

Arrays - I

Maths
→ Prime
→ GCD

- 1) Max absolute diff
- 2) 1st missing +ve integer
- 3) Merge Overlapping Interval
- 4) Rain water Trapping

Question: Max absolute difference

$$A = \begin{matrix} & 1 & 3 & -1 \\ & 0 & 1 & 2 \end{matrix} \quad N = 3$$

maximum value $|arr[i] - arr[j]| + |i-j|$

$$\max \left(\underbrace{|arr[i] - arr[j]|}_{\substack{(-2) \\ |1-3| \\ 2+1}} + \underbrace{|i-j|}_{\substack{1 \\ |0-1| \\ 1}} \right)$$

possibility $arr[i] = 1$ $arr[j] = 3$

$$|1-3| + |0-1| = 3$$

$i = 1, j = 2$

$$|3 - (-1)| + |1-2| = 5$$

$$A = \begin{matrix} -1 & 1 & 3 \\ 2 & 0 & 1 \end{matrix}$$

$$|3 - (-1)| + |1-2| = u+1 = 5$$

$\downarrow \quad \quad \quad \downarrow$

$i \quad \quad \quad j$

$$A = \begin{matrix} & 2 & 3 & 4 & 5 & 6 & 7 \\ 0 & & & & & & \\ 1 & & & 2 & 3 & 4 & 5 \end{matrix}$$

$$|6-1| + |5-4| \\ = 5+1 = 6$$

$$|6-2| + |5-0| = 6+5 = 11$$

Brute force:

consider all pairs of no. s

$$f(i, j) = |\text{arr}[i] - \text{arr}[j]| + |i - j|$$

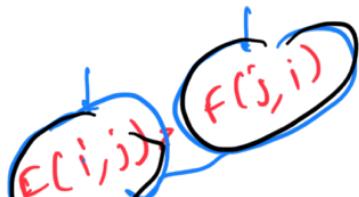
$$\begin{cases} f(i, i) = 0 \\ f(i, j) = f(j, i) \\ f(i, j) > f(i, k) \end{cases}$$

i	j	$ i-j + \text{arr}[i] - \text{arr}[j] $
0	0	$ 1-0 + 0-0 = 0$
0	1	$ 3-0 + 1-0 = 3$
0	2	$ 1-0 + 2-0 = 3$
1	0	$ 3-1 + 1-0 = 3$
1	1	$ 3-3 + 1-1 = 0$
1	2	$ 1-1-3 + 2-1 = 5$
2	0	$ 1-1-3 + 2-0 = 9$
2	1	$ 1-1-3 + 2-1 = 5$
2	2	$ 1-1+1 + 2-2 = 0$

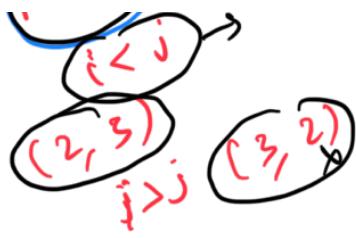
T.C: $O(n^2)$
S.C: $O(1)$

$$\text{Ans} = 5$$

```
for (i = 0 to n-1) {
    for (j = 0 to n-1) {
        ...
    }
}
```



1 1 ... 1



17

$$f(i, i) = |\text{arr}[i] - \text{arr}[i]| + |i - i| \\ 0 + 0 \quad \boxed{0}$$

$$f(i, j) = |\text{arr}[i] - \text{arr}[j]| + |i - j| \\ f(j, i) = |\text{arr}[j] - \text{arr}[i]| + |j - i|$$

 $T(i < j)$

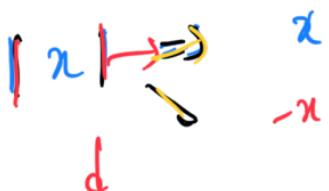
$$T(f(i, j)) = |\text{arr}[i] - \text{arr}[j]| + |i - j|$$

```
for(i=0; i<n; i++) {
    for(j=i+1; j<n; j++)
}
```

$$i = n-1 \\ j = n \\ (i < j) \\ i = n-1$$

$$n = 5 \\ |n| = 5 \\ |n| = n$$

$$T.C: O\left(\frac{n^2}{2}\right) = O(n^2) \\ S.C: O(1)$$

Observation $\boxed{O(n \log n)}$ 

If $x > 0$
If $x < 0$

$$x = -2 \\ |n| = 2 \\ = -(-2) \boxed{2}$$

$$f(i, j) = |\text{arr}[i] - \text{arr}[j]| + |i - j| \quad i < j$$

$$\max(A[i], A[j]) - \min(A[i], A[j]) \\ \boxed{\max(A[i], A[j])} \quad \boxed{\min(A[i], A[j])} \\ \boxed{A[2]} \quad \boxed{A[5]} \quad \boxed{A[0]} \quad \boxed{A[6]}$$

Case 1:

$$f(i, j) = \boxed{|\text{arr}[i] - \text{arr}[j]|} + j - i \quad (\text{arr}[i] + j) = x(i) \\ \text{Get rid of thus modulus?} \\ x[i] = \boxed{|\text{arr}[i] + j|} \\ x[j] = \boxed{|\text{arr}[j] + i|}$$

$$|\text{arr}[i] - \text{arr}[j]| \leq \text{arr}[j] - \text{arr}[i] \\ f(i, j) = \boxed{|\text{arr}[j] - \text{arr}[i]|} + j - i \quad \boxed{|\text{arr}[i] + i|} + j \\ \boxed{|\text{arr}[i] + i|} - \boxed{|\text{arr}[i] + i|} \quad \boxed{1}$$

$$\begin{aligned}
 & \text{if } arr[i] > arr[j] \\
 & \quad x_i = arr[i] + i \\
 & \quad x_j = arr[j] - j \\
 & \quad f(i, j) = \max(x_i - x_j) + i - j \\
 & \quad \text{ans} = \max(\text{case 1}, \text{case 2}) \\
 & \quad \text{max}(\text{max(case 1)} + i, \text{max(case 2)} + j)
 \end{aligned}$$

