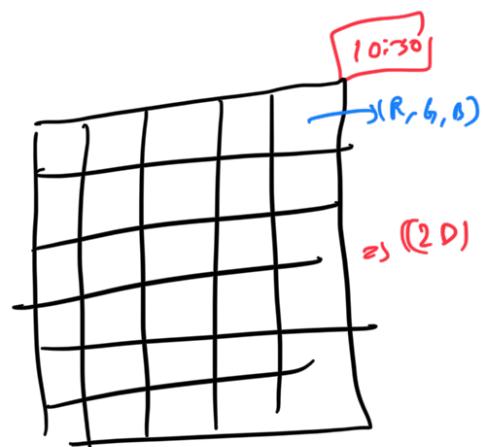


# Multi - Dimensional Arrays

Real life examples

- 1) Image
- 2) Chess Board  
↓  
 $8 \times 8$
- 3) Sudoku (area)
- 4) Snake
- 5) Tic tac toe
- 6) Candy crush



graphs  $\Rightarrow$  2D Array  
(Adjacency List)

2D Arrays  $\Rightarrow$  Matrix

$aor[n][m]$   
 $\Rightarrow$  2D Array with 'n' rows, 'm' columns

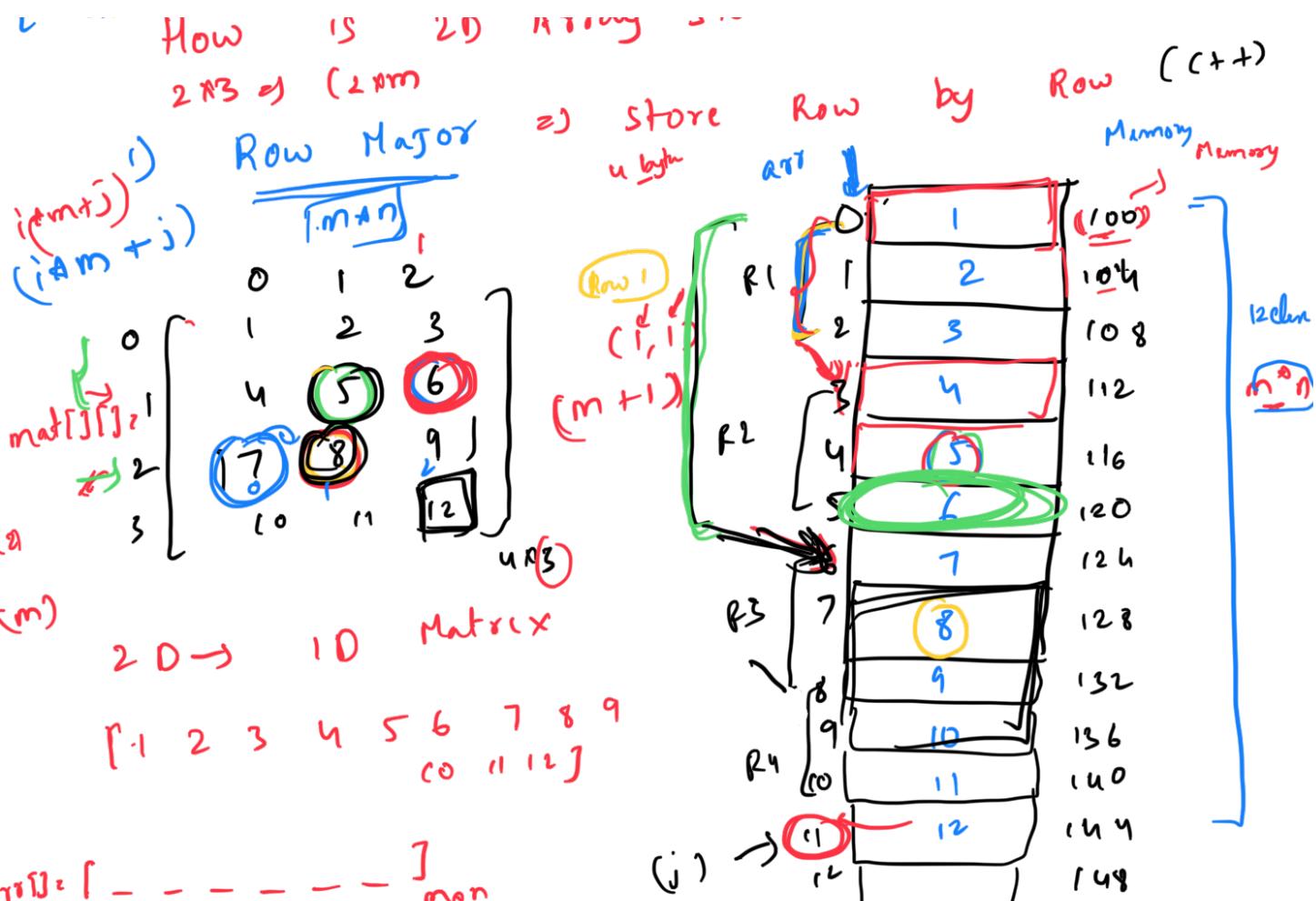
$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$   
 $\Rightarrow$  2D Index  $(i, j)$   $\Rightarrow$  2D Index  
Row No.      Column No.

1D Array  $\Rightarrow aor[i]$   
6  $\Rightarrow (1, 1)$   
4  $\Rightarrow (0, 2)$

Integer Array  $\Rightarrow (n \times m) \times u = u \cdot n \cdot m$

$\begin{bmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}_{n \times m}$   
 $\Rightarrow$  RAM  
 $\Rightarrow (n \times m)$  elements

$\{ \}$   $n \times m$  arrays stored in Memory!



$i \Rightarrow$  arr[ $\boxed{i}$ ][ $\boxed{j}$ ]  $8 \Rightarrow (2, 1) \Rightarrow (2^m) + 1$

$(i, j) \Rightarrow \boxed{(i, j)} \Rightarrow \boxed{(i * m + j)}$   
 $(i, j) \Rightarrow \boxed{1D}$  <sub>2D Indx.</sub>  $\boxed{1D Indx}$

$$\boxed{(3, 2)} \Rightarrow 3 \times 3 + 2 = \boxed{11}$$

2D  $(i, j) \Rightarrow \boxed{(i * m + j)}$   $\Rightarrow \boxed{(i * m + j)}$  <sub>No. of column</sub>  $\Rightarrow \boxed{1D Indx}$  <sub>Rows</sub>  
 $n \rightarrow$  Rows  
 $m \rightarrow$  Column

