

Trees - I

Amazon, M.S, Adobe

{ LL as Tree }

- Recursion
- Structure of the tree

}

- 5 session
- Trees 1, 2
 - B.S.T
 - LCA
 - Problems on Trees

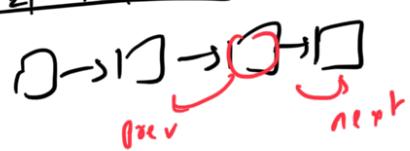
Linear Data Structures :

Arrays, L.L, stacks, Queues
↳ Elements are arranged in a linear / sequential manner.

Array:

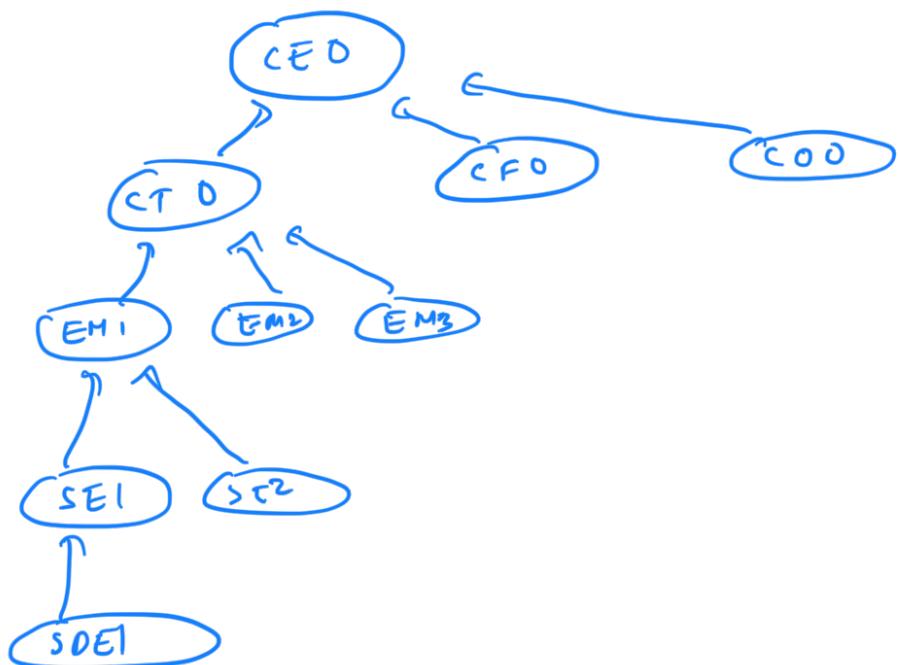


LL:



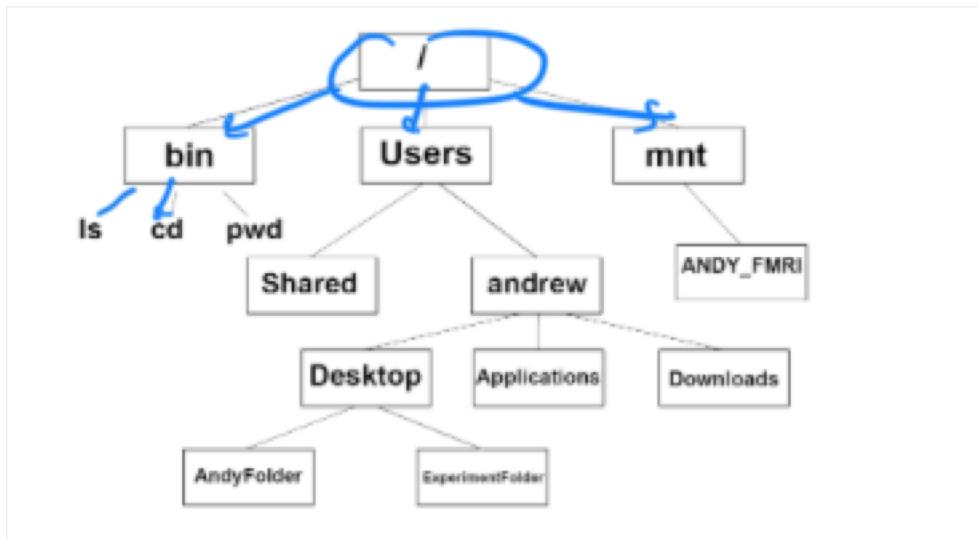
→ No Hierarchy

Trees → Come into picture



SDER

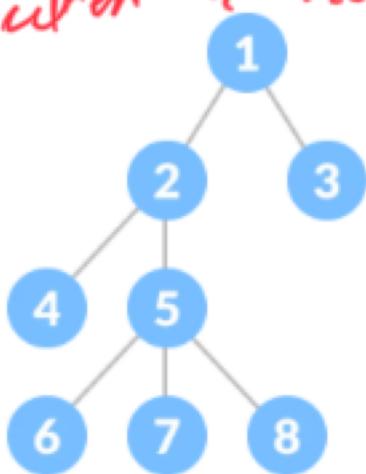
Directory:

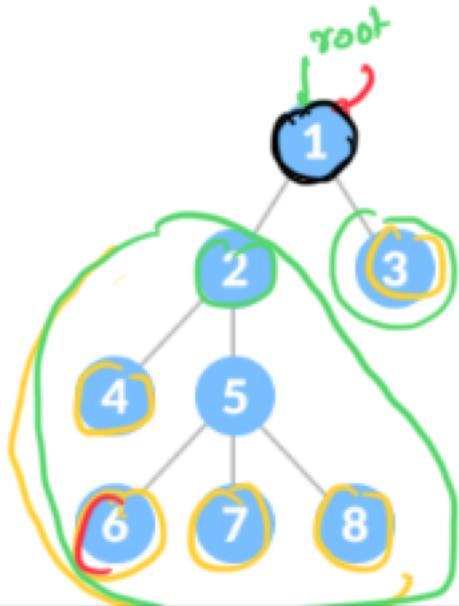


Non Linear D.S : Tree, Graph
Heap Segment Trees Try

DP?

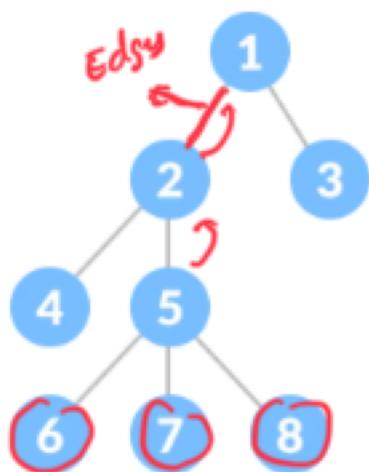
Collection of nodes/entities which are linked together to form a hierarchy





Terminology

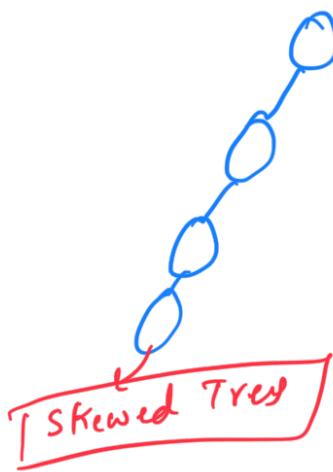
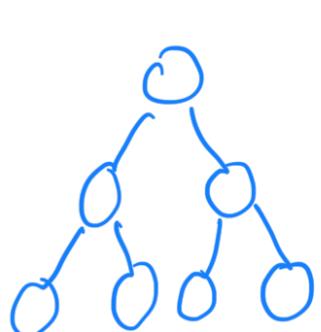
- Root $\Rightarrow 1$ [N : parent]
- 2, 3 are children to 1
- 1 is the parent of 2
- $\text{par}(1) = 5, \text{par}(2) = 5$
- Nodes with no children \Rightarrow leaf
- \rightarrow Left Subtree & Right Subtree are terms
- 6, 7, 8 are called siblings



$\text{grandP}(5) = 1$
 $\text{grandP}(8) = 2$
 $\text{grandC}(2) = 6, 7, 8$
 $\text{Ancestor}(7) = 5, 2, 1$
 $\text{Ancestors}(n) = \dots, 1$
 $\text{Successor}(2) = 4, 5, 6, 7, 8$

Binary Tree

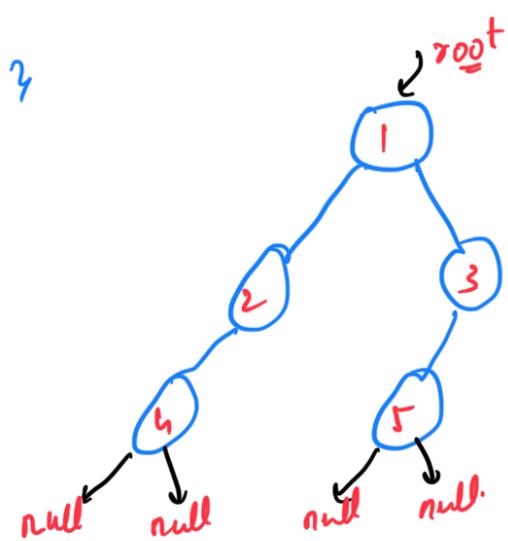
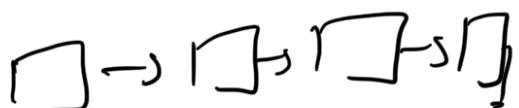
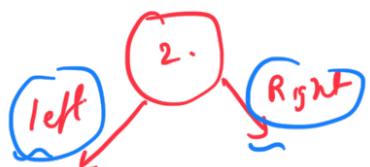
If it is a special tree where every node can have almost 2 children {0, 1, 2}