Case 1:

$$\begin{aligned}
 x_i &= arr[i] + i \\
 \max(x_j - x_i) &
 \end{aligned}$$

$$\begin{aligned}
 x &= 1, 2, 3, \dots, 5 \\
 \max(x_j - x_i) &+ i, j
 \end{aligned}$$

$$\begin{aligned}
 \text{Case 2: } y_i &= arr[i] - i \\
 \max(y_i - y_j) &+ i, j
 \end{aligned}$$

$$\begin{aligned}
 arr[i] &= 3, \quad arr[j] = 5 \\
 i &= 1, \quad j = 2 \\
 arr[i] &< arr[j]
 \end{aligned}$$

Case 1:

$$\begin{aligned}
 arr[i] &\leq arr[j] \\
 (arr[i] + i) - (arr[j] + j) &= 3 + 1 - (5 + 2) \\
 &= 4 - 7 = -3
 \end{aligned}$$

Case 2:

$$\begin{aligned}
 & (arr[j] - arr[i]) + (j - i) \\
 & (arr[i] - arr[j]) + (i - j) = 3 - 5 + 2 = -2
 \end{aligned}$$

Case 2:

C 3-522 Z -

$$(j-i) > 0$$

$$\text{Case 1: } 2 + (j-i) \\ \text{Case 2: } -2 + (j-i)$$

Case 1:

$$\left[\text{carry}[j] + j \right] - \left(\text{arr}[i] + i \right) \quad \text{max, my}$$

$$x_i = \text{arr}[i] + i$$

$$\Rightarrow \max_{i,j} (x_j - x_i) \quad \text{# } i, j$$

$$x_j = a \gamma \{j\} + j$$

$$X^{[i]} = AS^i + 1$$

Simple :

Array of numbers ↴

$$\max (\text{arr}[i] - \text{arr}^{\text{min}}[i])$$

Cox ^

$$\begin{array}{ccccccc} & & 5 & 2 & 0 & 1 \\ \downarrow & & & & & & \\ A^2 & & & & & & 187 \end{array}$$

$$A = \begin{pmatrix} 1 & 3 & -1 \end{pmatrix}$$

$$x_i = A[i] + 1$$

$$\downarrow x = \frac{1}{\sim} \approx \underline{4}$$

$$\max(x_i) = 4$$

$$m_{1,0}(x) = 1$$

Minus

Max for Case 1 : [3]

$u-1 = \boxed{3}$ for case 2
 main

Case 2:

$$(\text{arr}[i] - 1) - (\text{arr}[j] - 1)$$

$$y_i = \underbrace{\text{array}[j]}_{\max(\text{case2})} -$$

$$+ b_j$$

$$A = \begin{matrix} & 0 & 1 & 2 \\ 1 & & 3 & -1 \end{matrix}$$

$$x : \quad 1 \quad 2 \quad \rightarrow 3$$

$$\max(\gamma) = 2$$

$$\min(\gamma) \approx -3$$

$$\max - \min = 2 - (-3) \\ = [5]$$

$\max(\text{case 2}) = 5$

$$\text{Ans} = (3, 5) = \boxed{5}$$

$$|6-2| + |3-0| \\ n+s = \boxed{7}$$

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 3 & 1 \\ 6 & 1 & 3 \end{pmatrix}$$

$$y = 2^2 - 1^3$$

$$\max(x) - \min(x)$$

$g = 2 \in \boxed{7}$

$$\text{mark}(Y) = \min(Y)$$

$$\max(7, 4) = \boxed{7}$$

$$t_{\text{max}} \rightarrow \max \left(\text{arr}[j] + i \right) \quad \left[\begin{array}{l} \text{arr}[i] + j \\ \dots \end{array} \right]$$

$$x_{\text{min}} \rightarrow \min(\text{array}, i)$$

$$Y_{\text{max}} = \max_{n \in \text{array}} (array[n])$$

$$\gamma_{\min} = -\infty, \quad \gamma_{\max} = -\infty$$

$$\text{min}_2 \max\left(x_{\text{max}} - X_{\min}, y_{\text{max}} - Y_{\min} \right)$$

$x_{\min} = \text{INF}$, $x_{\max} = -\text{INF}$
 for ($i=0$; $i < n$; $i++$) {
 $x_{\max} = \max(x_{\max}, A[i] + 1)$
 $x_{\min} = \min(x_{\min}, A[i] + 1)$
 $y_{\max} = \max(y_{\max}, A[i] - 1)$
 $y_{\min} = \min(y_{\min}, A[i] - 1)$
 }
 return $\max(x_{\max} - x_{\min},$
 $y_{\max} - y_{\min})$;

T.C.: $O(n)$
 S.C.: $O(1)$

\Rightarrow

Small steps

Question: First Missing Non-Negative Integer with the $/-ve$ elements
 Given unsorted array with

$$A = \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 8 & 10 & 1 & -3 & 2 & -5 & 0 \end{matrix}$$

$$\text{Ans} = 3$$

$$A = \begin{matrix} 2 & 3 & 7 & 6 & 8 & -1 & -10 & 15 \end{matrix}$$

$$\text{Ans} = 0$$

- 1) max[array]
- 2) linear[1 array]

Brute Force:

0, 1, 2, 3

T.C.: $O(n \times n)$
 $O(n^2)$

\rightarrow for ($i=0$; $i \leq n$; $i++$) {
 check if ' i ' is there in array // or not

{ } " when
 for
 $\min(\text{ans}) = 0 \rightarrow$
 $\max(\text{ans}) \rightarrow N$
 $A = [0, 1, 2, 3, \dots, n-1]$
 $\underline{[0, n-1]}$
 $[0 \dots n-1] \rightarrow n$
 $N = 5 \rightarrow [0, 1, 2, 3, 4]$
 $N = 6 \rightarrow [0, 1, 2, 3, 4, 5, 6]$
 $\rightarrow > 6 ?$
Range: $[0, N]$
 T.C: $O(n^2)$
 S.C: $O(1)$

Approach 2:
 Use a hash set to check if element is present in array.