$$6 \Rightarrow (1, 2) \Rightarrow i * m + j$$

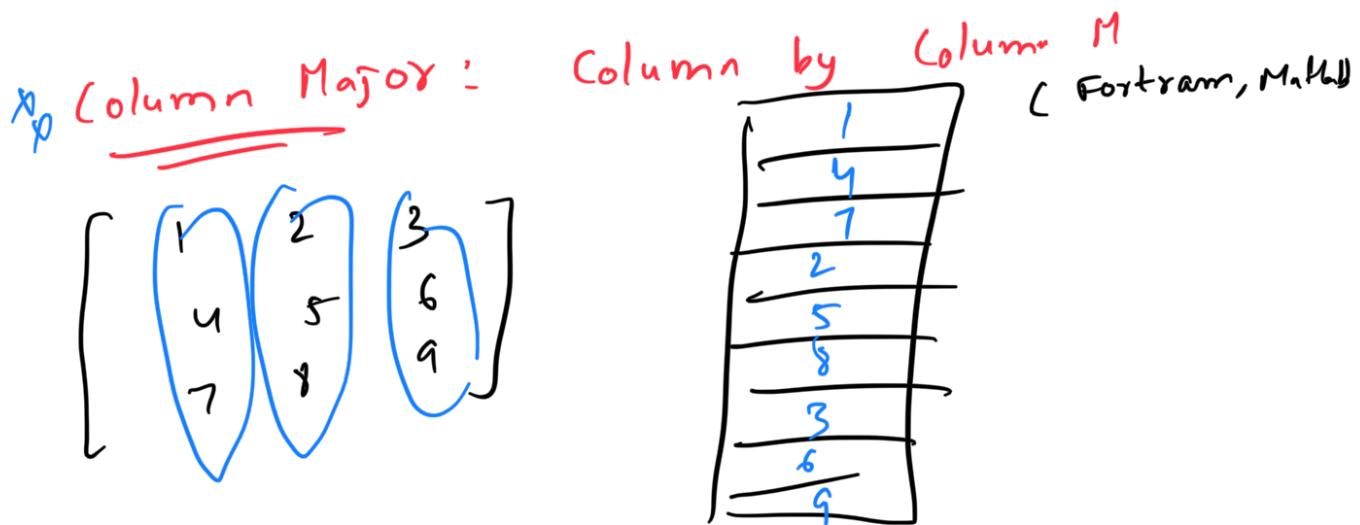
$$\begin{aligned} & \downarrow \quad \downarrow \\ i & \quad j \end{aligned}$$

$$(i * m + j) \quad \boxed{5} \Rightarrow \boxed{1D Indx}$$

$$(1 \times 3 + 2) = \boxed{5} \Rightarrow \boxed{1D Indx}$$

$\boxed{\text{base-address} + \text{index}}$

( i + m + j ) =  $\frac{(i-1) \cdot M + j}{M}$   
 ( INT : 4 )



~~Array of Arrays~~:  
 ( Java, JS, Python )  
 ( Tagged Array )  
 R & M

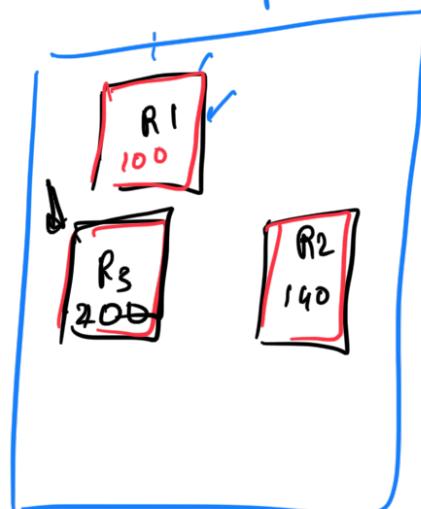
vector <vector>

$mat[3] = \begin{bmatrix} 2 & 4 \\ 1 & 6 \\ 7 & 9 \end{bmatrix}$

$R_1 \rightarrow [2, 4]$   
 $R_2 \rightarrow [1, 6]$   
 $R_3 \rightarrow [7, 9]$

$\downarrow$   
 $arr = [100, 140, 200]$

(1D Array)  
 → Storing References  
 in a different



These 1D arrays  
 1D Arrays

arr[2][1]

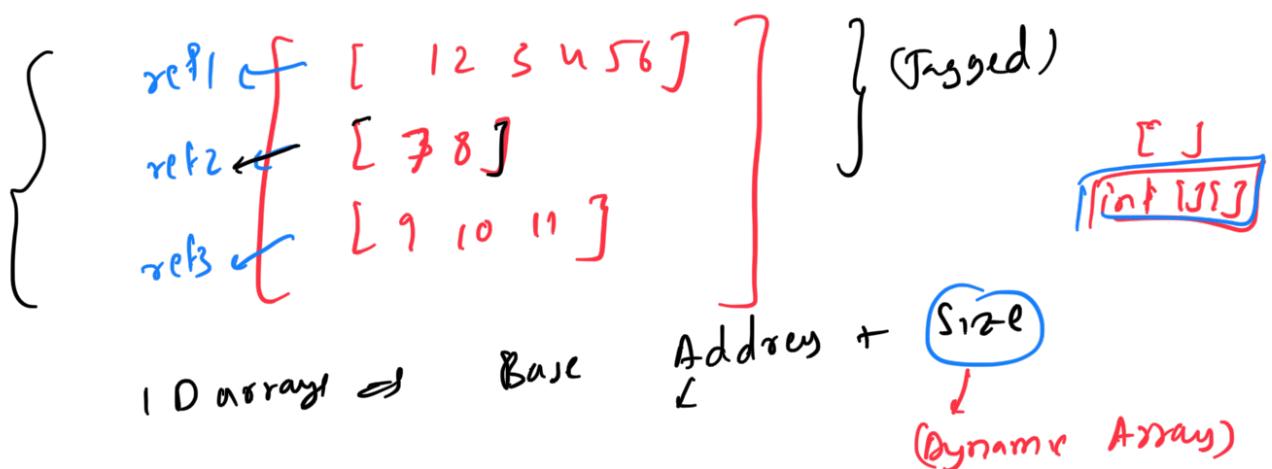
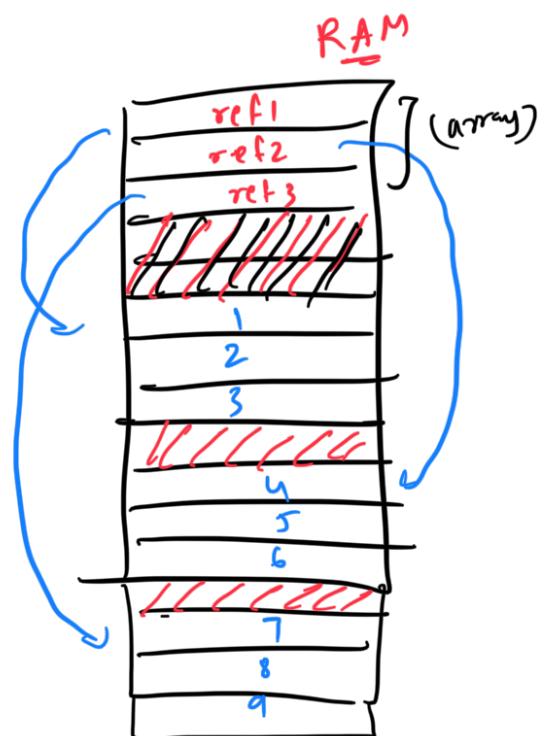
arr[i][j]

- First look at 1D array  $arr$
- references  $\Rightarrow O(1)$
- Using ' $j$ ' - find the element  $\Theta$   $O(1)$

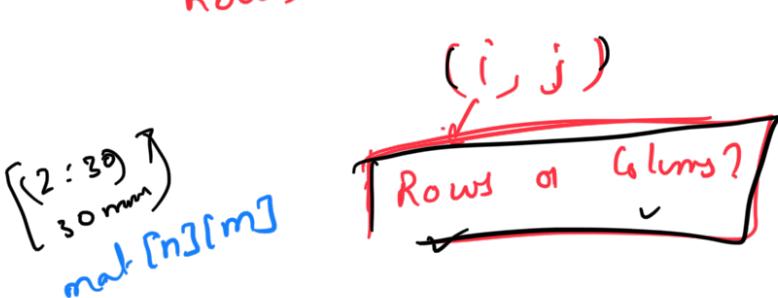
That 1D array  
⇒ All arrays are not in contiguous blocks



1D array (base address, size)



Rows or Columns?



