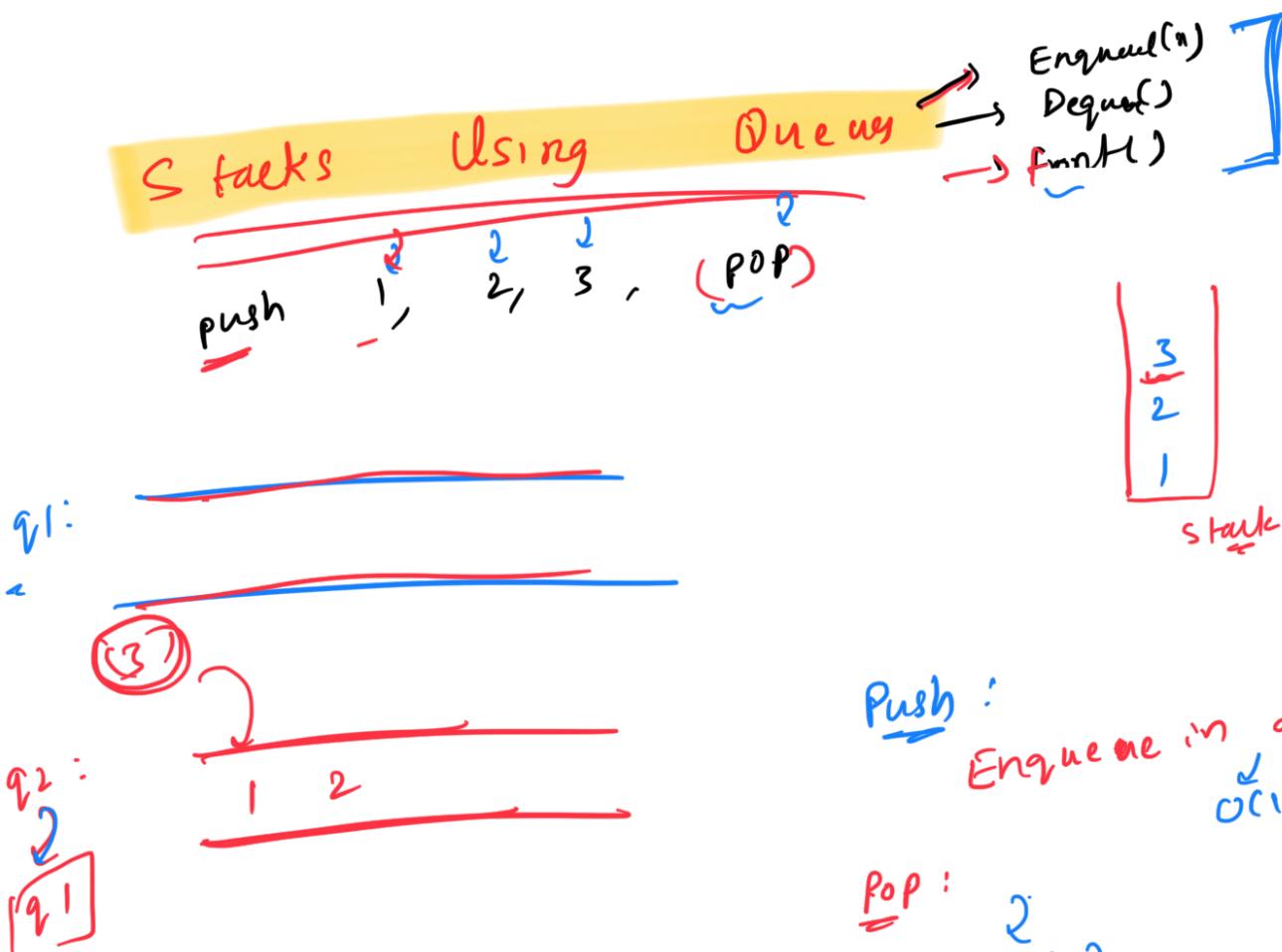
else
copy elements from stack1 to stack2
pop the top element.

\Rightarrow total T.C for all the n operations?