$\rightarrow \text{for } (i=0; i \leq n; i++) \{$
 if (i is not there in set) $\} O(1)$
 return ' $'$

$T.C: O(n \times 1) = O(n)$
 S.C: $O(n)$

$A = [-100, 0, 1, 100, \dots]$

$A = [1, 2, 3, 4, 5]$
 $6? \rightarrow [0]$
 $A = [0, 1, 2, 3, 4]$

$\{ -\infty, \infty \}$
 $\{ -99, -76, 0, 1, 1002, 105 \} \quad \boxed{\text{PS}}$
 $A = \{ -3, 0, 1, 9 \}$

Without Extra Space

$A = \begin{matrix} -5 & -3 & 0 & 1 & 2 & 8 & 10 \end{matrix}$
 \rightarrow

T.C: $O(n \log n)$

\downarrow
 $\boxed{\text{temp} = \emptyset \cup 23}$

$\boxed{\text{S.C}}$

T.C: $O(n \log n + n) = O(n \log n)$
 $\Rightarrow \boxed{O(n \log n)}$

S.C: $O(1)$

$A = \begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 2 & 2 & 3 \end{matrix}$
 \rightarrow skip all when moving index
 duplicate elements
 temp = $O(2)$

$\boxed{\text{T.C: } O(n \log n)}$
 $\boxed{\text{S.C: } O(1)}$

Efficient Approach:

ans: $[0, n]$
 Let us consider the range

array $\boxed{[0, n-1]}$
 elements only in range are present

Case 1: If all no.s in range are present
 answer = n

case 2: First missing num in $[0, n-1]$

Range: $[0, n-1]$

Hashset: $O(n)$ space

Can we do it in $O(1)$ space?