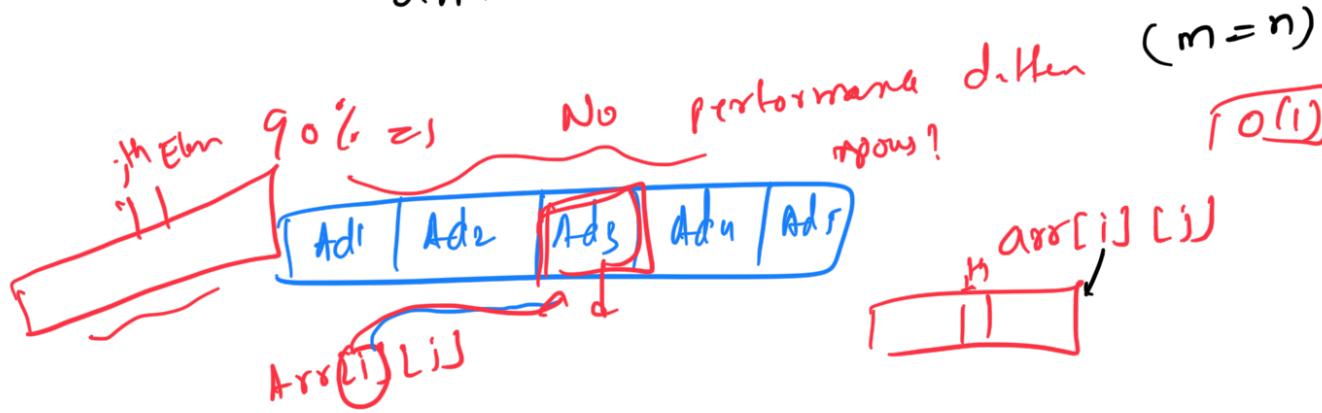
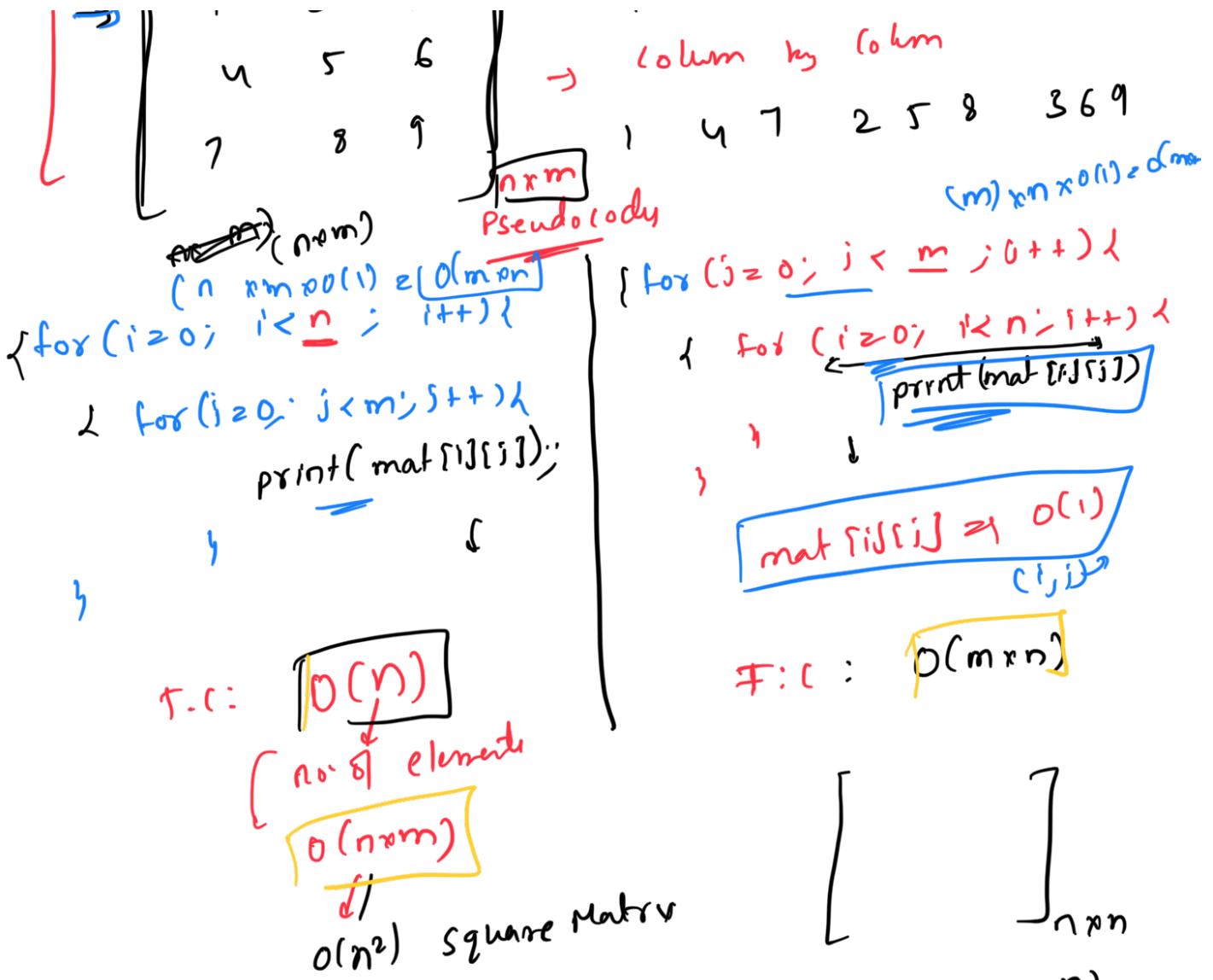
[]  
[] ...

