

Searching - I

- 1) Target (what to search)
- 2) Search Space (where to search)

target

T-shirt

Search Space

Room

Bed

wardrobe

Factors that affect the time taken to search?

size of search space

1) size of search space

2) order of elements in search space

Target

Word

Phone Number

Search Space

Newspaper

vs Dictionary

Hand written vs
List

phone book.
ordered

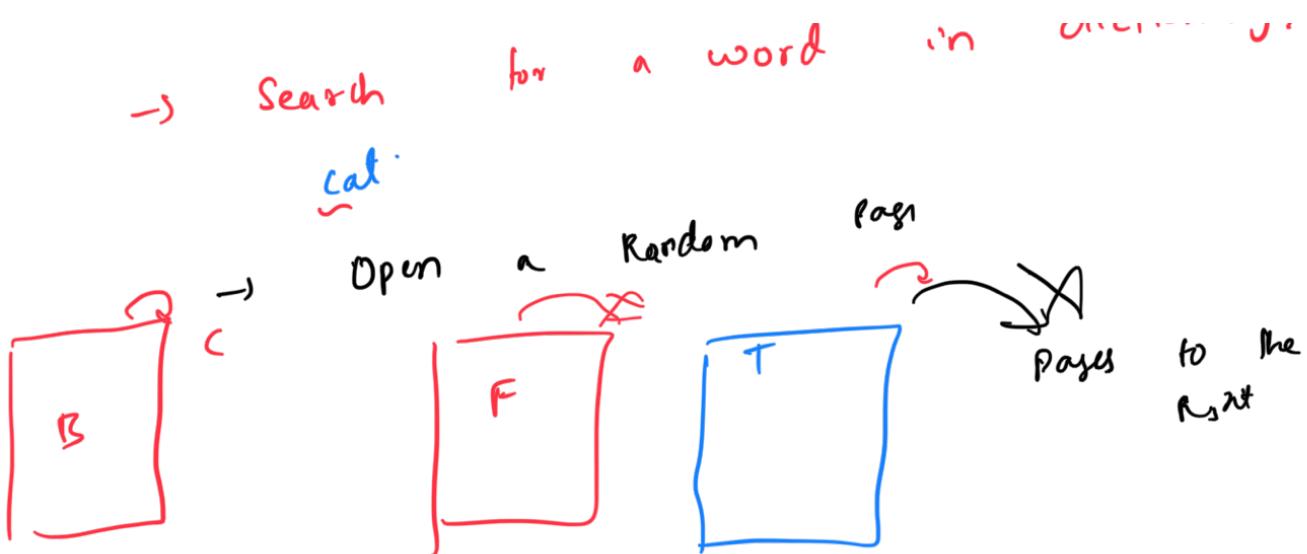
N words in newspaper

search in newspaper?
search word by word.

Linear search: $O(n)$

How is ordered arrangement helping us to
search faster?