→ Let's consider the range numbers which are not in $[0, n-1]$

$N = 7$

$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$

Range: $[0, 6]$

$\textcircled{2}$

i

$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$

if $(A[i] \geq 0) \wedge (A[i] \leq n-1)$

$O(1)$

Edge Case: Duplicate Elements

Range: $[0, 3]$

$N = 4$

$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 6 & 1 \end{bmatrix}$

infinit loop

→ Swap elements equal

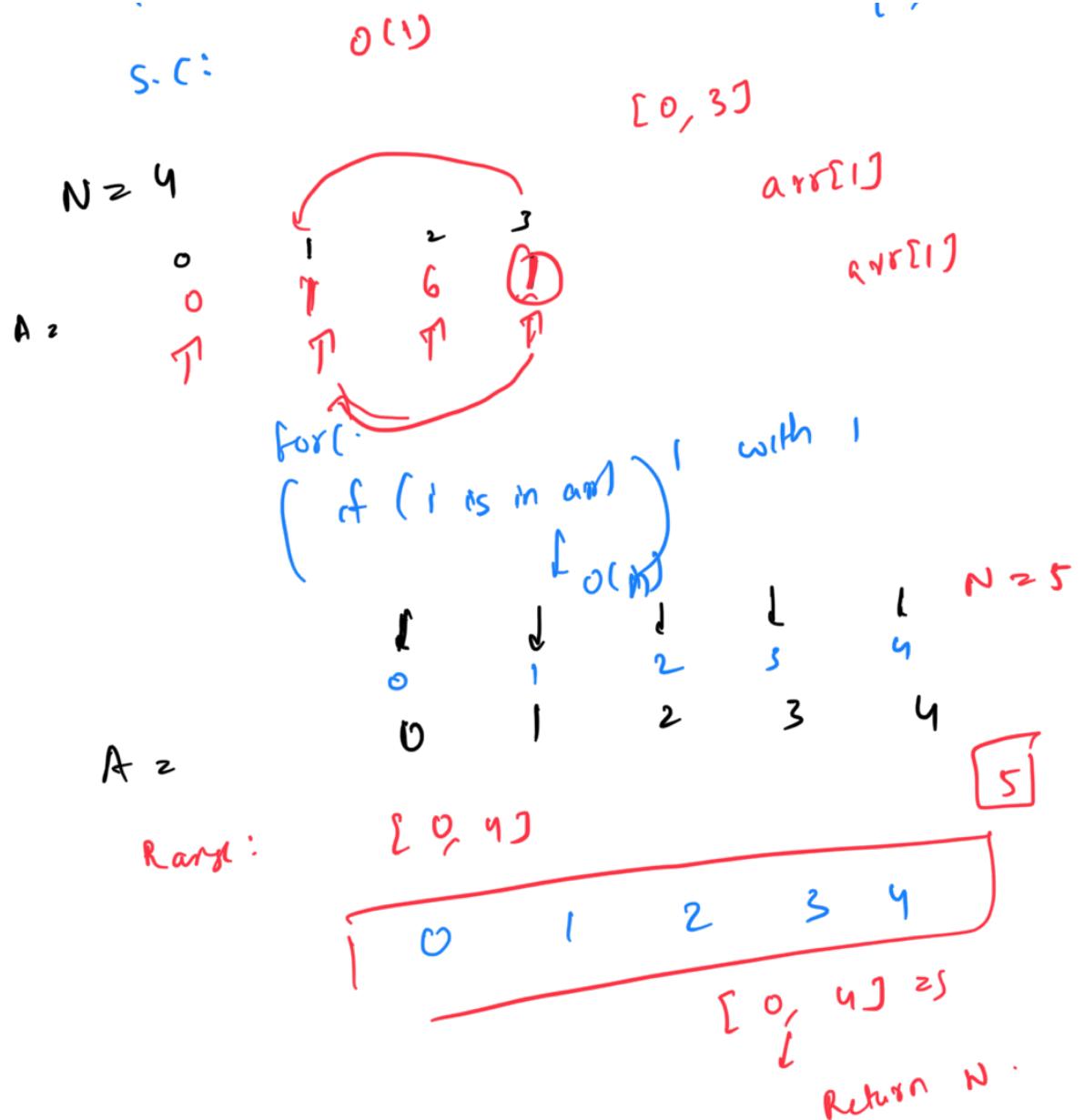
T.C: $O(n)$

only if they are not equal

$\text{if } A[i] == A[j]$

$\text{swap}(A[i], A[j])$

break



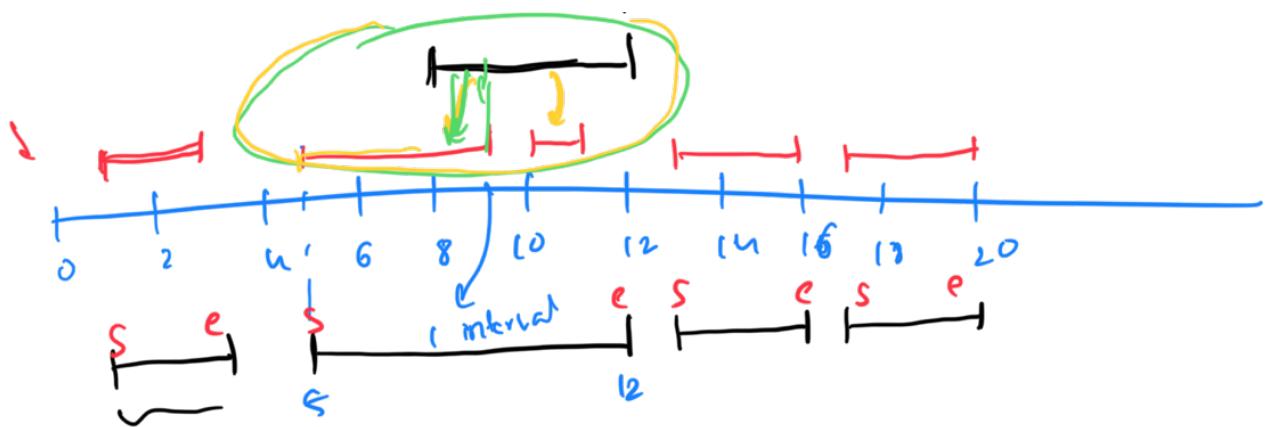
Question: Merge overlapping intervals

→ Given non-overlapping intervals in sorted order

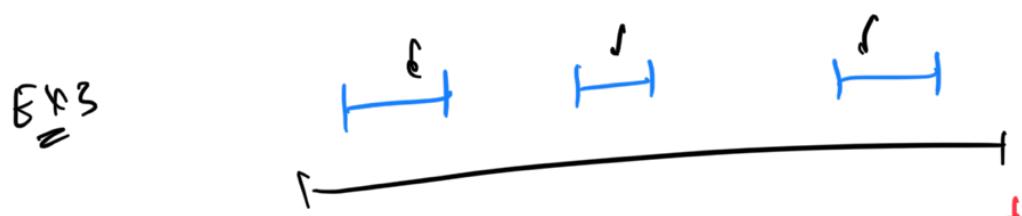
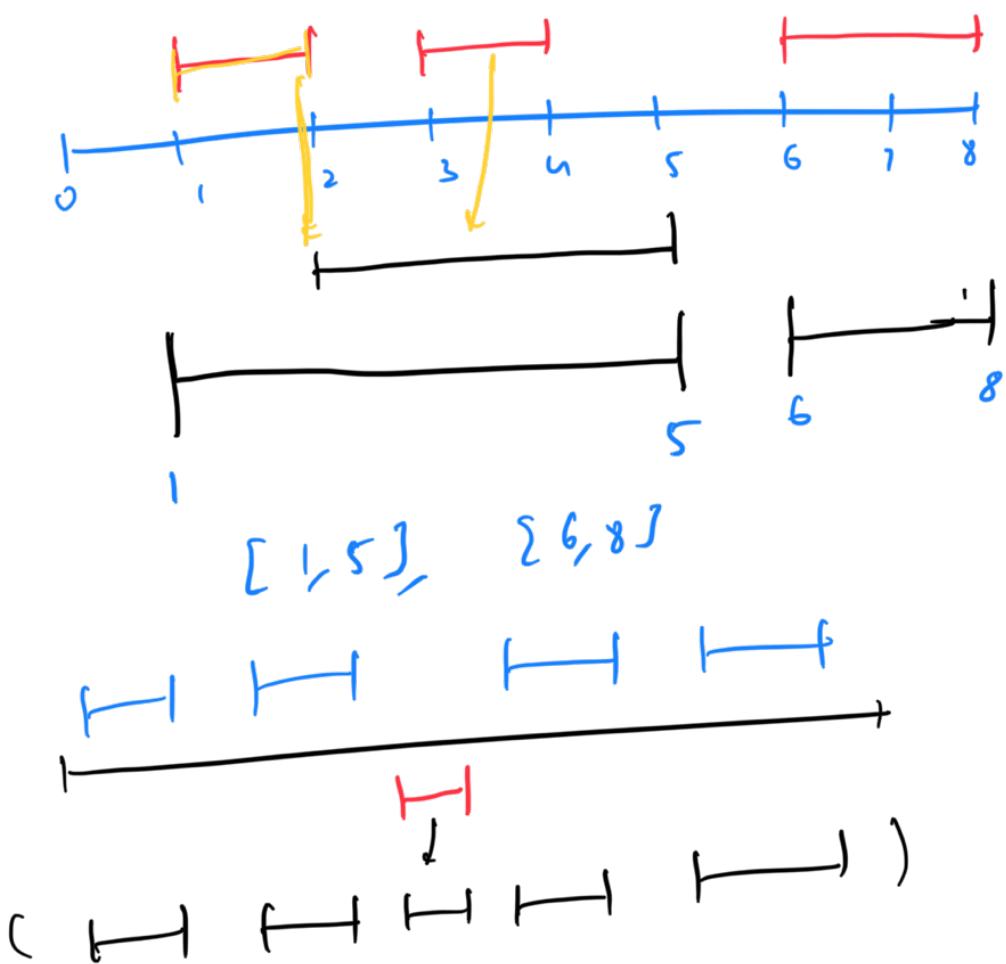
→ Given a new interval, merge this interval with existing interval