(MLE)  
 $\times 10^9$

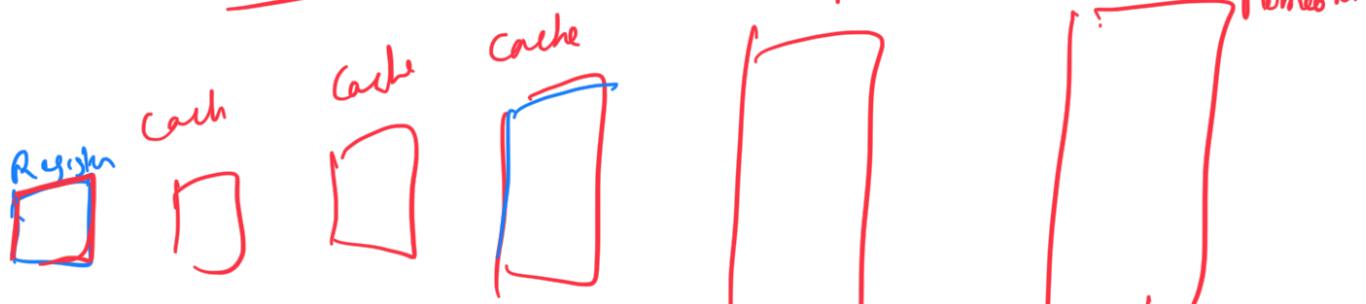
mat[i][j]

2) Iterating over 2D Matrix

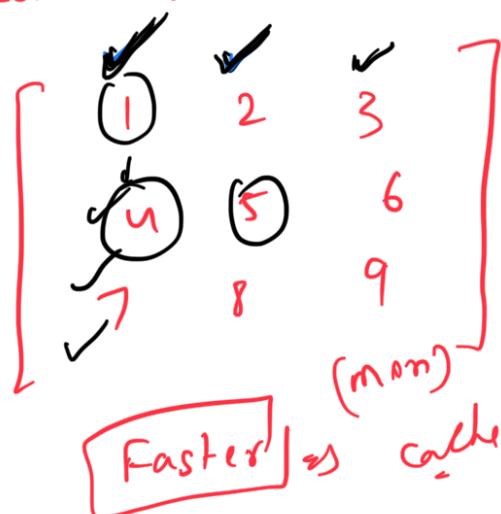




## Locality of Reference



Column by Column



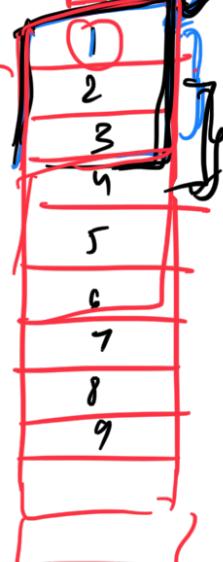
$\leftarrow$



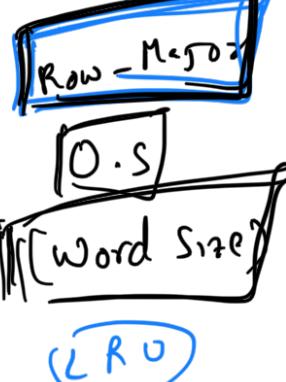
1  $\rightarrow$

LRU

RAM



Least Access



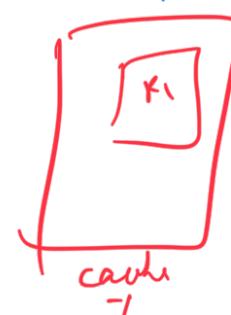
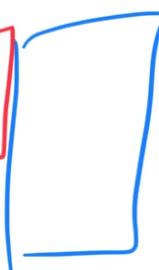
Row by Row  $\Rightarrow$  Faster Array time.

C++:

Tagged Array



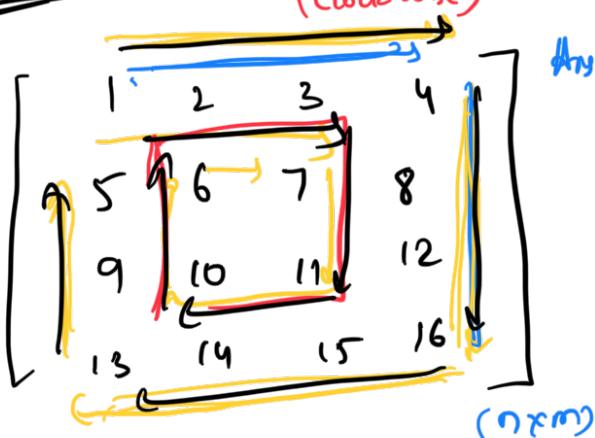
T.C:  $O(m \times n)$



Question:

Print Matrix in spiral way

(Clockwise)



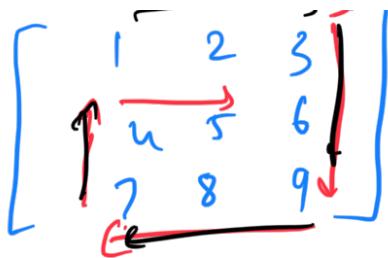
Ans:

1	2	3	4	8	12	16
15	14	13	9	5	6	7
11	10					



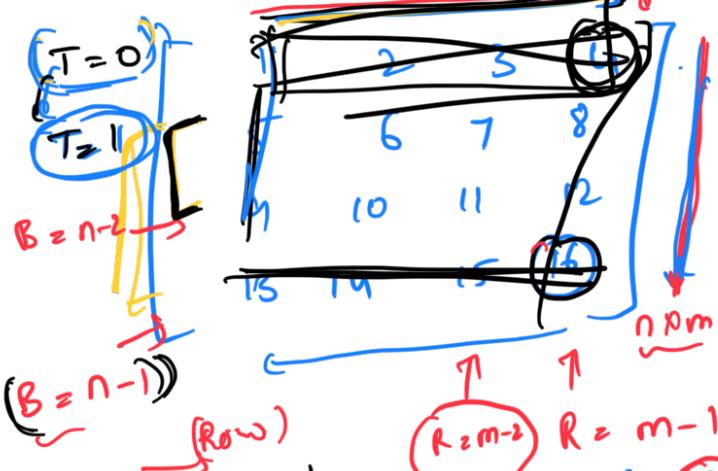
```
for(i=0; i<n; i++)
for(j=m-1; j>0; j--)
```

1 2 3 4 8 12 16  
15 14 13 9 5 6 7  
11 10  
...  
1 2 3 4 5 6 7 8 9

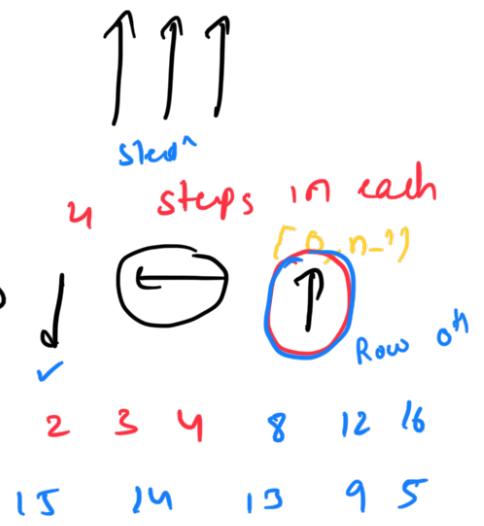


1 < s = ' Writing Code'

