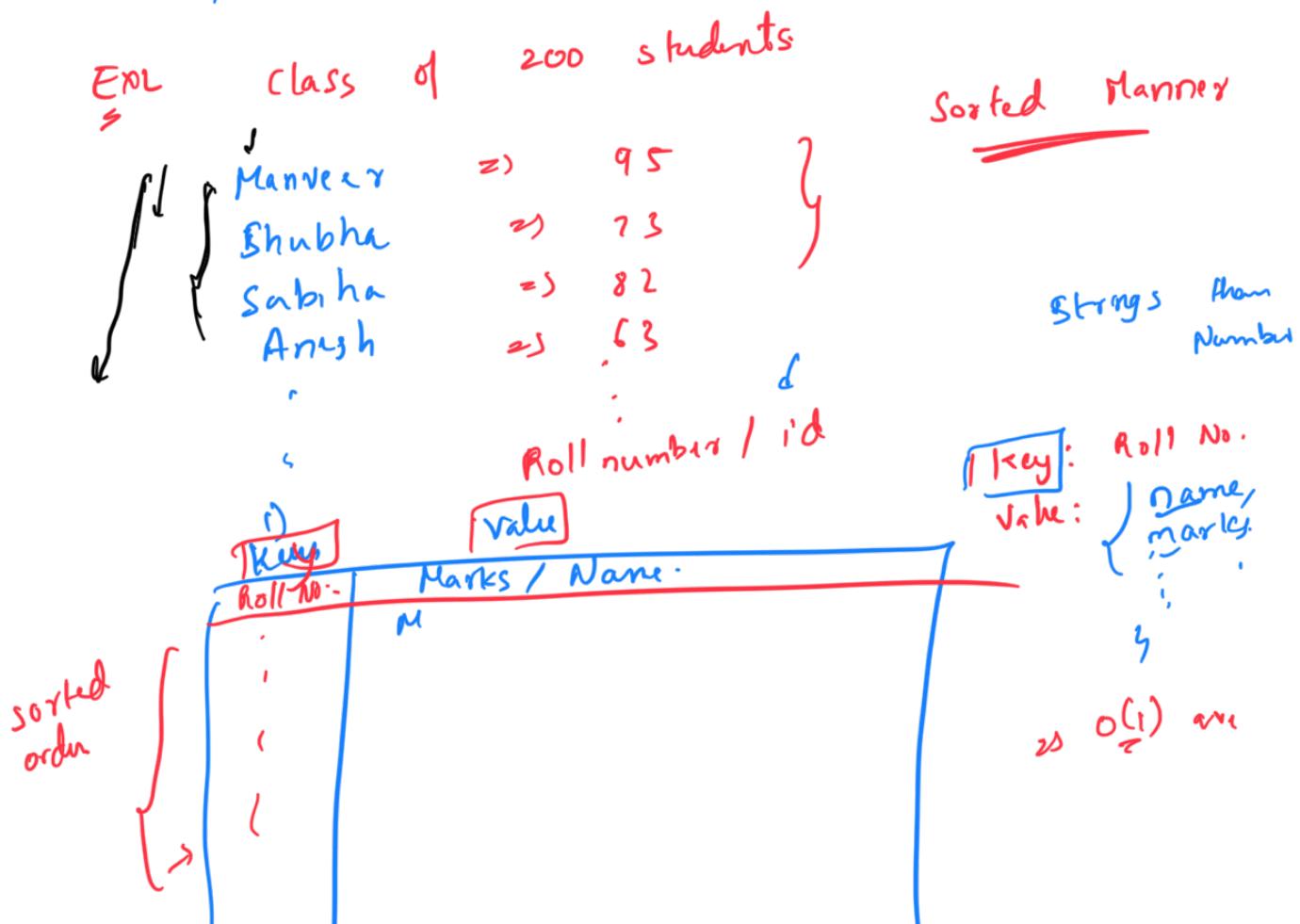
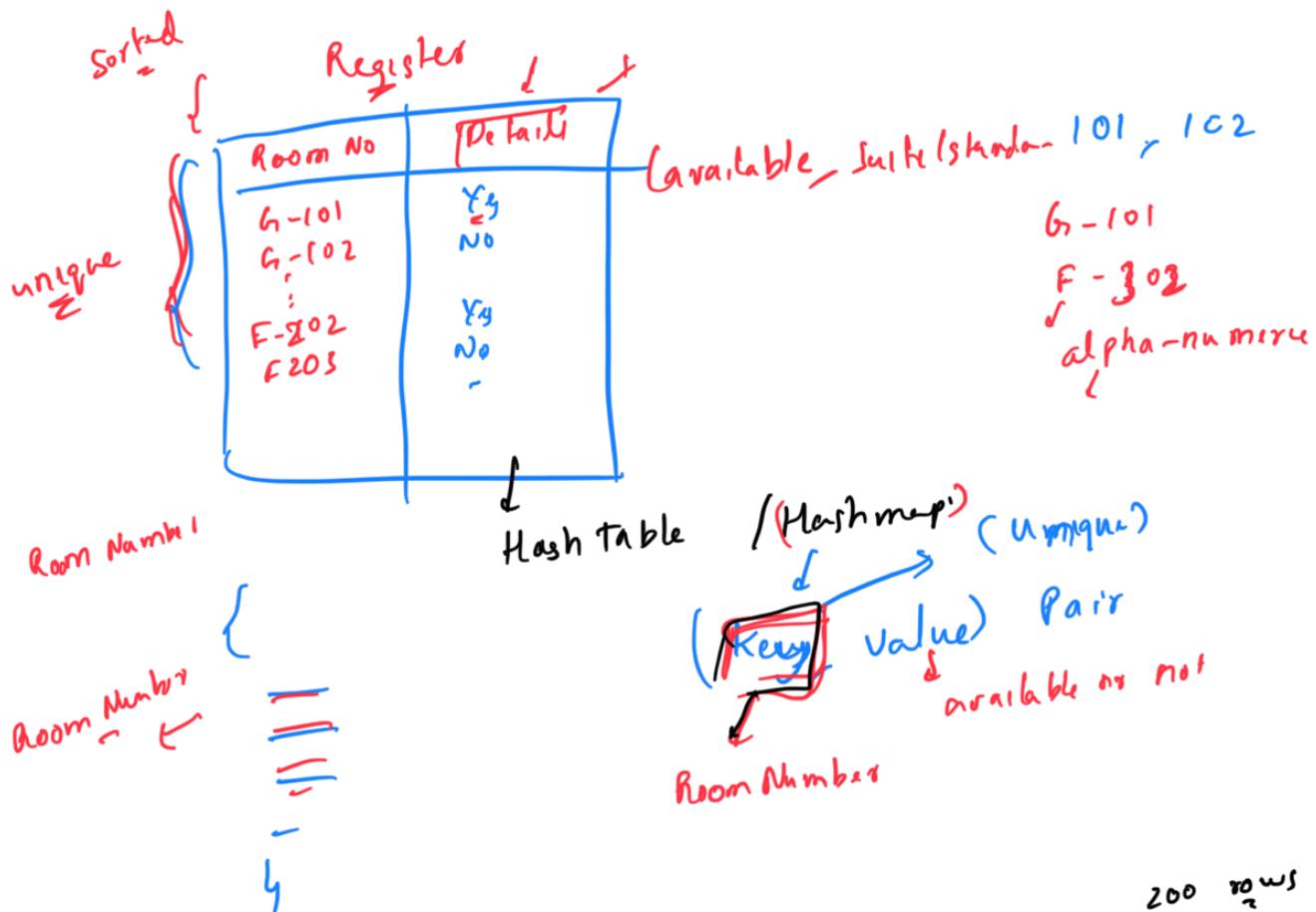