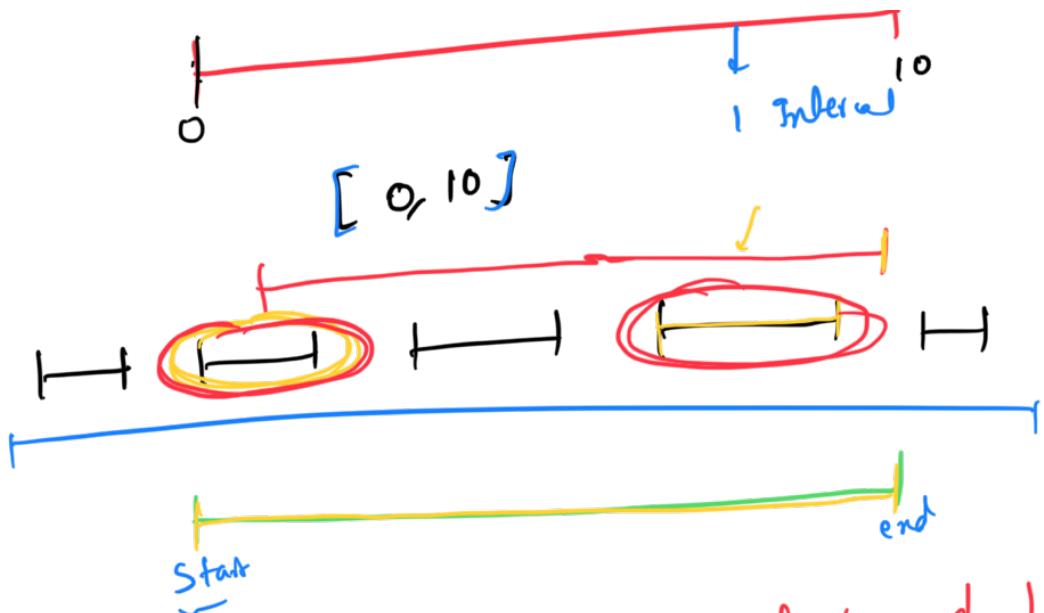
$S = \boxed{1}$
 $E = \boxed{3}$

$\boxed{8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 16 \quad 17 \quad 20} \quad [8, 12]$



$$\begin{array}{ccccccc}
 \text{Ex2} & s = & 1 & 3 & 6 \\
 & e = & 2 & 4 & 8 \\
 & \text{new interval: } & [2, 5]
 \end{array}$$



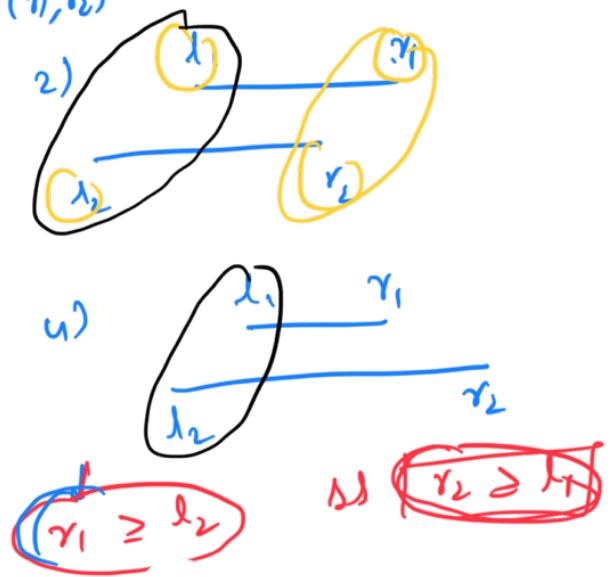
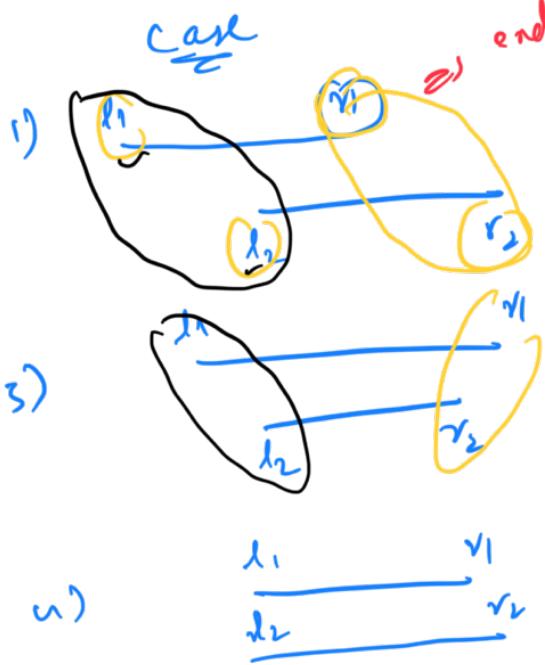


→ concerned with the first and last overlapping interval

How do we know if 2 intervals are overlapping:

$$\text{start} = \min(l_1, l_2)$$

$$\text{end} = \max(r_1, r_2)$$



→ One interval has to start before other

(l_1, l_2)

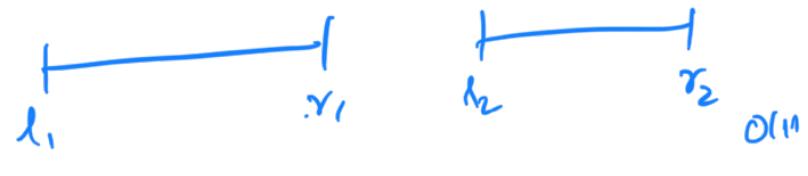
$$l_1 \leq r_1, r_2 \Rightarrow l_1 \leq r_2$$

$$l_2 \leq r_1, r_2 \Rightarrow l_2 \leq r_1$$

1) $(r_2 > l_1 \text{ and } r_1 \geq l_2) \Rightarrow \text{False}$

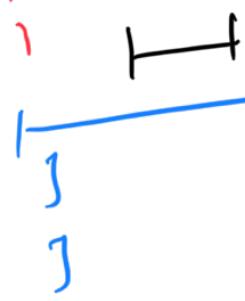
2) $(l_2 \geq l_1) \text{ or } (r_1 \geq l_2) \Rightarrow \text{True}$

~~pologn using B.S~~
 $O(n)$



$$S = \{r\}$$

$$e = \{l\}$$



first overlapping interval

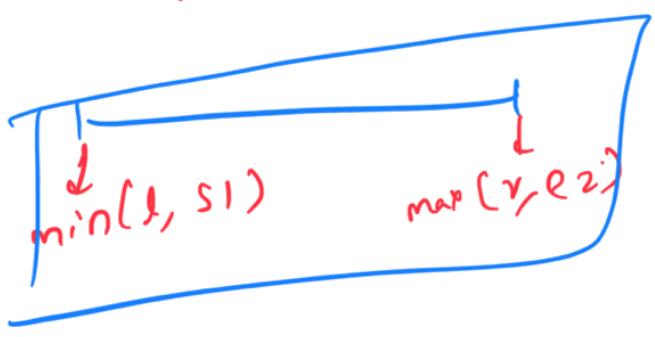
inp: {
out: {
for ($i=0$; $i < n$; $i++$) {
 if ($\text{end}[i] \geq l \text{ and } r \geq \text{start}[i]$) {
 break;
 }