2) we are able to discard some search
space in every dictionary

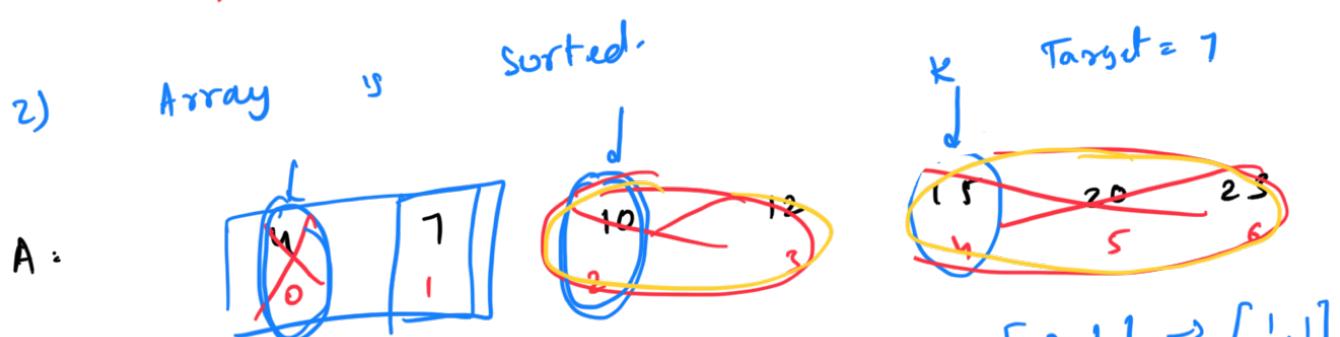


Searching for a number in an array

1) Array is not sorted
1) Sort and apply the above process

$O(n \log n)$

2) Linear Search : $O(n)$

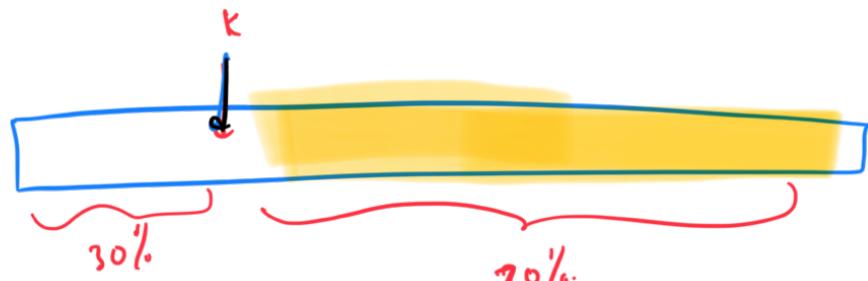


Search Spike: $[0, 6] \rightarrow [0, 3] \rightarrow [0, 1] \rightarrow [1, 1]$

Subarray \Rightarrow Start index and End index

~~Best~~ | ~~Best~~

What would be the best strategy for picking this random dictionary



Best Case: Discard 70% of search space
Worst Case: Discard 30%

Worst Case:
Maximize the search space to be discarded even in the worst case.



Best: Discard 60%.
Worst: Discard 40%.



Divide the array into 2 equal halves at every iteration

Defn: divide the search

the process where we make a decision to go either to the left or right and half.

into 2 equal halves, go either to the discard the other

binary search?

When can we apply binary search?
When array is sorted. ✗ (Blocked by question)

x1) Array

(Some sort)

1) Inherent order in the collection
(Not necessarily sorted)

2) Exploit this ordering to result half
of the possibilities by doing exactly
one comparison

Question:

E-A =

Target = 12 low = mid + 1

Search for

3 1 2 9

3 12 9 low = mid + 1

element in an array

3

4

5

6

7

8

19

26

29

35

39

mid

high = mid + 1

high

low

0

F(12) = 3

F(15) = -1

i) If our mid lands on {3, 5, 9}

2) Go Right : low = mid + 1

if (arr[mid] < Target),

low = mid + 1

2) If our mid lands on {19, 26, 29, 35, 39}

if (arr[mid] > target)

3) $\cdot \downarrow$ out mid lands on Target
 $\text{high} = \text{mid} - 1$

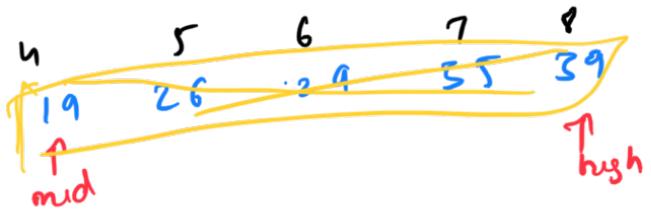
$\text{if arr[mid]} == \text{target}$
 and return mid.

$A =$



$T = 12$

and
 low
 mid
 high



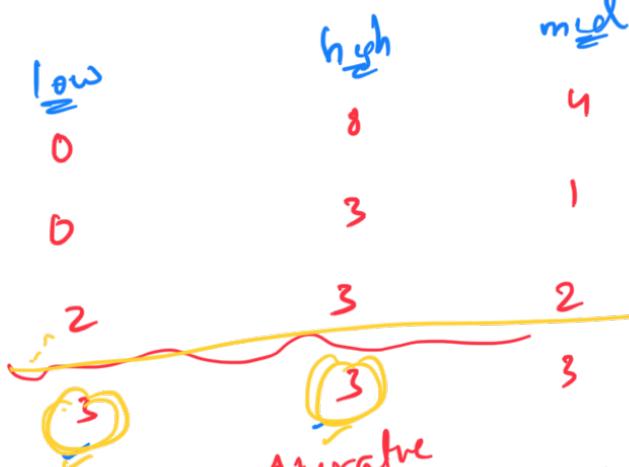
Action

$$\text{high} = \text{mid} - 1 = 3$$

$$\text{low} = \text{mid} + 1 = 2$$

$$\text{low} = \text{mid} + 1 = 3$$

Done : Return 3:



int binary search(arr, N, target) {

$\text{low} = 0, \text{high} = n - 1$
 $\text{while } (\underline{\text{low}} \leq \text{high}) \{$

$\text{mid} = (\text{high} + \text{low}) / 2;$
 $\text{if arr[mid]} == \text{target} \text{ return mid;}$
 $\text{if arr[mid]} < \text{target} \text{ low} = \text{mid} + 1;$
 $\text{else } \text{ high} = \text{mid} - 1;$

} returns -1;

Recursive code

(Lecture Notes)

$$\rightarrow \text{mid} = (\text{high} + \text{low}) / 2$$



Time Complexity

Search space was



$$\text{Step 1: } \frac{n}{2} = \frac{n}{2^1}$$

$$\text{Step 2: } \frac{n}{4} = \frac{n}{2^2}$$

$$\text{Step 3: } \frac{n}{8} = \frac{n}{2^3}$$

$$\text{Step K: } \frac{n}{2^K} = 1$$

$$n = 2^K$$

$$\log_2 n = K \log_2 2$$

$$K = \log_2 n$$

$$K = \log n$$

T.C: $O(\log n)$

$T(n) =$ time for searching in an array
of size N

$$T(n) = O(1) + T\left(\frac{n}{2}\right)$$

$$T(n) = T\left(\frac{n}{2}\right) + O(1)$$

↓
Master's theorem

Question : Given a sorted array which contains duplicates and a target, find the frequency of target element.

$$A = \begin{matrix} & & & & & & & & & & & & & & & \\ & 0 & 1 & 2 & 5 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ -5 & -5 & -3 & 0 & 0 & 1 & 1 & 1 & 2 & 5 & 5 & 5 & 5 & 10 & 10 & \\ \end{matrix}$$

$T = 5$

$\#$

$P_1 = 9$

$P_2 = 12$

$|P_2 - P_1 + 1|$

$12 - 9$

Brute Force:

T.C: $O(n)$

S. (1) $\sigma(1)$

```

for (fz or i < n; i++))  

    if (array[i] = = target)  

        count++;

```

Approach 2:

Approach 2: → Do a binary search to find any occurrence of target

Time Complexity :

A 2

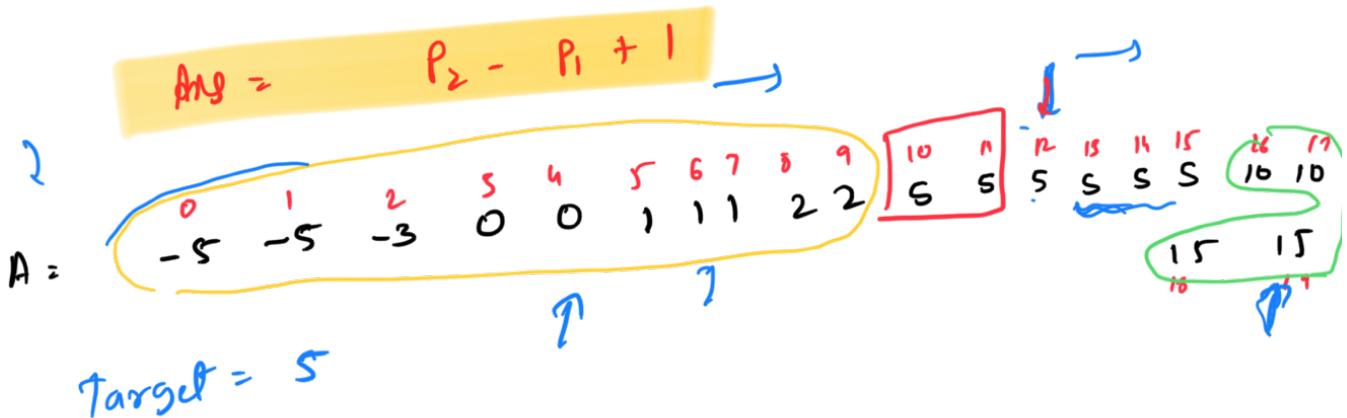
$$T_{\text{start}} = 2$$

$\pi_0 \rightarrow \eta(n)$

7-02

Approach 3 :