# INTRODUCTION TO HASHING



Binary search  $\Rightarrow O(\log n)$

Hash table / Hashmap:

Hashing:

Arrays

Do this operation in average  $O(1)$

- 1) Insert
  - 2) Update
  - 3) Delete
  - 4) Search
- 2)  $O(1)$  amortized

{ 100 operations take  $O(1)$  time  
- → 99 operations take  $O(n)$  time.  
- → 2 operations take  $O(1)$  Amortized  
Each Operation:  $O(1)$  Amortized

Question:

Getting numbers in range  $(1-500)$

No. of times this number has occurred before.

Ex:	1	1	19	3	19	2	19	3	1	19
(size: 500)	0	0	0	1	0	2	1	1	3	
	0	1	2	3	4	5	6	7	8	9
Freq =	0	1	0	1	0	2	1	1	0	0

Range

was  $(0 - 500)$   
create an array of size 500

an array

$(0 - 10^8) \Rightarrow$  Memory limit error  
number (Range 0-500)

15 99 ...  
1...100

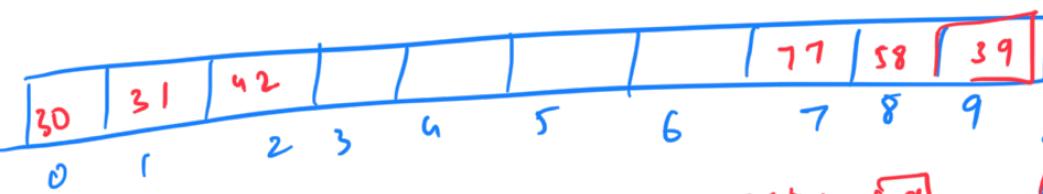
Key: Number  
Value: Frequency  
(Almost 10 different)  
 $(1-100)^2$

Question:

Numbers in range  $[1-100]$  can be  $10^2$   
Array size  $\Rightarrow$  constraint:  $10^2 = 100$

$100 \rightarrow 10$

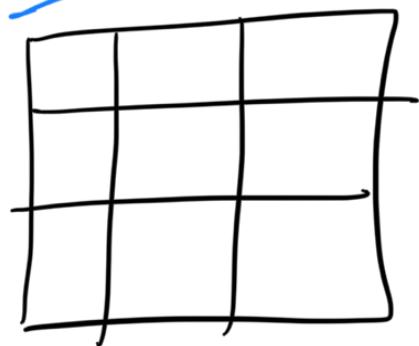
modulo:  $x \% 10 \rightarrow [0, 9]$   
Map 100 elements  $\rightarrow$  10 elements  
 $x \% 10$  Hash Code  
 $\begin{array}{l} 58, 31, 42, 1, 6, 5, 7, 77, 58, 39 \\ | | | | | | | | | | \end{array}$ 
 $\begin{array}{l} 58 \\ 31 \\ 42 \\ 1 \\ 6 \\ 5 \\ 7 \\ 77 \\ 58 \\ 39 \end{array}$ 
 $\begin{array}{l} 1 \\ 3 \\ 9 \\ 7 \\ 8 \\ 9 \\ 1 \\ 10 \end{array}$

$\rightarrow$    
Search for 39?  
 $31 \% 10 = 1$   $\rightarrow$  slot 1

Ex:  $58, 31, 42, 1, 6, 5, 7, 77, 58, 39$   
 $\rightarrow$   $58 \% 10 = 8$   $\rightarrow$  slot 8

100 values  $\rightarrow$  10 values?

Pigeon Hole Principle: 9 slots



$\rightarrow$  10 pigeons

smaller Range,  $\rightarrow$  1, 2, 3, 4, 5, 6, 7, 8, 9

21/10

map

there will be  
 $\lceil \frac{n}{10} \rceil$

collisions (clashes)

Deal with collisions?