↳ i^{th} interval \rightarrow the 1st overlap

for ($i=n-1$; $i \geq 0$; $i--$)
 New merged interval!

$$l, r$$

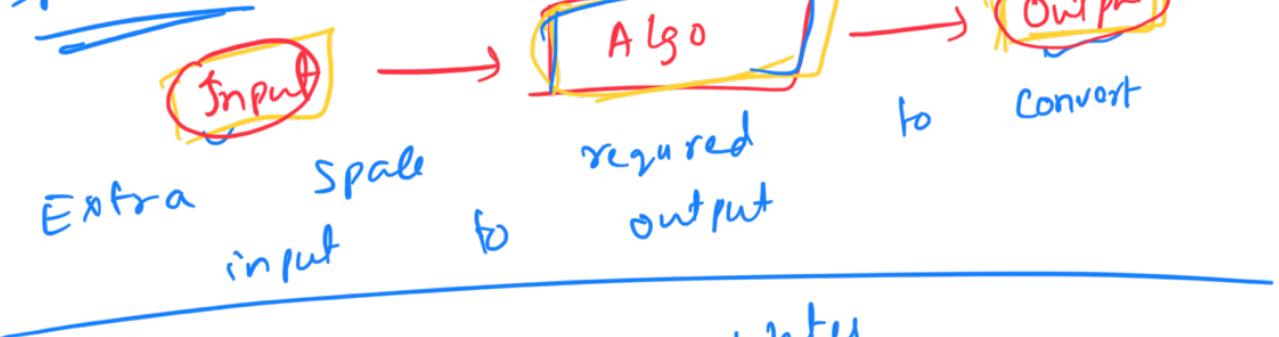
$$s_1, e_2$$



T.C

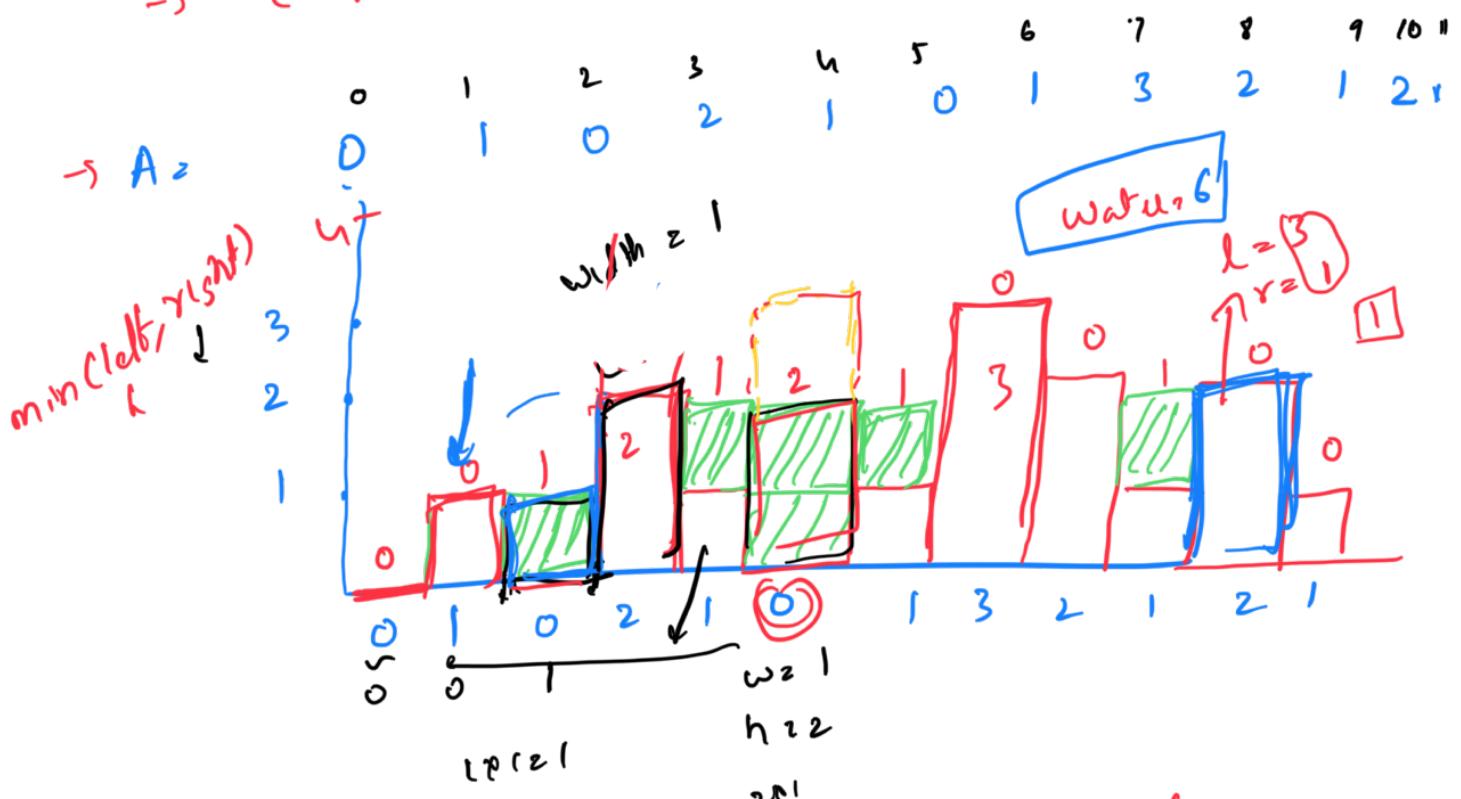
S.C

Space Complexity



Ques:

Given an array representing height of n elements each building. Compute the water trapped after raining.



$$0 + 0 + 1 + 0 + 1 + 2 + 1 + 1 = 6$$

Water stored above 1st building

$$w_{ij} = \sum w[i]$$

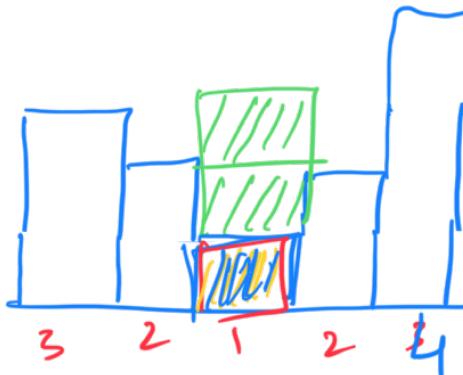
\rightarrow (Compute contribution of each element by adding them up
the array and answer) \rightarrow technique.