- Find the first occurrence of 5 : p_1
- Find the last occurrence of 5 : p_2



- If you land on $\{-5, -5, -3, 0, 0, 11, 11, 22\}$
Go Right $\Rightarrow low = mid + 1$
- If you land on $\{10, 10, 15, 15\}$
Go Left $\Rightarrow high = mid - 1$
- If you land on $\{5, 5, 5, 5, 5\}$
Update the answer,
Go Left $\Rightarrow high = mid - 1$

Last occurrence (Go Right $\Rightarrow low = mid + 1$)

```

int firstOccurrence ( Arr, n, target ) {
    low = 0, high = n - 1, p1 = -1;
    p2 = -2;
    while ( low <= high ) {
        mid = ( high + low ) / 2;
        if ( arr[ mid ] < target ) low = mid + 1;
        else if ( arr[ mid ] > target ) high = mid - 1;
        else {
            p1 = mid;
            high = mid - 1;
        }
    }
}

```

\lfloor return
 y
 $(P_2 - P_1 + 1)$ as the answers.

$$P_2 = -1$$

$$P_1 = -1$$

$$P_2 = -2$$

$$P_1 = -1$$

$$\begin{aligned} & (-2) - (-1) + 1 \\ & = -2 + 1 + 1 \boxed{= 0} \end{aligned}$$

$$P_2 - P_1 + 1$$

$$(-1) - (-1) + 1$$

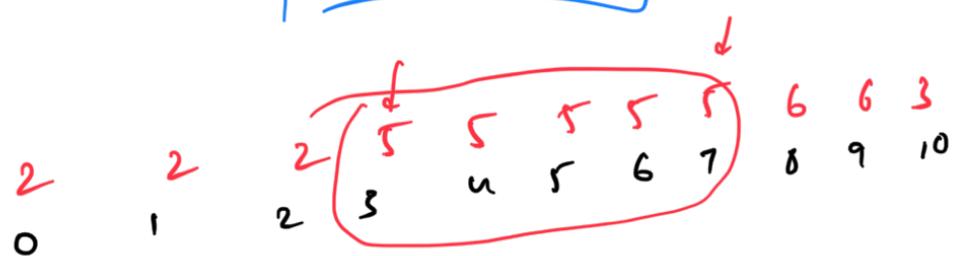
$$e^{P_1 + P_2 + 1} = \boxed{1}$$

$\text{if } P_1 == -1 \text{ or } P_2 == -1$
 return 0;

- 1) If we land on invalid part: we go towards the valid part.
- 2) If we land on valid part, update the answer, and search for a better answer.

1) Find first occurrence ($O(\log n)$)
 2) " last occurrence ($O(\log n)$)