21/10 = 1

Chaining:

1, 1, d, d, 51, 63, 33,  
23, 12, 42

BS-S.T.  
 $\Theta(\log n)$

Store the References

Maintain a Linked /  
Dynamic Array

$\lceil \frac{n}{10} \rceil = 1$

$\Theta(n)$



Extra Space  
for storage

10/10

1, 11, 21, 31, 41, 51, 61, 71, 81, 91  
1 4 0 3 6  
 $91/10 = 11$

21/1

T.C:  $\Theta(n)$

→ Uniform Distribution  
→ Less No. of collisions

Array: T  
searching:  $\Theta(1) \times \Theta(n)$

Sorted Array:



$\Theta(\log n)$

Insertion:

2 3 6 7 9 10

$\dots$   $n - 1$   $n$   $\dots$   $n + 1$   $\dots$   $n + m - 1$   $\dots$   $n + m$

$\Theta(n)$

$\Theta(1) \times$

Can we do better than this?

Java 7 → Java 8

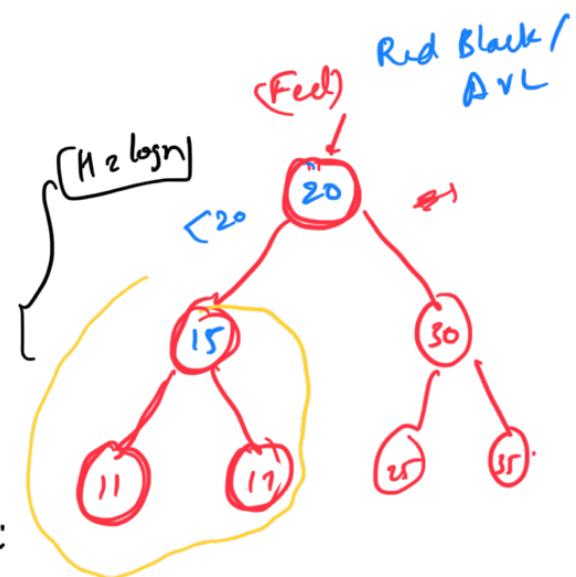
~~Balanced~~

Binary Search Tree

Binary Search:

Insertion, Deletion, Search:

$O(\log n)$



get Min:

$O(1)$

$O(n)$

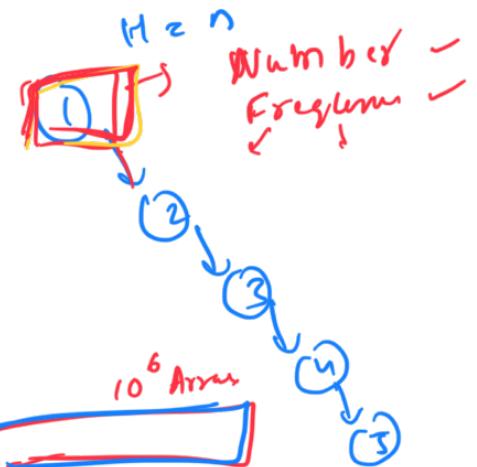
Heap / Priority

Pointers to left, right

Balanced BST

$H = \log n$

B. S. T

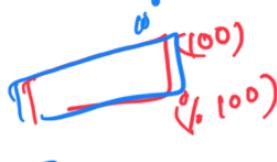


{ Question:  
Getting

Key: String  
Value: Frequency  
Hashcode: Index  
random strings

a b c

0



b z y

0

b c a

0

a b c

1

a b c

2

b z y

1

Anish Anish

$n \% 10$   
 $(str) \% 10$

integer to string

↓ (Index)

9/6/10  
String Hash

Random Access / Quick lookup  $\Rightarrow$  Array  
Task: Map thus string to integer!  
Hash Function: string  $\rightarrow$  Number

Option 1:	Consider	ASCII value of 1st character
a	= 97 -	⑨)
ab	= 97 +	1) Two many collisions
abc	= 97 + 98 -	2) Not using space
bca	= 98 + 97 -	
abb	= 97 + 97 +	

26 entries

$$(a - z) \Rightarrow 26$$

Toabb  $\Rightarrow$   
ASCII: int  
(abb b c x n ... )

Option 2:	Sum of ASCII value.	String $\rightarrow$
a ab	$97 + 97 = 294$	
ab b	$97 + 98 + 97 = 294$	}
b c a	$98 + 99 + 97 = 294$	
abb	$97 + 98 + 97 = 293$	

Anagrams:

abld, bcad, adbl ... same hash val

ac, bb  $\Rightarrow$

Option 3: Polynomial Rolling Hash Function

str =  $s[0] + s[1]p^1 + s[2]p^2 + s[3]p^3$

$\Rightarrow$  First char  $\downarrow$  o: Prime Number  $> 2^6$

10-15

$s[i] \rightarrow$  ASCII value of  $i^{\text{th}}$  character

Java:  $| p[23] |$

$p(n) = 22n + 1$

$$F(iab|d) = (97 + 98 \times i + 99 \times j + 100 \times k \dots)$$

$= a + b \times p + c \times p^2$

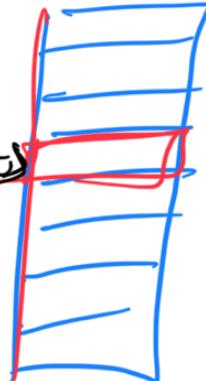
(very big)

~~Prime Number:~~

- 1) Lesser Collision
- 2) Uniform Distribution of keys.