(top)  $\Rightarrow L=0$  Do all iteration



steps above m columns ( $m-1$ )  $\rightarrow T++$

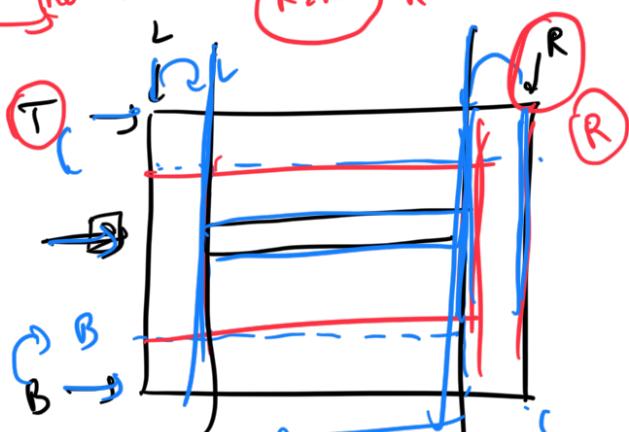


( $\rightarrow$  rows) ( $n-1$ )

Discarding  $(n-1)^{th}$  row  
 $(n-2)^{th}$  row

$n+1-1$

Collapsing the matrix  
peeling layers of onion



$T=0, B=\underline{n-1}, L=0 \rightarrow R, R=m-1$

$L \leq R$   
 $T \leq B$

$T, B$  (if  $L \leq R$ )

$L \leq R$

$((T \leq B) \& (L \leq R))$

$11 \quad L \leq R$   
 $L=5$   
 $R=3$

for ( $K = L ; K \leq R ; K++$ ) {  
    print (mat[T][K]);

for ( $K = \underline{top} ; K \leq \underline{bottom} ; K++$ ) {  
    print (mat[T][K]);

B -  
B  
T

print( mat [L][J] ) ; //

R--;  
for ( k = R; k ≥ L; k-- )  
print( mat [B] [k] );

B--;

for ( k = B; k ≥ T; k-- ) {  
print( mat [k] [L] );

)

L+f;

Way2: Count no. of printed elements  
(count)  
, if (count == m+n) {  
break;

T.C: O(m+n)

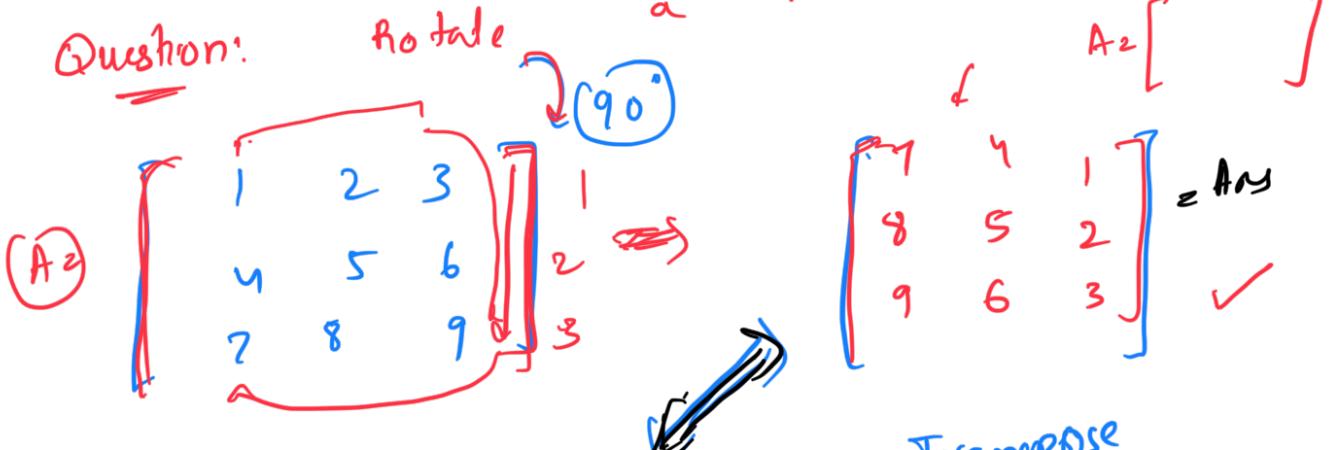
(Amortized)

Question:

rotate

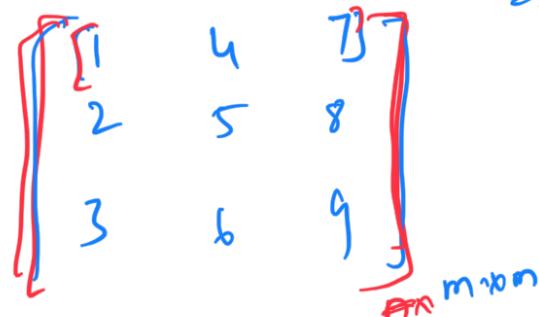
a

matrix by (90°)



A<sup>T</sup>

(2D Matrix  
of Fixed Size)



z) Transpose

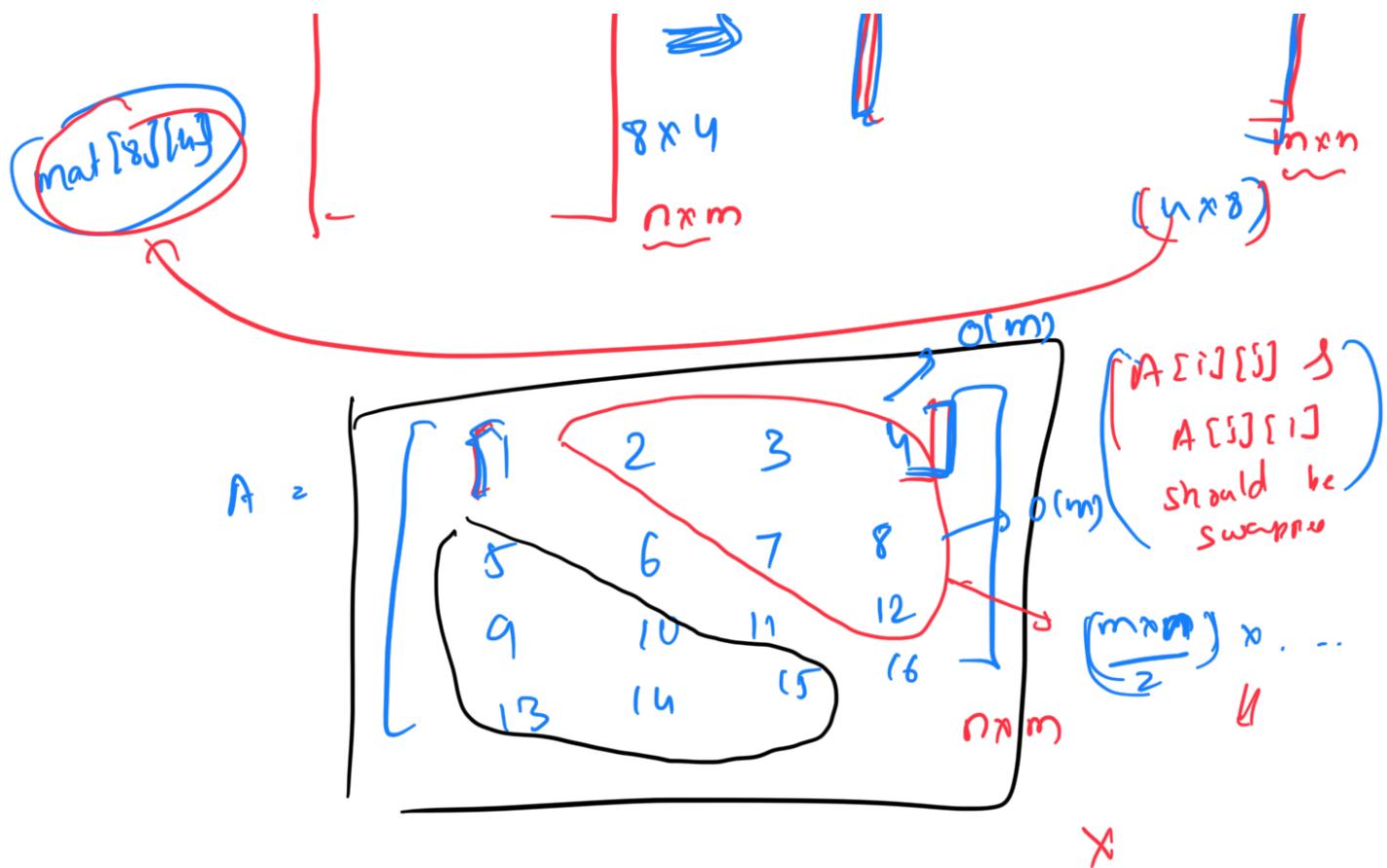
→ Rows become columns  
→ Columns become rows