$\lceil T.C: O(\log n) \rceil$



$T_{P_1 = 3}$

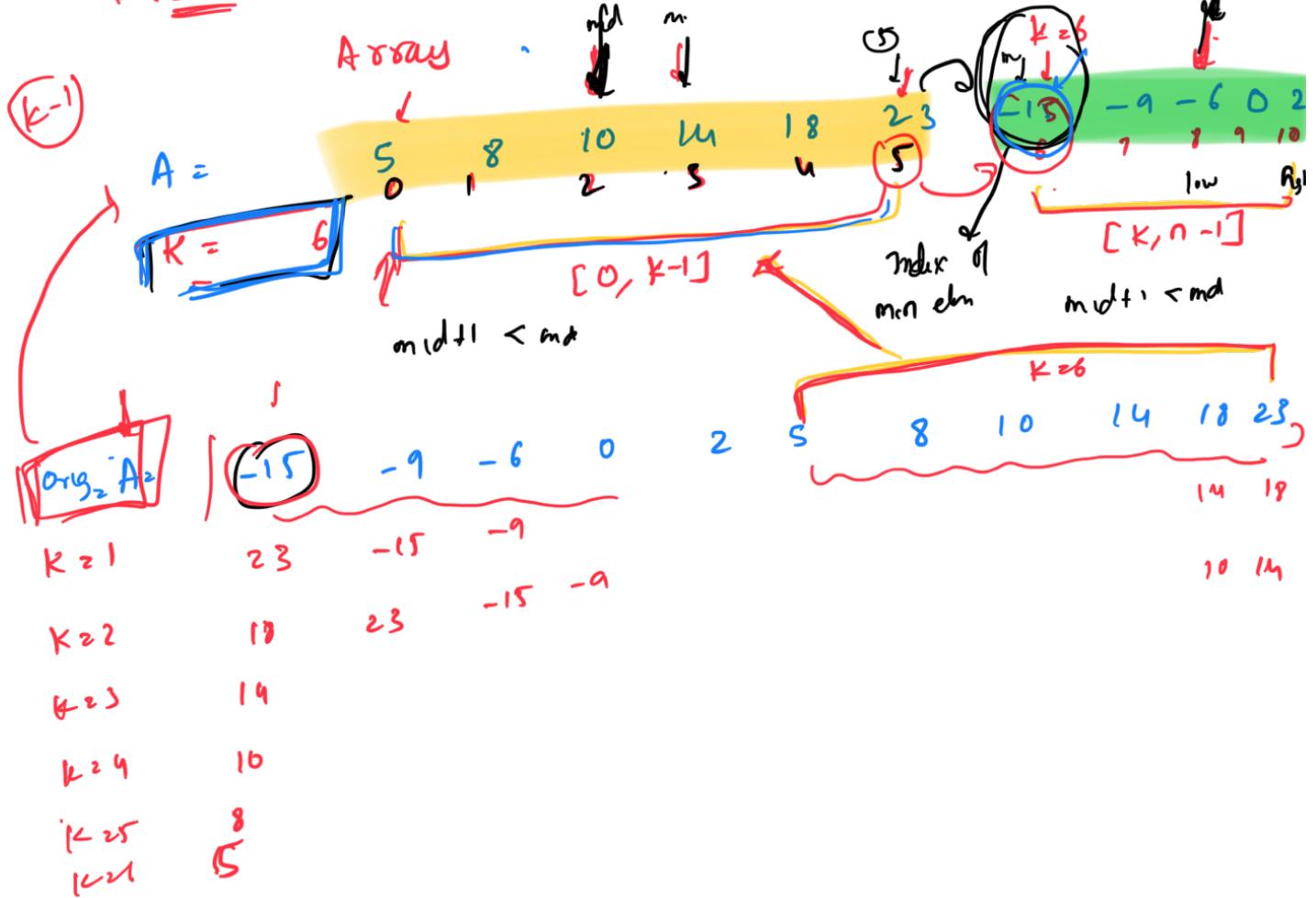
$\lceil 7 - 3 \rceil = 4$

$\text{arr}[\text{mid}] < \text{arr}[\text{mid}+1]$

$$\begin{array}{l} P_2 = 1 \\ P_2 - P_1 = 4 \end{array}$$

$\{ 3, 7 \}$
 $\{ 3, \dots, 7-1 \}$
 $\boxed{3, 4, 5, 6}$

Question: Search ^ for target
in a sorted and rotated array



Brute Force :

Linear search : $O(n)$

Approach 2 :

Binary search on

Arr : $[0, k-1]$

arr : $[k, n-1]$

2 sorted arrays

BS (arr, 0, $k-1$, target)
BSL (arr, k , $n-1$, target)

Time

T.C: $O(\log n)$

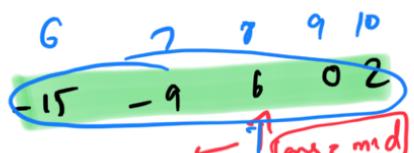
What if K is not given

Approach 1:
Iterate and find the first element which is less than its previous.
 $T.C: O(n) + O(\log n) =$