1)  $p > 26$   
 2)  $p \geq 31 \Rightarrow$  prime

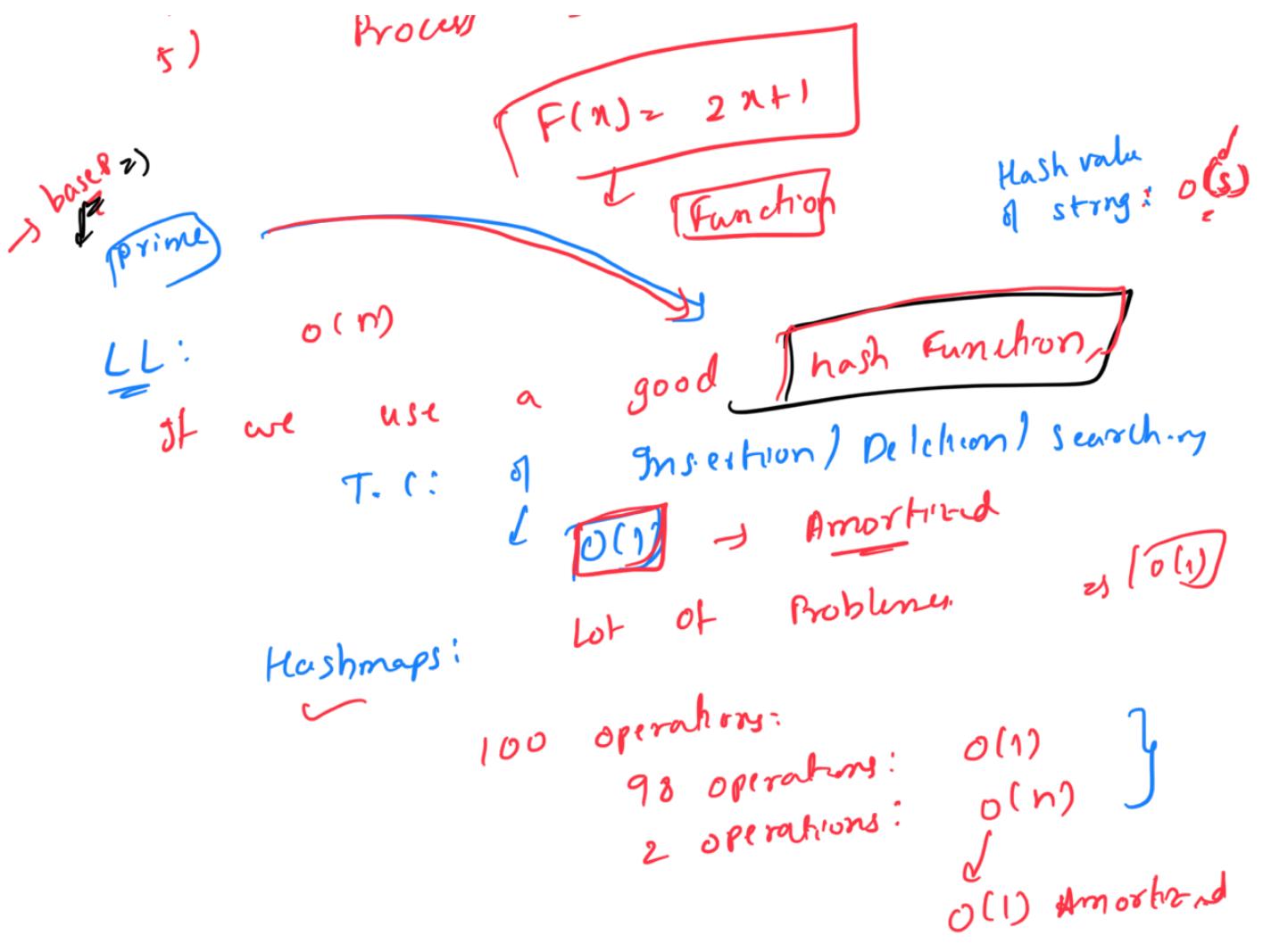
Characteristics of Good Hash Function

- 1) No / less collisions  
 Deal with it  $\langle \text{key}, \text{value} \rangle$   $\xrightarrow{\text{hash code}}$  hashable
  - 2) Fast computation  
 $n \% 10 \Rightarrow O(1)$
  - 3) Even distribution  
 $F(n) = 2^n + 1$   
 $\Rightarrow$  odd indices will be occupied
- 

$O(M)$

Summary:

- 1) Hash Map: Key - Value pair
- 2)  $\text{int} / \text{string} \Rightarrow$  int to use as an index of array
- 3) Hash Code: Hash Function
- 4) function  $\rightarrow$  is called Hashing



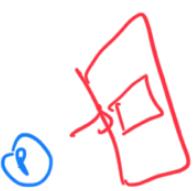
## Hashing Libraries

HashMap :  $\langle \text{key}, \text{value} \rangle$

Frequently of string

key: string  
value: int

HashMap<string, int> mp1;



Hashset :  $O(1)$

$O(1)$   $n^2 \dots$

Question:

Give us

you some  
occurred

number, tell  
or not.

$O(1) \Rightarrow$   
 $O(1)$

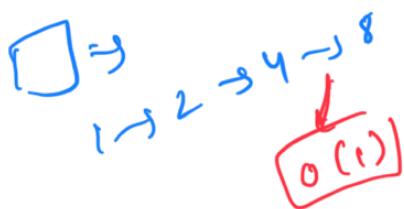
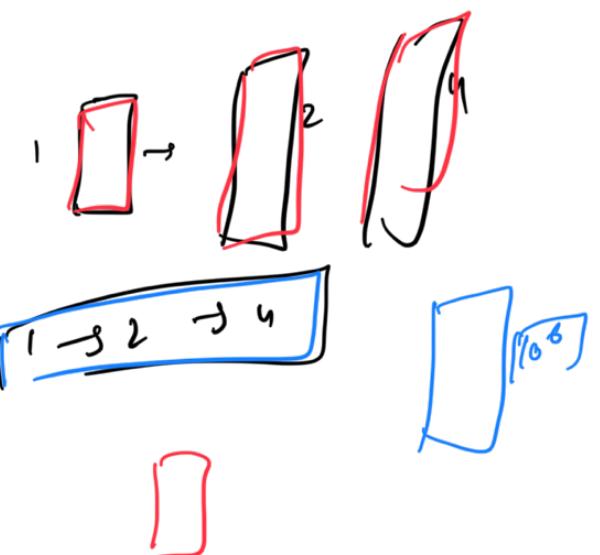
if it is  $\Rightarrow$