Out of bound error

T<sup>n</sup>

T

T

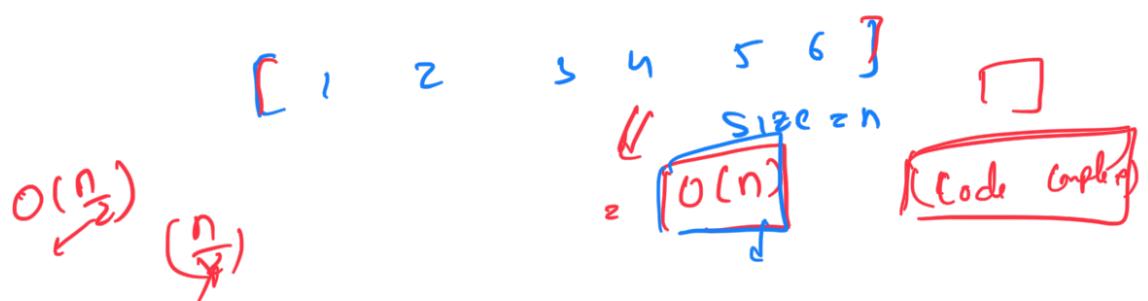


Algo

{ 1) Find Transpose  $\rightarrow O(m \cdot n)$   
 2) Reverse the rows  $\rightarrow O(m \cdot n)$

Rewriting:  $O(m) \times O(n) = O(n \cdot m)$

T.C:  $O(m \cdot n)$



Question:

→ Given a matrix of 0's and 1's  
 → All the rows are sorted!  
 → All the rows with maximum

Task: Find the sum no. of 1's [Increasing]

$$\text{mat} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

ans = 2  
(ans = 2)

Approach 1:

For every row, count no. of 1's. Find the row with maximum 1's.

T.C:  $O(n \times m)$

Approach 2:

For every row, find the sum. Find the row with maximum sum.

T.C:  $O(m \times n)$

Approach 3:

Iterate Column by Column

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\in O(1)$

Array is sorted

ans = 0

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T.C:  $O(m \times n)$

Best be  $O(1)$

T.C: Worst Case  
B.S

Approach 4:

	0	1	2	3
0	0	1	1	1
1	0	0	1	1
2	1	1	1	1
3	0	0	0	0

Binary Search

(ind = 1)

N = 4

((N - index) / 2)

Find index of first 1

B.F. :-

T.C: Linear Search  
T.C.  $O(m)$

Ans  $\rightarrow$   
(sorted)

→

[0, 9]  
[5, 9]  
[5, 6]

0 0 0 0  
0 1 2 3

0	0	1	1	1	1
0	0	1	1	1	1
5	6	7	7	8	9

(ind = 7)  $\rightarrow$   
ind = 6

$$\left\lceil \frac{n}{2} \rightarrow \frac{n}{4} \rightarrow \frac{n}{8} \dots 1 \right\rceil \text{ (logn)}$$

int left;  
int right;

For i Row  $\geq$  (logm)

T.C:  $n \times \log m$

$\Rightarrow O(n \log m)$

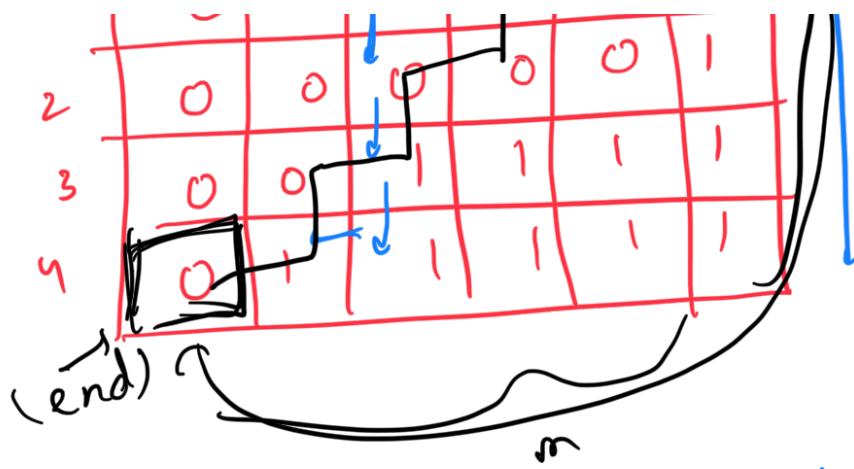
Approach 5:

	0	1	2	3	4	5
0	0	0	0	1	1	1
1	0	0	1	1	1	1
2	1	1	1	1	1	1
3	0	0	0	0	0	0

ans = 6

{ ans = 2 \* k  
row = 0 }  
5  
4

d

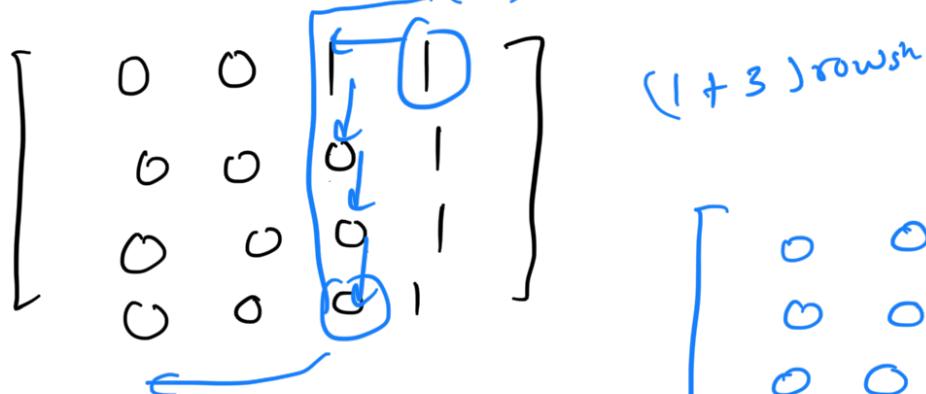


T.C:  
 1) Left  
 2) Bottom

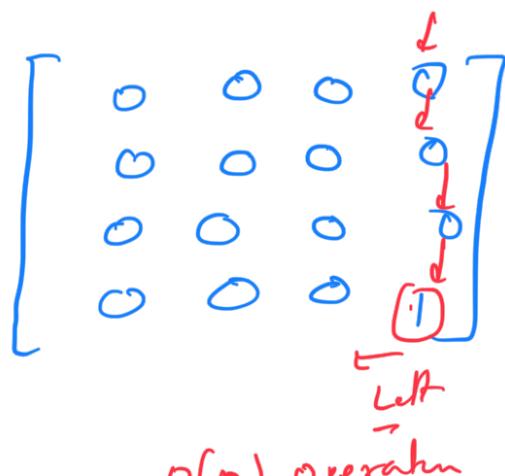
$(m+n)$  steps

T.C:  $O(m+n)$

$0^{th}$  row  $\rightarrow$   $n^{th}$  row  
 $(m-1)^{th}$  column  $\rightarrow$   $0^{th}$  column  
 $n+m$  steps  
 $(n$  steps)  
 $(m$  steps)



$(1+3)$  rows



$O(n)$  operations

Question:

Search in a sorted row-wise matrix.

→ column-wise

$T = 14 \Rightarrow$  True

$T = 35 \Rightarrow$  False

mat

Brute Force Approach

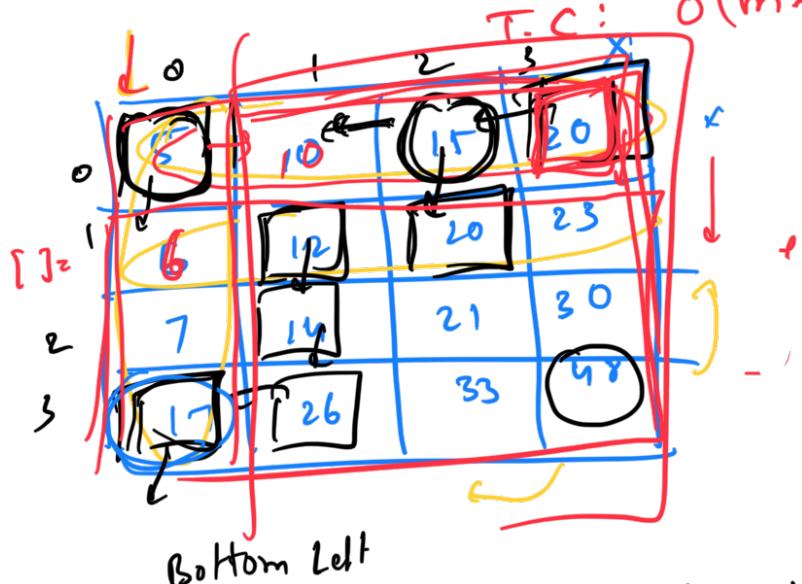
Iterate the whole matrix

T.C:  $O(n \times m)$

Approach 2: B.S on rows  
( $\log m$ )

T.C:  $n + \log m$

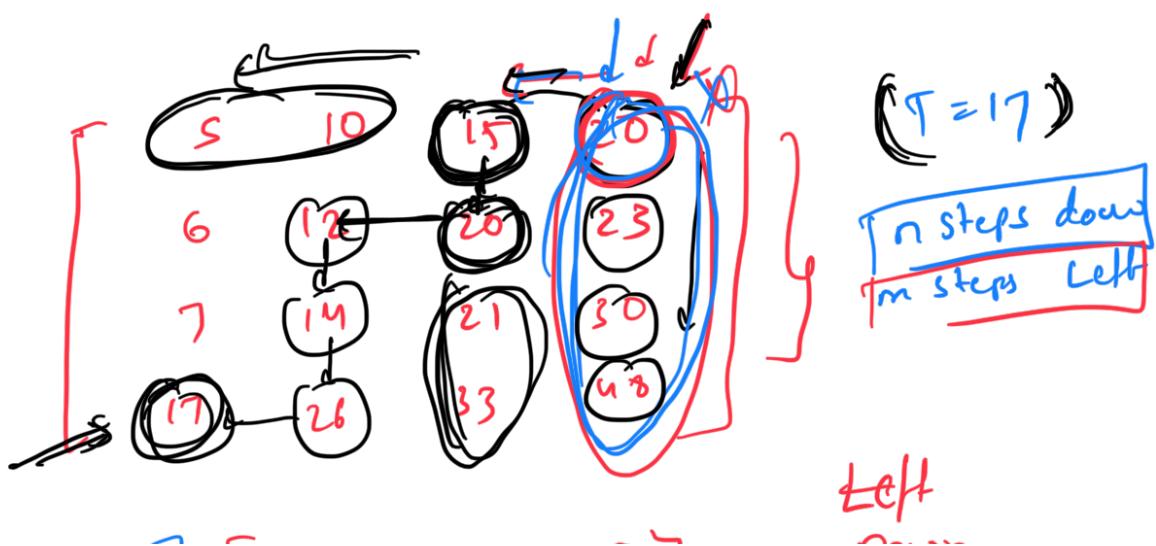
Approach 3: B.S on columns  
 $\log n \rightarrow 1$  column

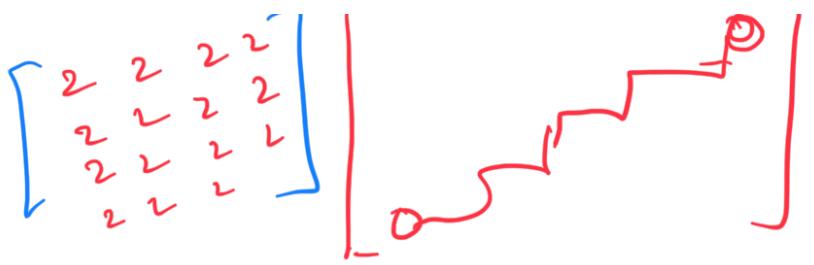


Binary Search Tree

BST

(Min Heap / Priority-queue)  
Columns are sorted





Down  
(m+n) steps

(6 - 7 fast cell)

Input:

Output:

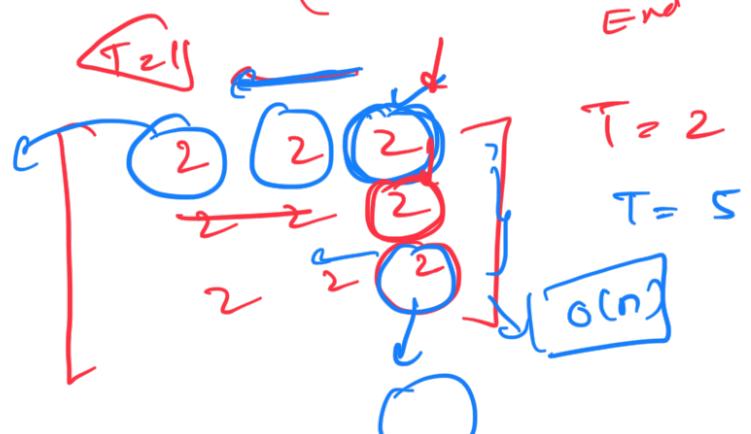
Differ

Corner Case

Arr. length  
 $O(1)$

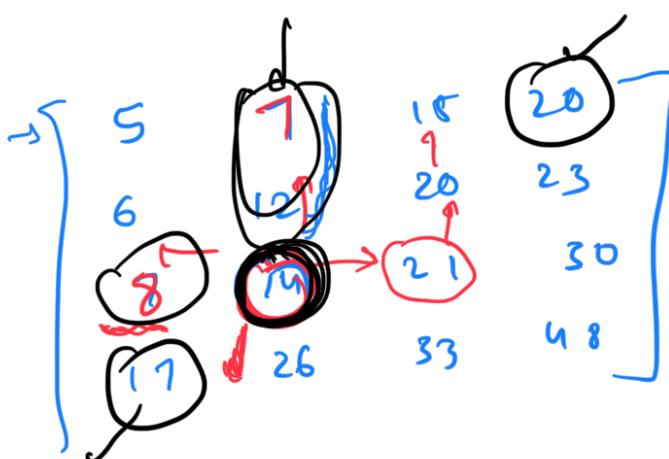
(meta data)  $\Rightarrow$  length  
= Start Address  
End

Add flat



cross the border  
&  
 $O(n)$

$T^*$



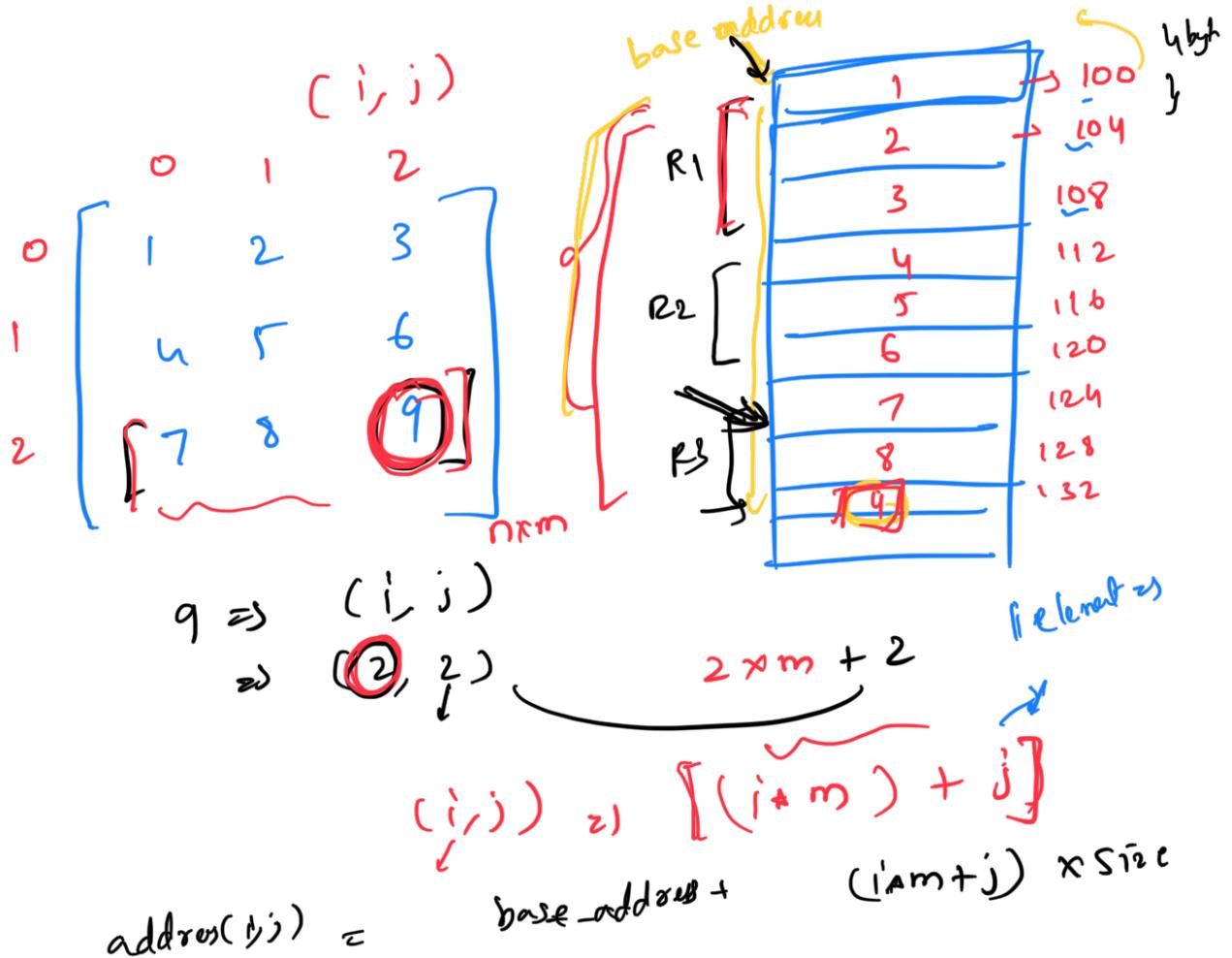
$T = 5$   
 $T = 7$



$T = 7$

T.C:  $O(mn)$

Russian



## Print Matrix in Spiral Way

```
printSpiral(int A[N] [M] {  
    T = 0, B = N-1, R = m-1, L = 0;  
  
    while(L <= R && T <= B) {  
        for(int k = L; k <= R; k++)  
            print(A[T] [k]);  
        T++;  
        for(int k = T; k <= B; k++)  
            print(A[k] [R]);  
        R--;  
  
        for(int k = R; k >= L; k--)  
            print(A[B] [k]);  
        B--;  
  
        for(int k = B; k >= T; k--)  
            print(A[k] [L]);  
        L++;  
    }  
}
```

## Transpose of a Matrix:

```
void transpose(int A[N] [M])  
{  
    for (int i = 0; i < N; i++)  
        for (int j = i+1; j < M; j++)  
            swap(A[i] [j], A[j] [i]);  
}
```

## Find a row with max 1s

```
int findRowWithMax1s(int arr[N] [M]) {  
    int row = 0, i, j = M-1;  
    for (i=0; i<N; i++) {  
        while (arr[i] [j] == 1 && j >= 0) {  
            row = i;  
            j--;  
        }  
    }  
    return row;  
}
```

## Search for target in Sorted Matrix

```
bool searchInMatrix(int arr[n][m], int target){  
    row = 0, col = m-1;  
  
    while(row < n && col >= 0){  
        if(a[row][col] == target)  
            return true;  
        else if(a[row][col] < T){  
            row++;  
        }  
        else  
            col--;  
    }  
    // If element is not found, return false  
    return false;  
}
```