Approach 2:

Find K in efficient time
 $K = \text{Index of } 1^{\text{st}} \text{ elements of the Right sorted Array}$
Task: Find index of 1st element in min element of Right sorted Array

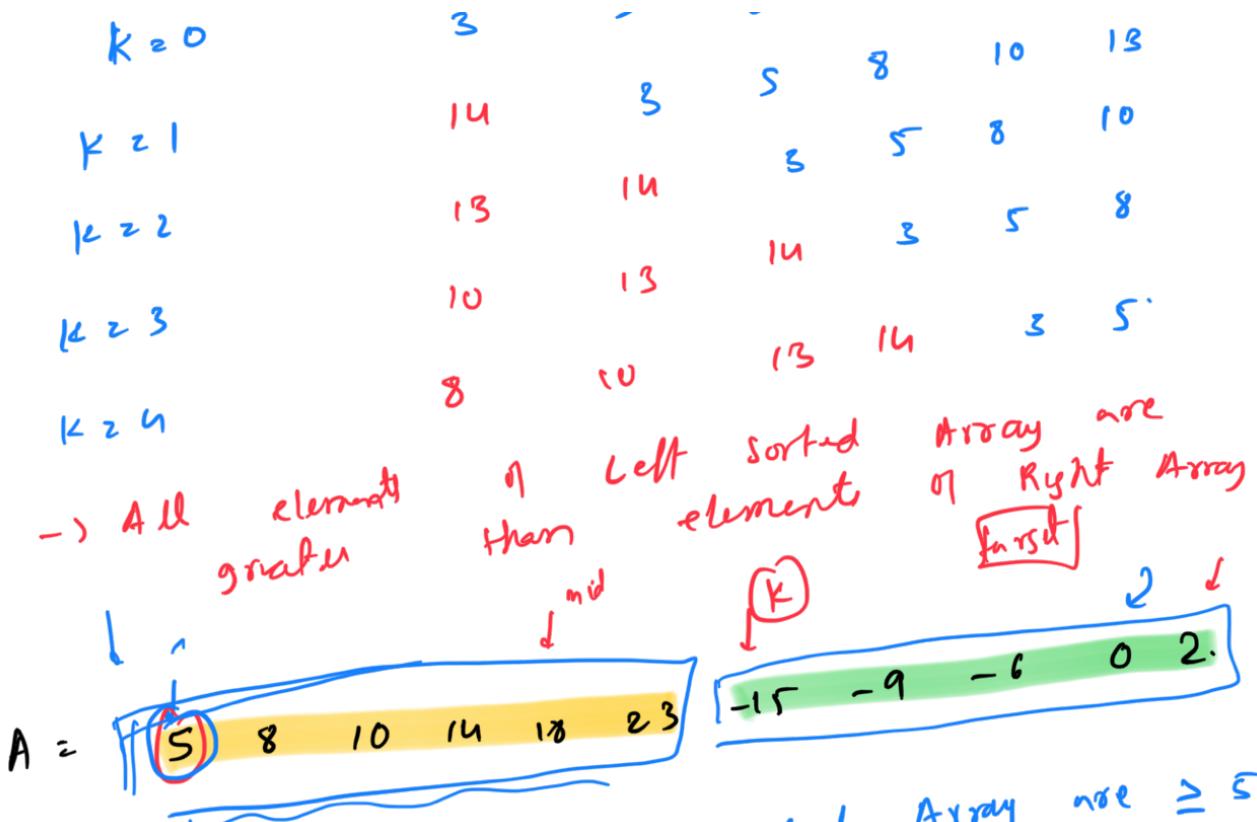
$A = [5, 8, 10, 14, 18, 23]$



- 1) If mid pointer lands on Left sorted Array Go Right $\Rightarrow low = mid + 1$
- 2) If mid pointer lands on Right sorted Array
 $\{ -15, -9, 5, 0, 2 \}$
Update the answer.
Go Left $\Rightarrow high = mid - 1$