$\Rightarrow 10$

$\text{Arr} = \{5, 10, 11, 5\}$   
 $\text{Set} = \{5, 10, 11\}$   
 $\rightarrow$  store only key  
 Time Complexity:  $O(1)$   
 Space Complexity:  $O(n)$   
 (Hashmap)

(++)  
 gone:  
 unordered\_set  
 Hashset, Flashmap } [Flash Map]

Dynamical Array



Hashmap: implemented using Hash table.

map = {2: 3, 3: 2, 4: 1}



Question:  
 $\rightarrow$  Array = [2, 3, 2, 2, 3, 4, 5, 7, 2, 5]  
 Size  $n$

$(-10^9) \leq q \leq 10^5$   
 Queries: 2, 7

Hashmap <int, int>  
 ↓, ↓  
 Frequency

$[ \quad ]$ : 2 → Numbers  
GFB  
 $B = 5$   $n = 3$   
 $\alpha^{-1}$   
 $\gamma$  times

```

    Hashmap<int, int> mp;
    for(int i=0; i<n; i++) {
      O(1) ← [Check if a[i] in mp]
      O(1) ← [mp[a[i]]++]
      else { put {a[i], 1} in mp; }
    }
  
```

$\Theta(n)$

```

    for(i=0; i<q; i++) {
      ele = B[i];
      if ele in mp return mp[ele];
      else return 0;
    }
  
```

$\Theta(q)$

queries:  $[5 | 2 | 1 | 1 | 6 | 2 | 9 | 10]$   $q = 9$

T.C:  $(O(n) + O(q))$

Step:  $O(n+q)$

T.C:  $O(n+q)$

$$q = N^2$$

$$\therefore O(n+q) \approx O(n+n^2) \approx O(n^2)$$

Question:

$A = [2, 0, 1, 2, 10, 3, 20, 20, 1, 5, 6, 7, 8]$

$O(1)$

$\downarrow$

$ele = [0, 3, 5]$

$\downarrow$

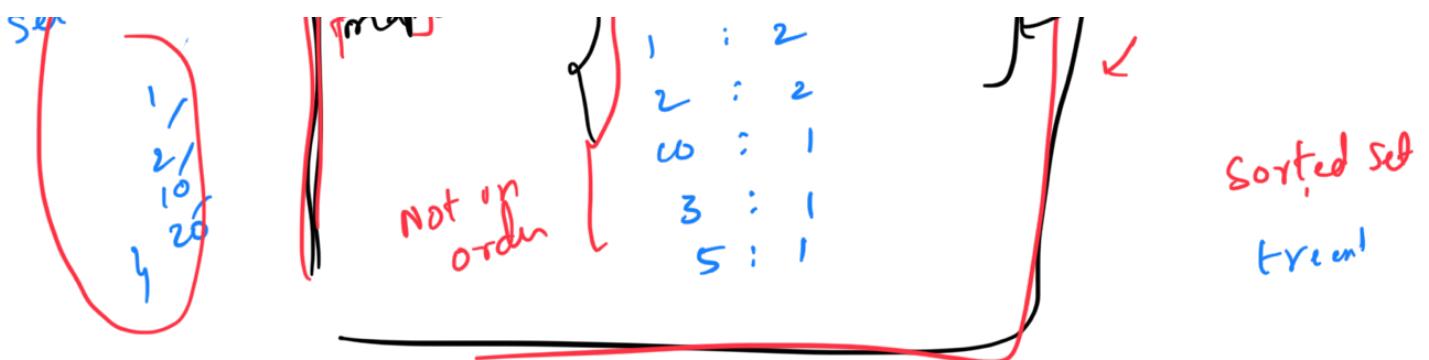
$\boxed{[2, 0, 1, 2, 10, 3, 20, 20, 1, 5, 6, 7, 8]}$  (Key: 10)

$\boxed{[20: 2]}$

$\boxed{4 / }$

First Non-Repeating element

Freq = 1

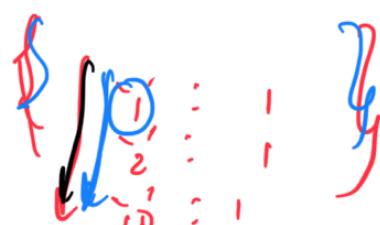


$O(1)$   
Not store in sorted order

T.C.:  $O(n) + O(n)$

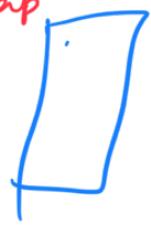
20 1 2 10 20 1 5 25

Tree set: sorted order



1 2 10

Map  
Hash Function



T.C:  $O(n) + O(n)$   
:  $O(n)$

Hashmap

2) 20 1 2 10 20 1 1 3 25

Set = {1, 2, 10} Hash map

Question:

Given  
(No)

2 arrays  
Duplicates

$\rightarrow O(n^2)$

$N \leftarrow A = 3 2 10 7 5 15 9$

$M \leftarrow B = 7 1 18 2 16 10 23$

Ans = 7 10 2

$\rightarrow O(N)$

Hashset:

setz { 3, 2, 10, 7, 5, 15, 9 }

T.C:  $O(n+m)$

Duplicity:

Hashset

(Hashmap)?

