Matrix expo

1) What is a matrix
2) Matrix multiplication
3) $a^n \Rightarrow \log n$

$a^{n/2}$

n odd

n even

$a^n \Rightarrow a^{n/2} \times a^{n/2}$ n even

$a^n \Rightarrow a \times a^{n/2} \times a^{n/2}$

```
int pow ( int a, int n ) {
    if (n==0)    return 1
    int p = pow( a, n/2 )
    if ( n is even )
        return p*p
    else
        return a*p*p
}
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^n$$

```
matrix pow (matrix a, int n) {
    if (n==0)     return identity matrix
    int p = pow(a, n/2)
    if ( n is even )
        return p*p
    else
        return a*p*p
}
```

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q1 Fibonacci no.s.

$$F(n) = F(n-1) + F(n-2)$$

Using matrix expo. $\Rightarrow \log n$

$$F(n) = F(n-1) + F(n-2)$$

$$F(n) = 1 * F(n-1) + 1 * F(n-2)$$

$$F(n-1) = 1 * F(n-1) + 0 * F(n-2)$$

$$\underbrace{\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix}}_{M(n)} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{x} \times \underbrace{\begin{bmatrix} F(n-1) \\ F(n-2) \end{bmatrix}}_{M(n-1)}$$

$F(1) = 1$ $\qquad$ $F(2) = 1$

$$M(n) = x\, M(n-1)$$

$$M(n-1) = x\, M(n-2)$$

$$M(n) = x^2\, M(n-2)$$

$$M(n-2) = x\, M(n-3)$$

$$M(n) = x^3\, M(n-3)$$

$$\vdots$$

$$M(n) = x^k\, M(n-k)$$

$$M(2) = \begin{bmatrix} F(2) \\ F(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$n-k = 2 \qquad k = n-2$$

$$M(n) = x^{n-2}\, M(2)$$

$$M(n) = x^{n-2} \begin{bmatrix} F(2) \\ F(1) \end{bmatrix}$$

$$M(n) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-2} \begin{bmatrix} F(2) \\ F(1) \end{bmatrix}$$

$$\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Calc using pow function for matrices.

$n = 4$

$$\begin{bmatrix} F(4) \\ F(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix} \qquad \begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix} \qquad \begin{matrix} 2 & 1 \\ 1 & 1 \end{matrix}$$

$$\begin{matrix} F(4) \\ F(3) \end{matrix} = \begin{matrix} 2 & 1 \\ 1 & 1 \end{matrix} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{matrix} F(4) \\ F(3) \end{matrix} = \begin{matrix} 3 \\ 2 \end{matrix}$$

1 1 2 3 5

We can extend it to all linear sequences.

$$F(n) = F(n-1) + 3F(n-2) + 2F(n-3)$$

$$\begin{matrix} f(n) \\ f(n-1) \\ f(n-2) \end{matrix} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} F(n-1) \\ F(n-2) \\ F(n-3) \end{bmatrix}$$

Q Find the sum of all fibonacci no.s uptill N.

$$F(n) = F(n-1) + F(n-2)$$

$$S(n) = F(1) + F(2) + F(3) + \cdots + F(N)$$

$$\begin{matrix} S_N \\ F_N \\ F_{N-1} \end{matrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{matrix} S_{N-1} \\ F_{N-1} \\ F_{N-2} \end{matrix}$$

$$S_N = S_{N-1} + f_N$$

$$4 f_{N-1} + f_{N-2}$$

$$\left.\begin{matrix} S_N \\ S_{N-1} \\ S_{N-2} \end{matrix}\right\}$$

S_{N-2}

f_{N-1}

f_{N-2}

Q

$$f_n = 4 f_{n-1} + 2 g_{n-1}$$

$$g_n = 3 g_{n-1} + 2^{n-1}$$

Calc f_n

$$\begin{bmatrix} f_n \\ g_n \\ 2^n \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} f_{n-1} \\ g_{n-1} \\ 2^{n-1} \end{bmatrix}$$

Q. You are standing at n; you can jump from $n-1$ cost 1

$n-2$ cost 3

$n-4$ cost 5

$$f_n = f_{n-1} + 3f_{n-2} + 5f_{n-4}$$

$$\begin{bmatrix} f_n \\ f_{n-1} \\ f_{n-2} \\ f_{n-3} \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 0 & 5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} f_{n-1} \\ f_{n-2} \\ f_{n-3} \\ f_{n-4} \end{bmatrix}$$

i=0 i<26 i++

j=0 j<26 j++

int x = x<n x++

$26^2 n$

26×26

$[\quad]^n$