Brute force:

for ($i = 0$; $i < n$; $i++$)
 $\quad \quad \quad l = \text{fallest building to left}$

Any building i : $\min(l, r)$

for ($j = i$; $j \geq 0$; $j--$)
 $\quad \quad \quad L = \max(L, a[\Sigma j])$



for ($j = i$; $j < n$; $j++$)
 $\quad \quad \quad r = \max(r, a[\Sigma j])$

y

$l = 3$

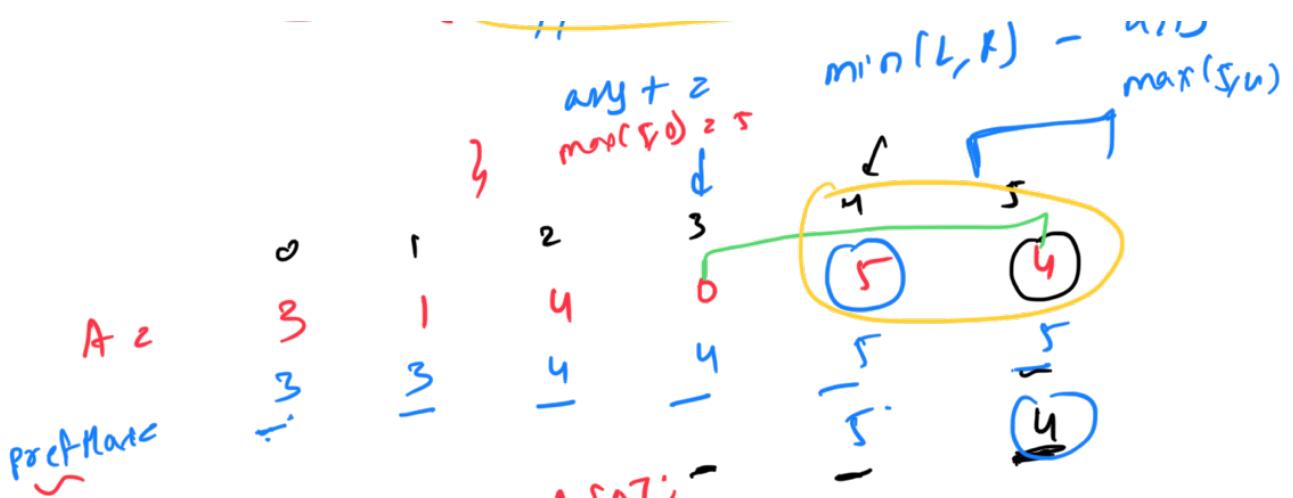
$r = 4$

$\min(4, 3) = 3$
 $\min(l, r) - a[\Sigma i]$

T.C: $O(n^2)$

for ($i = 0$; $i < n$; $i++$)
 $\quad \quad \quad$

$O(1)$ { $\begin{cases} O(n) & \text{for finding } L \\ O(n) & \text{for finding } R \\ O(n) & \text{for } \Sigma i \end{cases}$



$$\text{prefMax}[0] = A[0];$$

for ($i = 1; i < n; i++$) {

$$\text{prefMax}[i] = \max(\text{prefMax}[i-1], A[i])$$

suffix Max

T.C: $O(n)$

S.C: $O(n)$

[
prefMax & suffMax

$$\text{suffMax}[n-1] = \text{ans}[n-1];$$

for ($j = n-2; j \geq 0; j--$)