Hashmap = <key, value>

Question:

Goldman, Sahni, Google, Deshpande

$1 + (K-1)$

$K=9$

A = [ 7, 9, 10, 2, 5, 4, 16, 3 ]  
0 1 2 3 4 5 6

$$a[i] + a[j] = c_K \quad \text{if } i \neq j$$



$O(n^2)$

$O(n)$

Brute Force:

for  $a[i]$ ,

$(K - a[i])$

T.C:  $O(n^2)$

Efficient Approach

A = [ 1, 4, -2, 8, -9, 14, 25, 22, 17, 13 ]  
 $K = 22$

setz { 1, 4, -2, 8, -9, 14, 25, 22, 17, 13 }  
(14, 14) Yes

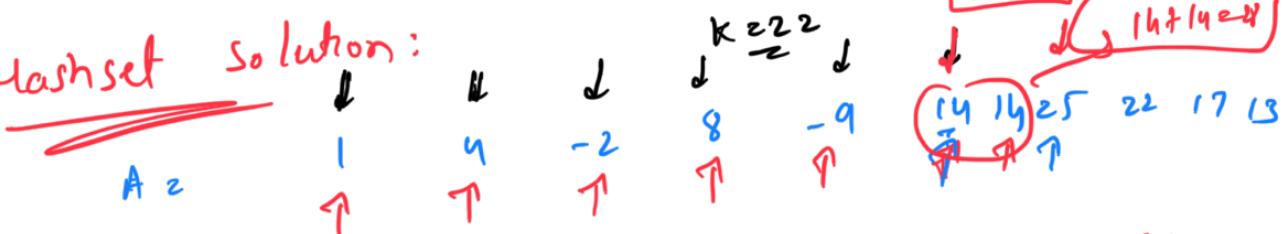
$(1, \infty)$   
 $[4, 18) \times$   
 $(-2, 24) \times$   
 $[8,$

Flashmap:  $\Rightarrow$   $\langle \text{Num, Frequency} \rangle$   
 Frequency

$$a_{ij} \approx K - a_{ij}$$

$a_{ij} = K - a_{ij}$   
 only if  $\text{freq}(a_{ij}) \geq 2$  True

Hashset Solution:



set =  $\{1, 4, -2, 8, -9, 14\}$

$\circ$  HashFunction  
 {intnum}

$(14, 8)$

$K = 28$

(2)  
G.W.

-9,

1) Pair s.t.  $a_{ij} - a_{jj} = K$   
 2) pair s.t.  $a_{ij} + a_{jj} = K$

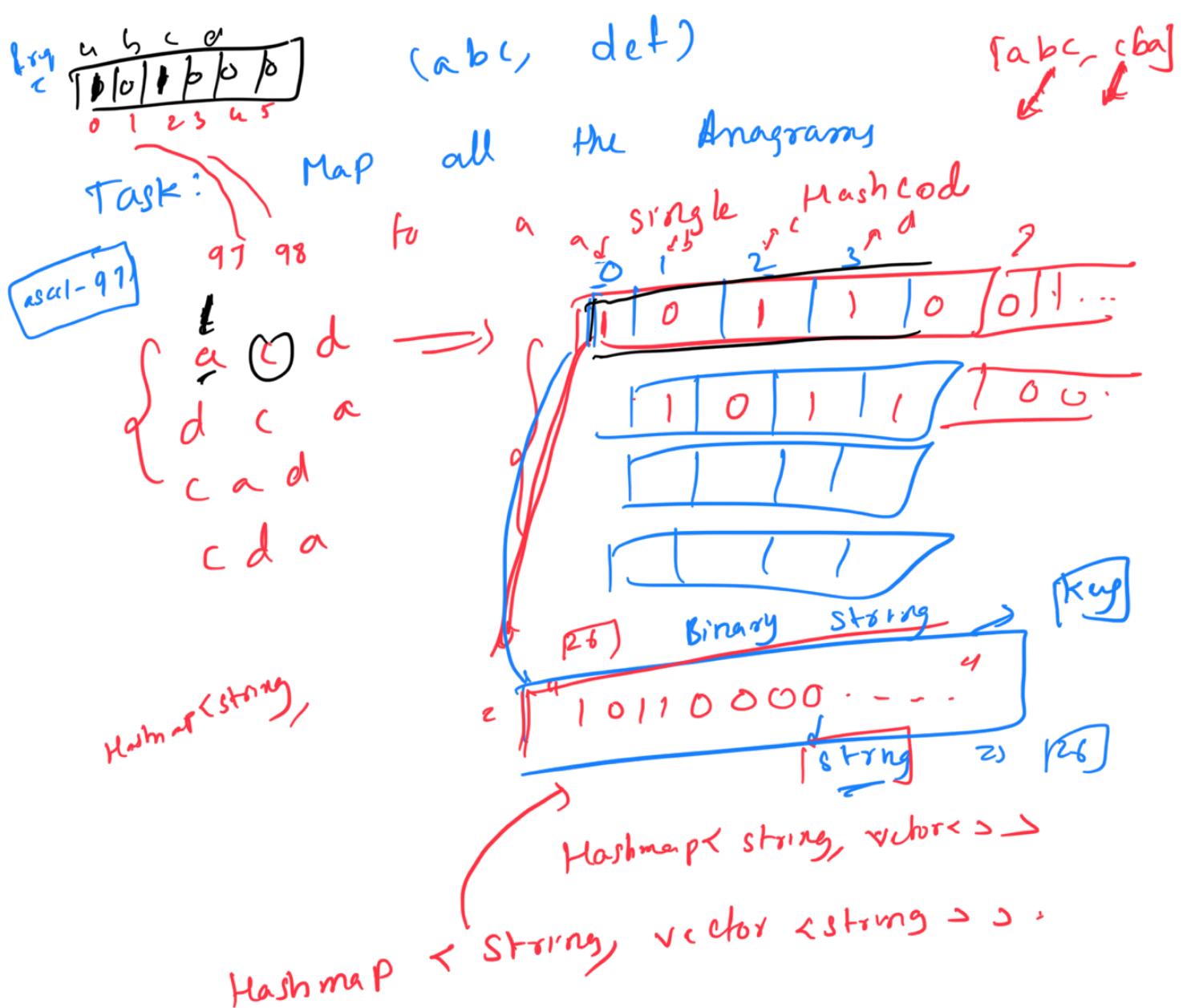
[Goldman, M.S., Amazon]