Consider any element
 → Pushed to stack 1
 → Pop from stack 1
 ...
 stack 2

} 4 operations

\rightarrow Push n from stack 2
 \rightarrow Pop n from stack 2
 For n elements $\Rightarrow n$ operation
 For one element \Rightarrow $O(1)$
 Amortized T.C : $O(1)$



Issues with Queue

eng 4 5 6 7

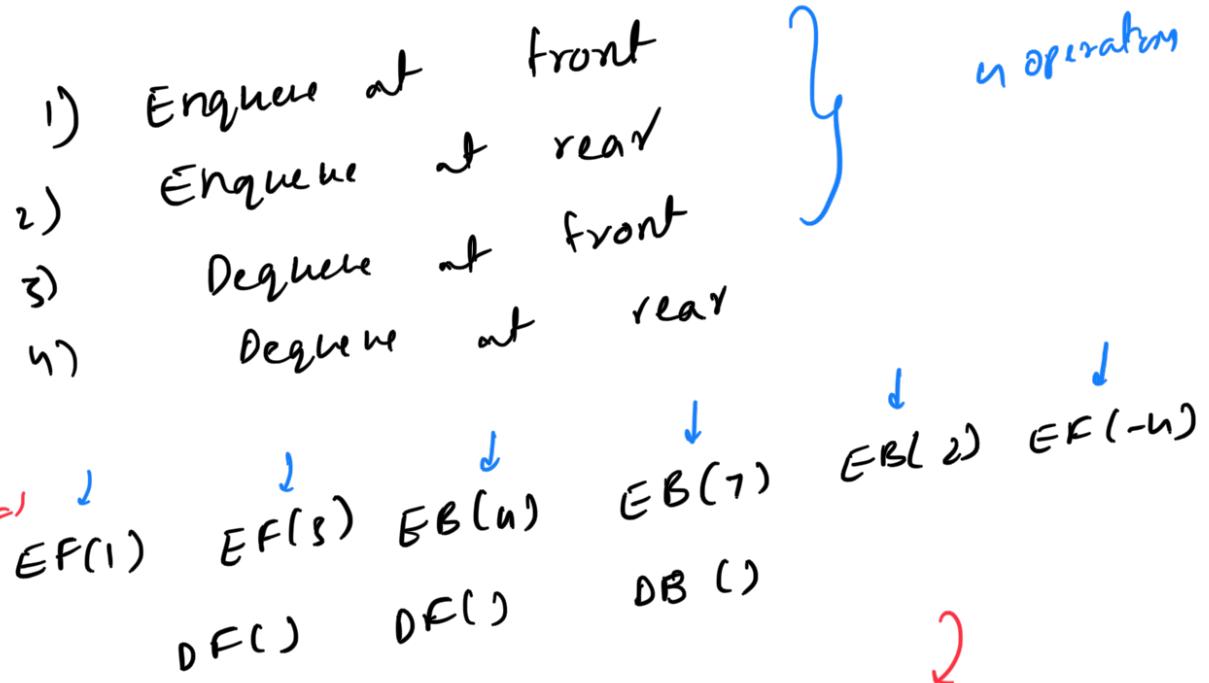
\Rightarrow Cannot undo a wrong operation

Enqueue / Dequeue

enque() deque() front()

Doubly Ended Queue

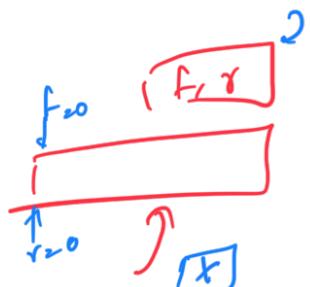
→ Operation are done at both the ends
 → Insert from both the ends
 → delete from both the ends



1 4 7

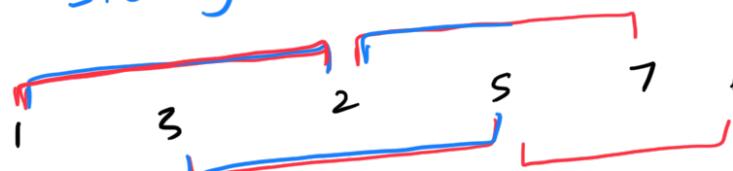
Implementation

(HW)



Question: Sliding window Max

A =



$$\begin{matrix} k = 3 \\ N = 6 \end{matrix}$$

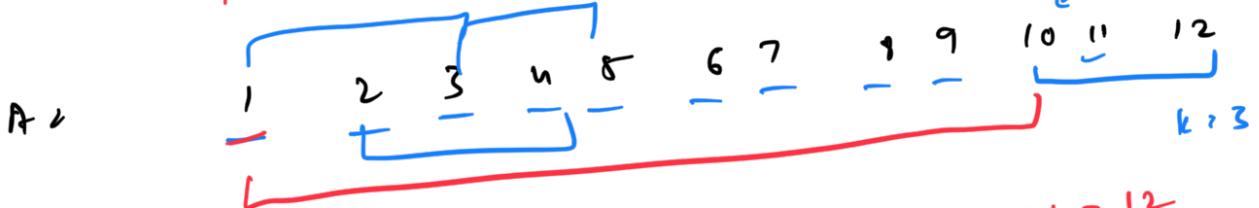
Output = [3 5 7 7]