→ How do we know we are on Left or Right sorted Array

5 8 10 13 14



$=>$ All elements of left sorted array \geq mid
 $=>$ All elements of right sorted array $< mid$

```

if( arr[mid] < arr[0] )
  RIGHT SORTED ARRAY
else
  LEFT SORTED ARRAY
  
```

int rotationFactor(int arr[], int n) {

low = 0, high = N - 1;

while(low \leq high) {
 mid = (high + low) / 2;

→ if(arr[mid] < arr[0]) {
 $K = mid$

high = mid - 1;

→ } else {
 $low = mid + 1$;

y

$$arr[mid] \geq 5$$

$$arr[0] \geq 5$$

... low ... high

↴ return i ;
 1) Find $K \rightarrow O(\log n)$
 2) search($0, K-1$) $\rightarrow O(\log n)$
 3) search($K, n-1$) $\rightarrow O(\log n)$

↴ Binary Search Cells
 if ($arr[0] \leq \text{target}$) {
 BSC($0, K-1$); (Left sorted Array)

↴
 else {
 BSC($K, n-1$) (Right sorted Array)

↴
 Ex: $0 \ f(0) \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$
 low = 0, high = 6
 mid = 3
 arr[mid] = 5
 arr[0] = 5
 low = 4, high = 6

Question: → Given an array of elements, all elements repeat twice except 1 element.
 → Repeated elements are adjacent to each other.
 → Find the non-repeating element.

A: 3 3 1 1 4 6 6 8 8

<u>low</u>	<u>mid</u>	$\gamma \Rightarrow 6$	$low = mid + 2$
0	4		$high = mid - 1$
8	4	11	
8	10	9	$high = mid - 1$
8	8	8	

```

int uniqueElement( arr, N) {
    low = 0; high = n-1;
    while( low <= high ) {
        mid = (low+high)/2;
        if( arr[mid] != arr[mid-1] )
            arr[mid] = arr[mid+1];
        return arr[mid];
        // update mid to first occurrence
        if( arr[mid] == arr[mid-1] )
            mid = mid-1;
    }
    if( mid%2 == 0 )
        low = mid+2;
    else
        high = mid-1;
}
    
```

$\rightarrow arr[mid] \neq arr[mid-1] \quad \text{if } arr[mid] == arr[mid+1]$
 $mid-1, mid, mid+1$

Base Cases

- 1) Array is single Element Array
 $A = [3]$
 \downarrow mid.
 \downarrow if($n == 1$) return $arr[0]$
 \dots 1, 4

2) 6 5 3 1 1 ...

$\text{arr}[\text{mid}+1]$

$\text{arr}[-1]$

$\text{if } (\text{arr}[0]) \neq \text{arr}[1]$

return $\text{arr}[0]$:

mid

3)