Question:

Given array of strings, group all the  
 anagrams together. (Permutations)

$A = \{nlmo, abc, xyz, cab, lmn, \dots, 7\}$

$\text{pqrs} \quad \text{rpqr} \quad \text{omln} \quad \text{nmos}$   
 array of arrays (vector of vector)  
 $\Rightarrow \{ \text{nlmo}, \text{lmno}, \text{omln}, \text{nmos} \}$   
 $[abc, cab]$   
 $[pqrs, rpqr]$   
 $[xyz]$   
 i) sum of ASCII  
 $ac = bb$  (block)



## Approach 2:

a cd,      dca,      cad,      da  
   ↓            ↓            ↓            ↑  
   acd        acd        ~~acd~~        acd

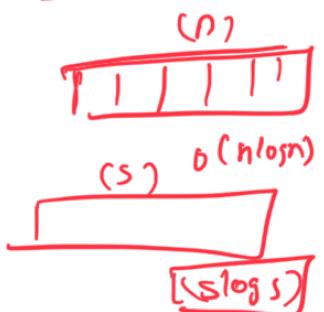
use sorted string as the key:



hashmap<string, vector<int>>

key: sorted string

size's string:  $\lceil \log_2 n \rceil$   
                     -  $O(n \cdot \log n)$



Can we sort faster?

Count Sort      (Bucket Sort)

Count Sort

$S = b\ da\ b\ cde$        $\xrightarrow{\text{Count Sort}}$   $\xrightarrow{\text{O}(S)}$

Freq.  $\begin{array}{cccccc} a & b & c & d & e & f \\ \hline 1 & 1 & 2 & 1 & 2 & 1 \end{array}$        $\xrightarrow{\text{Bucket Sort}}$   $\xrightarrow{\text{O}(S)}$

$T = ab\ b\ c\ d\ d\ e$

Total Cost:  $O(S) + O(S) \approx O(S)$

T.C.:  $O(S \cdot N)$

$O(26) \approx O(1)$

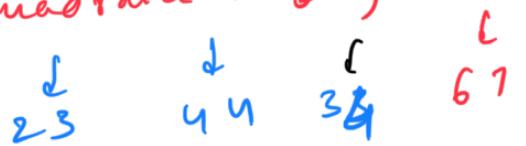
$(26)^N$

$26^N$

## Chaining

Linear Probing

Quadratic Probing



$$\lceil \frac{n^2}{6} \rceil, \lceil (x+1)^2 \rceil, \lceil (x+2)^2 \rceil, \lceil (x+3)^2 \rceil \rceil / 6 \text{ slots}$$

Delete : 13

13

13

13/10

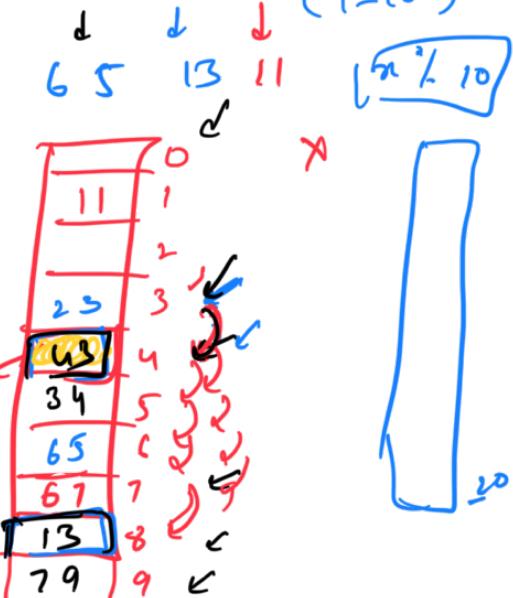
Search : 13

Flag +

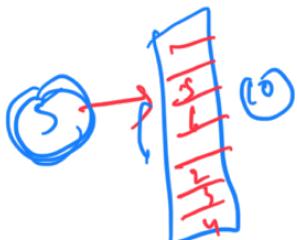
$$\lceil \frac{n^2}{6} \rceil, \lceil (n+1)^2 \rceil + \lceil (n+2)^2 \rceil / 6 \text{ slots}$$

Array = 10

$$(1-100)$$



Delete : 44



GCD / HCF

H(n) =

$$\lceil \frac{n}{12} \rceil$$

$$\lceil \frac{n}{6} \rceil$$

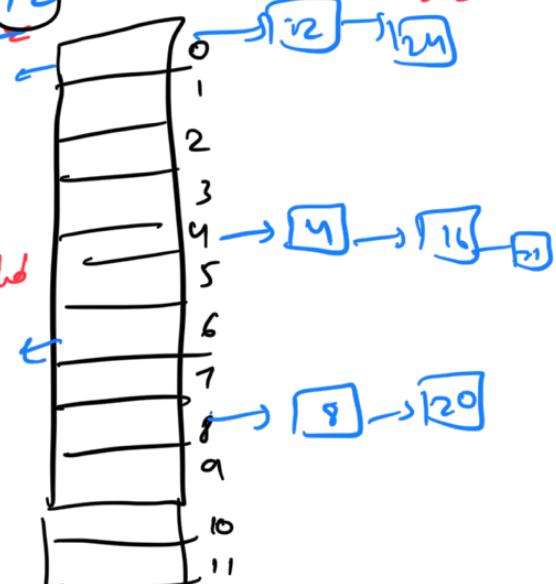
→ Less Collisions  
→ Uniform Distribution