Brute Force:

for every subarray of size K, iterate and find the max value

T.C:

No. of subarrays of size K = $\frac{N-K+1}{K}$



T.C:

$$O((n-k) \times k)$$

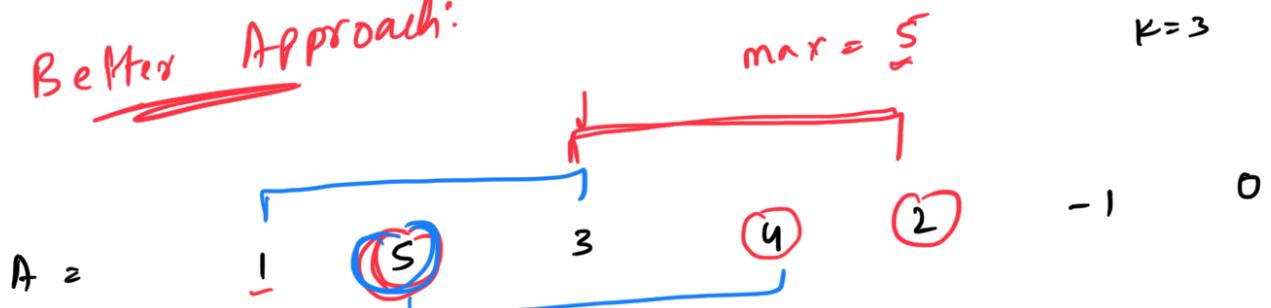
$$k = \frac{N}{2}$$

$$O\left(\frac{n}{2} \times \frac{n}{2}\right) \approx O(n^2)$$

$$\begin{aligned} N &= 12 \\ k &= 3 \\ N - k + 1 &= \\ 12 - 3 + 1 &= 10 \end{aligned}$$

$$\begin{aligned} k &= N \\ O(n - k + 1) \times k &= \\ O(N - N + 1) \times N &= O(N) \end{aligned}$$

Better Approach:



Does it make sense to have some sort of queue to get the next max element when current max goes out of window.

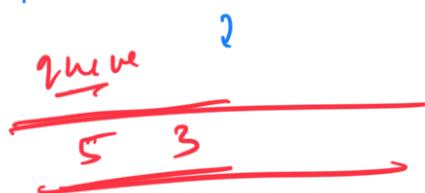
when the ...

Observations:

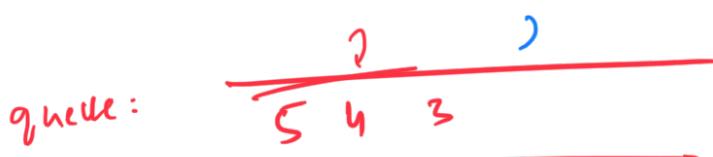


1) cannot be future in any window

2) 3 can be the max of a window future



Ex A = [1, 2, 5, 4, 3] max = 5 K = 5

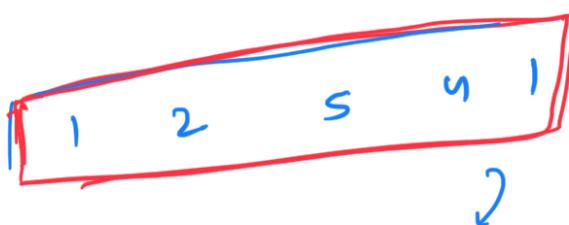


queue is in descending order

↓ A = [1, 2, 5, 3, 4] K = 5



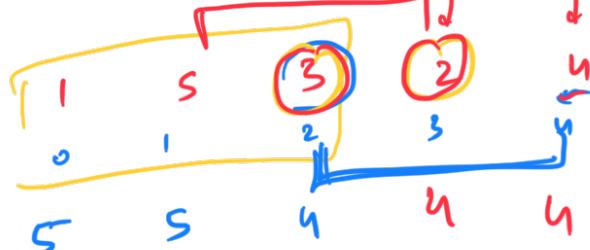
↓ A = [1, 2, 5, 4, 1] K = 5



queue:



$A =$



Output

queue:



store indices in the queue

What all operations are we doing?