3 3 1 1 4 4 6

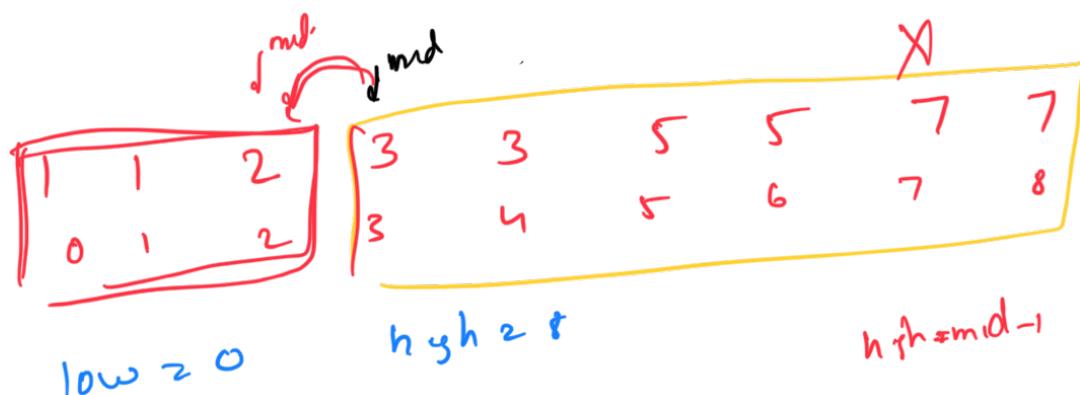
$\text{arr}[\text{mid}+1]$

$\text{if } (\text{arr}[n-1]) \neq \text{arr}[n-2]$

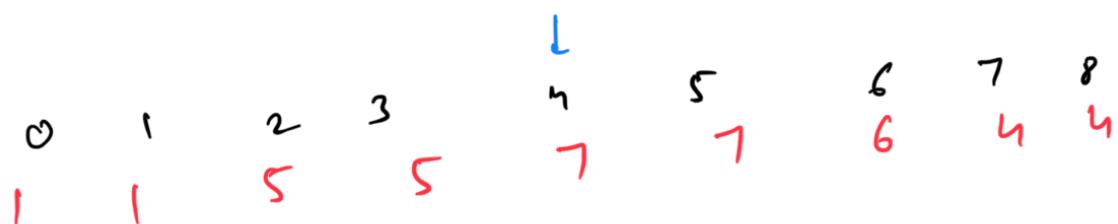
return $\text{arr}[n-1]$;

$O(n^2)$

T.C:



mid = 4



Low

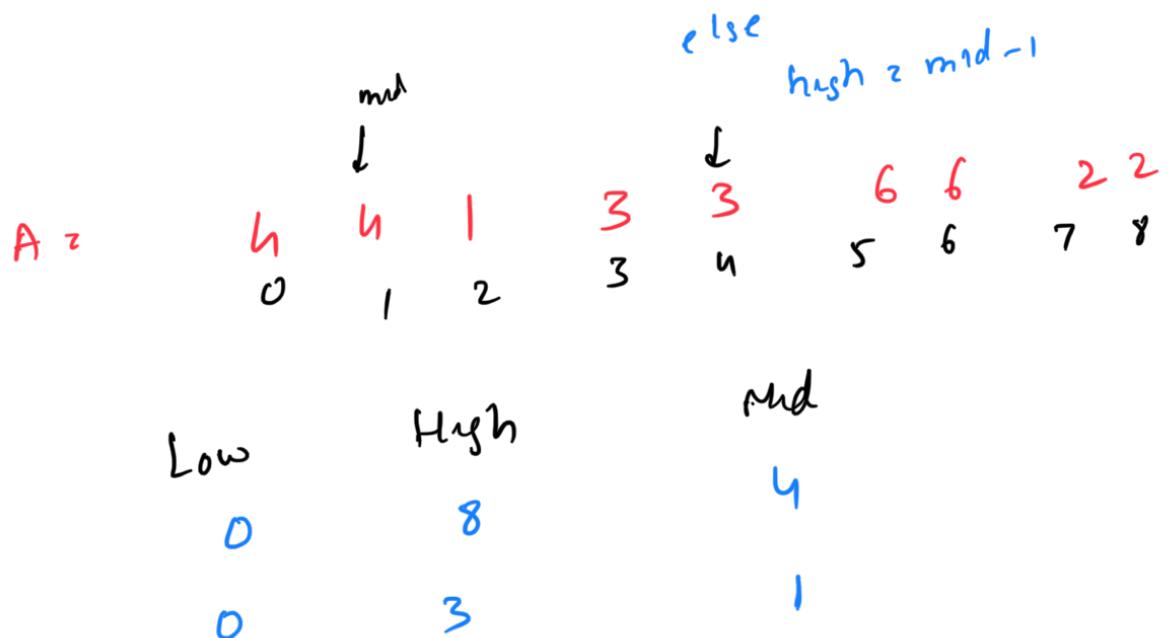
High

mid
4

high = mid - 1

6 8

$\text{if } (\text{arr}[\text{mid}]) \neq \text{arr}[\text{mid}+1]$
 $\text{low} = \text{mid} + 1$



Recursive code:

```

int BSR(int arr[], int low, int high, int val{
    if(low > high) return -1;

    mid = (high + low)/2

    if(arr[mid] == k)
        return mid;

    // We are on right side; we should go left
    if(arr[mid] > val)
        return BSR(arr, low, mid-1, k);

    // We are on left side; go right
    else
        return BSR(arr, mid +1, high, k);

}

```