Multiply of 4:

$$4, 8, 12, 16, 20, 24, 28$$

$$\frac{12}{\gcd(12, n)}$$

Only 3 buckets very filled



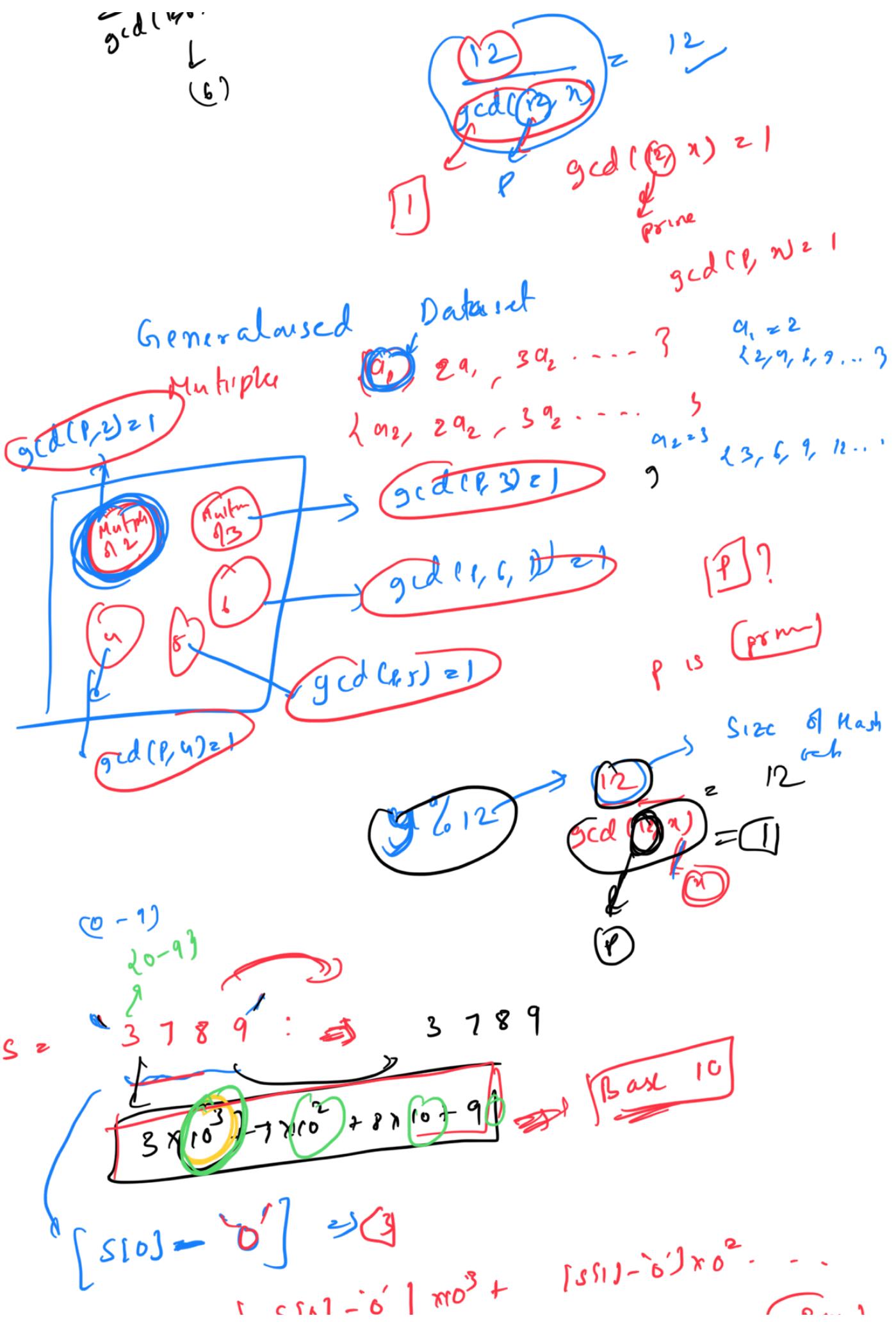
2) Multiply of 6:

$$6, 12, 18, 24, 30, 36$$

$$\{0, 6\}$$

$$\frac{12}{6} = 2 \quad \gcd(12, n) = 4$$

$$\frac{12}{4} = 3$$



1 > 100

$$st: 'a-z' \quad (26)$$

$$abcd = ('a' * 26^3 + b * 26^2 + c * 26 + d) \quad \text{mod } p$$

(p > 26) Base 26

$p \geq 26$

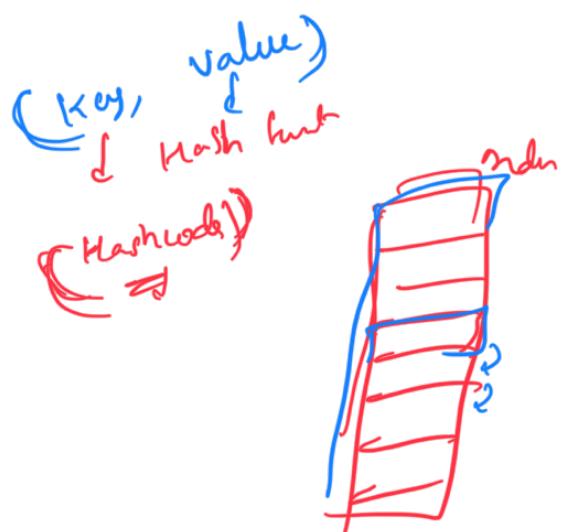
$\% \text{ Mod}$

$P$

10 | 3 | u | t |

2 3 u t

↳ Leads to error!



### **Pair with sum = k**

```
bool checkSum(int a[], int n, int k){  
    unordered_set<int> st;  
    for(int i=0;i<n;i++){  
        if(st.find(k - a[i]) != st.end()) {  
            return true;  
        }  
        else  
            st.insert(a[i]);  
    }  
    return false;  
}
```