1) Enqueue at back

2) Dequeue at back

3) Dequeue from front

w) Front()

back

back

front

$k = 3$

$A =$



Output

(ind, val)

queue:

(5, 5) (6, 3)

top = 1

$i = 3$

$3 - 3 + 1 = 1$

$start = i - k + 1$

$4 - 3 + 1 = 2$

... 1 2 3 4

slidingMax(int arr[], int N, int k)

vector<int> ans;

deque<int> dq;

for(i=0; i < k; i++)

[while(!dq.empty())
 dq.pop_back();
 dq.push_back(i);

ans.push(a[dq.front()])

for(i=k; i < n; i++) {

start = i - k + 1;

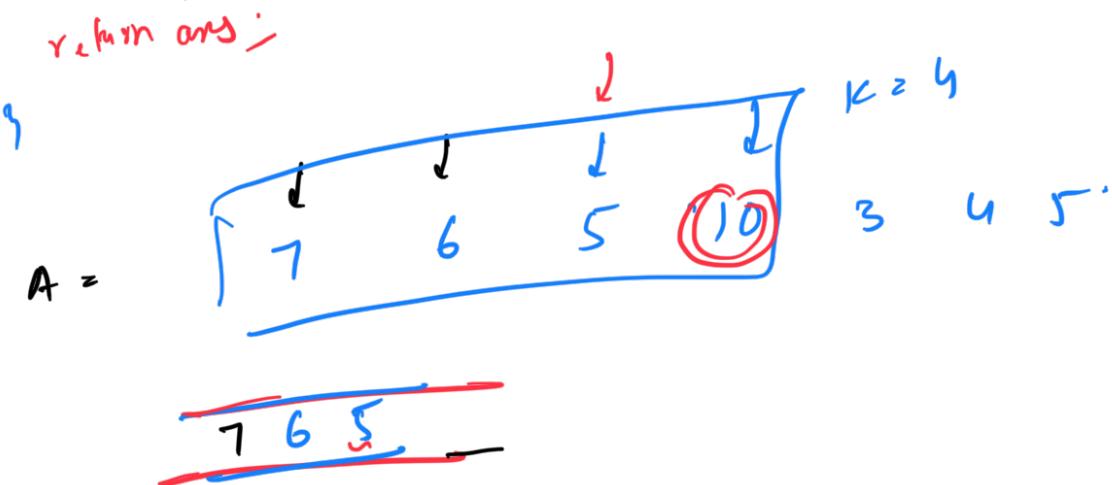
while(!dq.empty())
 if(a[i] ≥ a[dq.back()])
 dq.pop_back();

 dq.push_back(i);

 while(!dq.empty())
 if(dq.front() < start)
 dq.pop_front();

 ans.push(a[dq.front()]);

return ans;



Queue Implementation using arrays

```
MAX_SIZE = 10;
int arr[SIZE];
f = 0, r = 0;

bool isEmpty() {
    if(f == r)
        return true;
    return false;
}

void enqueue(x) {
    if(r == MAX_SIZE) {
        print('Overflow');
        return;
    }
    arr[r] = x;
    r++;
}

void dequeue() {
    if(isEmpty()) {
        print('Underflow');
        return;
    }
    f++;
}
```

Circular Queue

```
int arr[MAX_SIZE];
int l = 0, r = 0;
int flag = 0;
bool isEmpty() {
    if(f == r && flag == 0)
        return true;
    return false;
}
bool isFull() {
    if(f == r && flag == 1)
        return true;
    return false;
}
```

```
void enqueue(x) {
    if(isFull()) {
        print('Overflow');
        return;
    }
    arr[r] = x;
    r = (r + 1)%N;
    if(f == r) flag = 1;
}

void dequeue() {
    if(isEmpty()) {
        print('Underflow');
    }
    f = (f + 1)%N;
    if(f == r) flag = 0;
}
```

Queues using stacks

```
stack<int> stk1, stk2;
void enqueue(int x) {
    stk1.push(x);
}
void dequeue() {
    if(!stk2.isEmpty())
        stk2.pop();
    else{
        while(!stk1.isEmpty()){
            top = stk1.top();
            stk1.pop();
            stk2.push(top);
        }
        if(!stk2.isEmpty())
            stk2.pop();
        else
            return "Underflow"
    }
}
```

Stacks using queues

```
queue<int> q1, q2;
void push(int x){
    q1.push(x);
}

void pop(){
    while(!q1.isEmpty()){
        int f = q1.front();
        q1.pop();
        if(!q1.isEmpty())
            q2.push(f);
    }
}
```

Question : Sliding window maximum

```
slidingMax(int a[], int n, int k) {
    vector<int> ans;
    if(n < k)
        return ans;
    deque<int> dq;
    // First window of size k
    for(int i = 0 ; i < k; i++){
        while(!dq.empty() && a[i] >= A[dq.back()])
            dq.pop_back();
        dq.push_back(i);
    }
    ans.push_back(a[dq.front()]);

    for(i = k; i < n; i++){
        while(!dq.empty() && a[i] >= A[dq.back()])
            dq.pop_back();
        dq.push_back(i);

        start = i - k + 1;
        while(!dq.empty() && dq.front() < start)
            dq.pop_front();
        ans.push_back(a[dq.front()]);
    }

    return ans;
}
```