$$\text{suffMax}[j] = \max(\text{suffMax}[j+1], A[j])$$

any = 0;
for ($i = 0; i < n; i++$) {

$L = \text{prefMax}[i]$

$R = \text{suffMax}[i]$

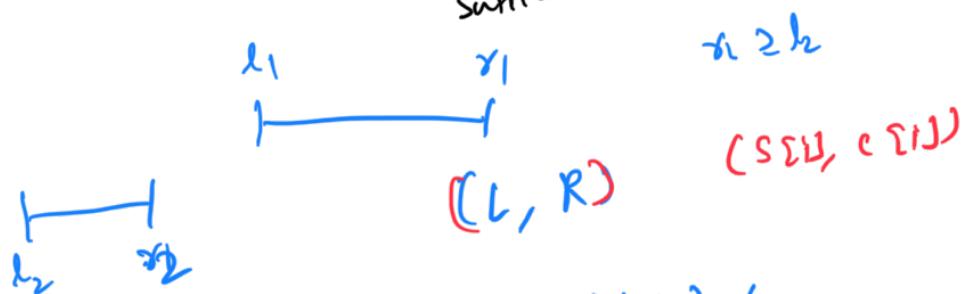
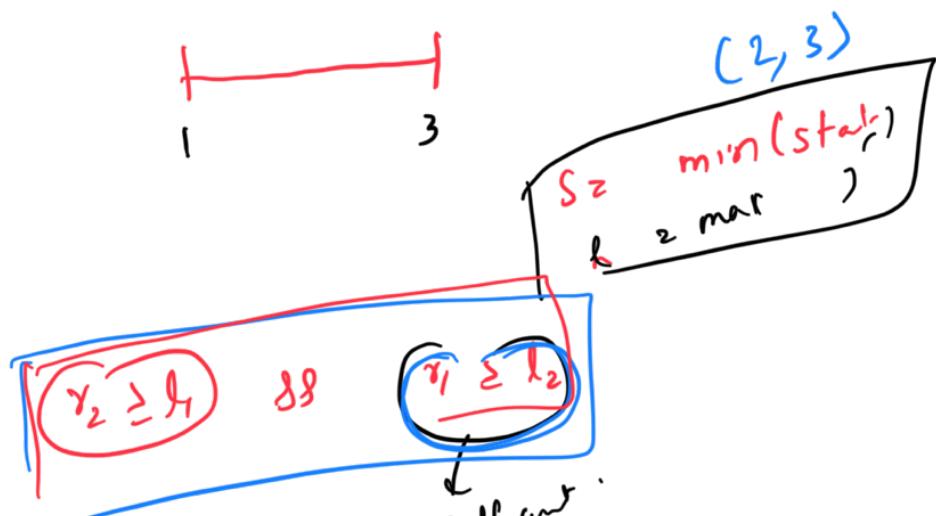
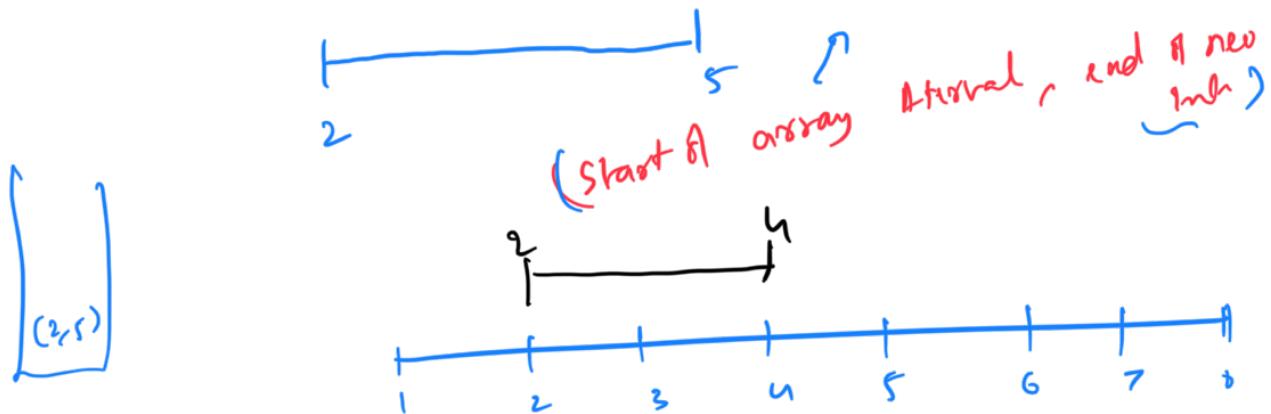
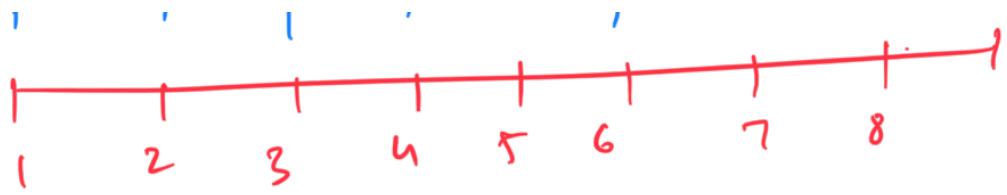
any = ans +

$\boxed{\min(L, R) - \text{ans}}$

return any;

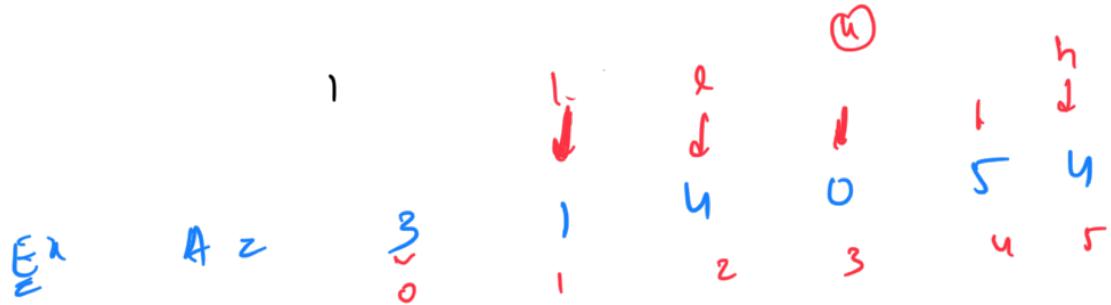
1 2 3 4

6 7 8 9



```
for (i = 0; i < n; i++) {
    if (e[i] >= L) {
        if (y2 >= L) {
            first_overlap = i;
            break;
        }
    }
}
```

```
for (i = n - 1; i >= 0; i--) {
```



while ($low \leq high$) {

$low = 0$

$high = n-1$

T.S.C: O(1)

Ans

Max Absolute Difference

```
X_max = INT_MIN, X_min = INT_MAX;
Y_max = INT_MIN, Y_min = INT_MAX;

for(i = 0; i < n; i++){
    X_max = max(X_max, A[i] + i);
    X_min = min(X_min, A[i] + i);
    Y_max = max(Y_max, A[i] - i);
    Y_min = min(Y_min, A[i] - i);
}
ans = max(X_max - X_min, Y_max, Y_min)
```

First Missing Positive Integer

```
i = 0;
while(i < n) {
    if(arr[i] >= 0 && arr[i] <= N-1) {
        if(arr[i] != arr[arr[i]]) {
            temp = arr[i];
            arr[i] = arr[arr[i]];
            arr[arr[i]] = temp;
        }
    }
    if(arr[i] not in range [0, N-1])
        i++;
    // else we'll again swap this number to its index
}
// Return the first instance where arr[i] != i
for(int i = 0; i < n; i++)
    if(arr[i] != i)
        return i;
// Return n if we have all numbers in [0, n-1]
return n;
```

Rainwater trapping

```
// Prefix Max
int L[];
L[0] = A[0]
for(int i = 1; i < n; i++){
    L[i] = max(L[i-1], A[i])
}

// Suffix Max
int R[];
R[n-1] = A[n-1]
for(int i = n-2; i >=0 ; i--){
    R[i] = max(R[i+1], A[i])
}

ans = 0;
for(int i = 0; i < n; i++){
    ans += min(L[i], R[i]) - A[i];
}
```

Merge Intervals

```
for(int i = 0; i < n; i++){
    if(r >= s[i] && e[i] >= l){
        // First interval
        first_start = s[i];
        break;
    }
}
for(i = n-1; i >= 0; i--){
    if(r >= s[i] && e[i] >= l){
        // Last interval
        last_end = e[i];
    }
}
newMergedInterval = [ min(l, first_start)      max(r, last_end) ];
```