∅ NULL
Empty tree is a Binary Tree

Structure

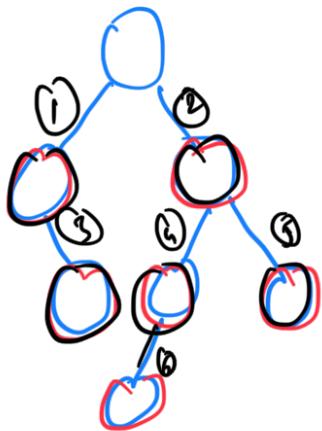
```
class Node {
    int data;
    Node left;
    Node right;
}
```



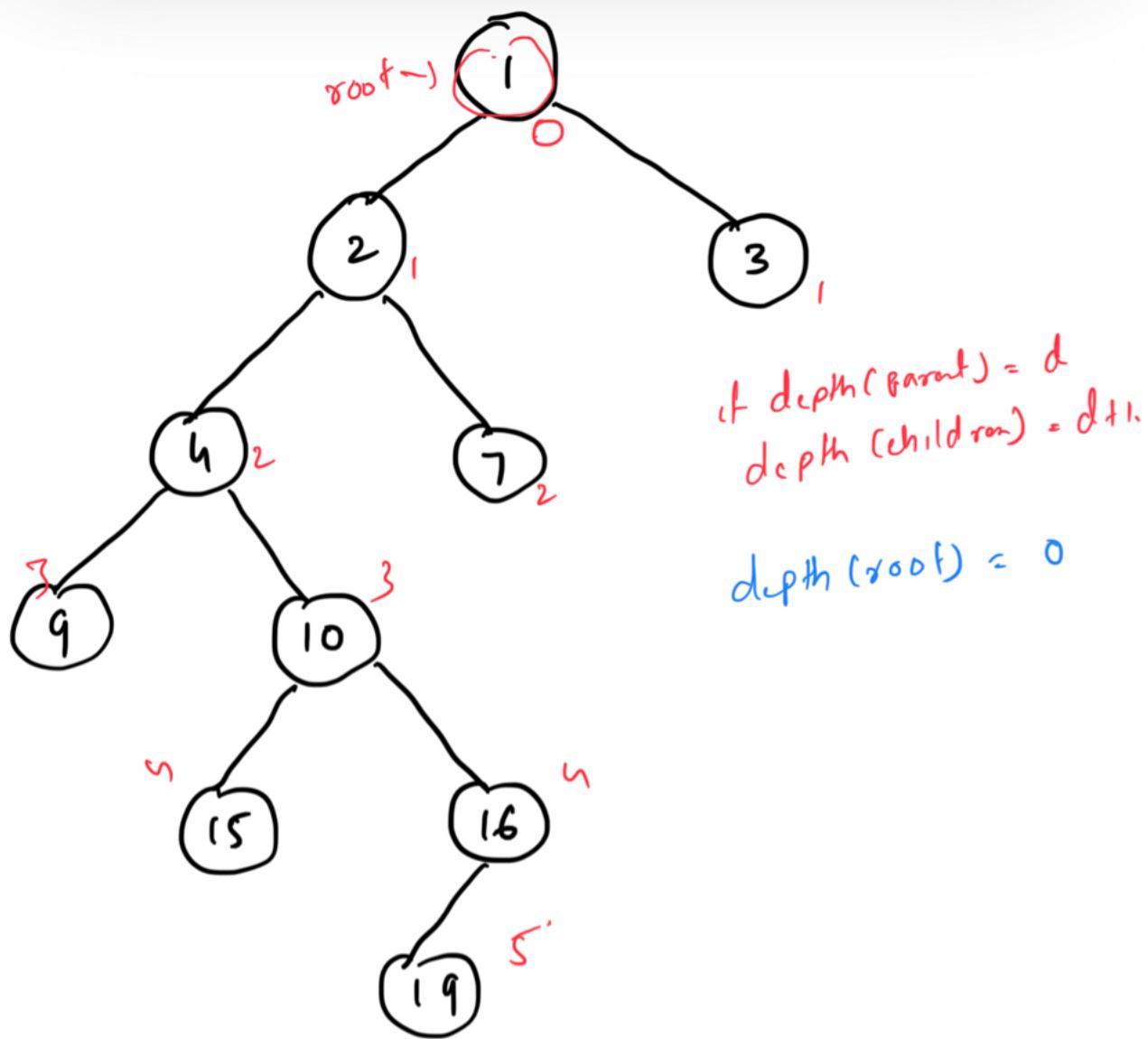
Properties:

- N nodes \Rightarrow $(N-1)$ edges

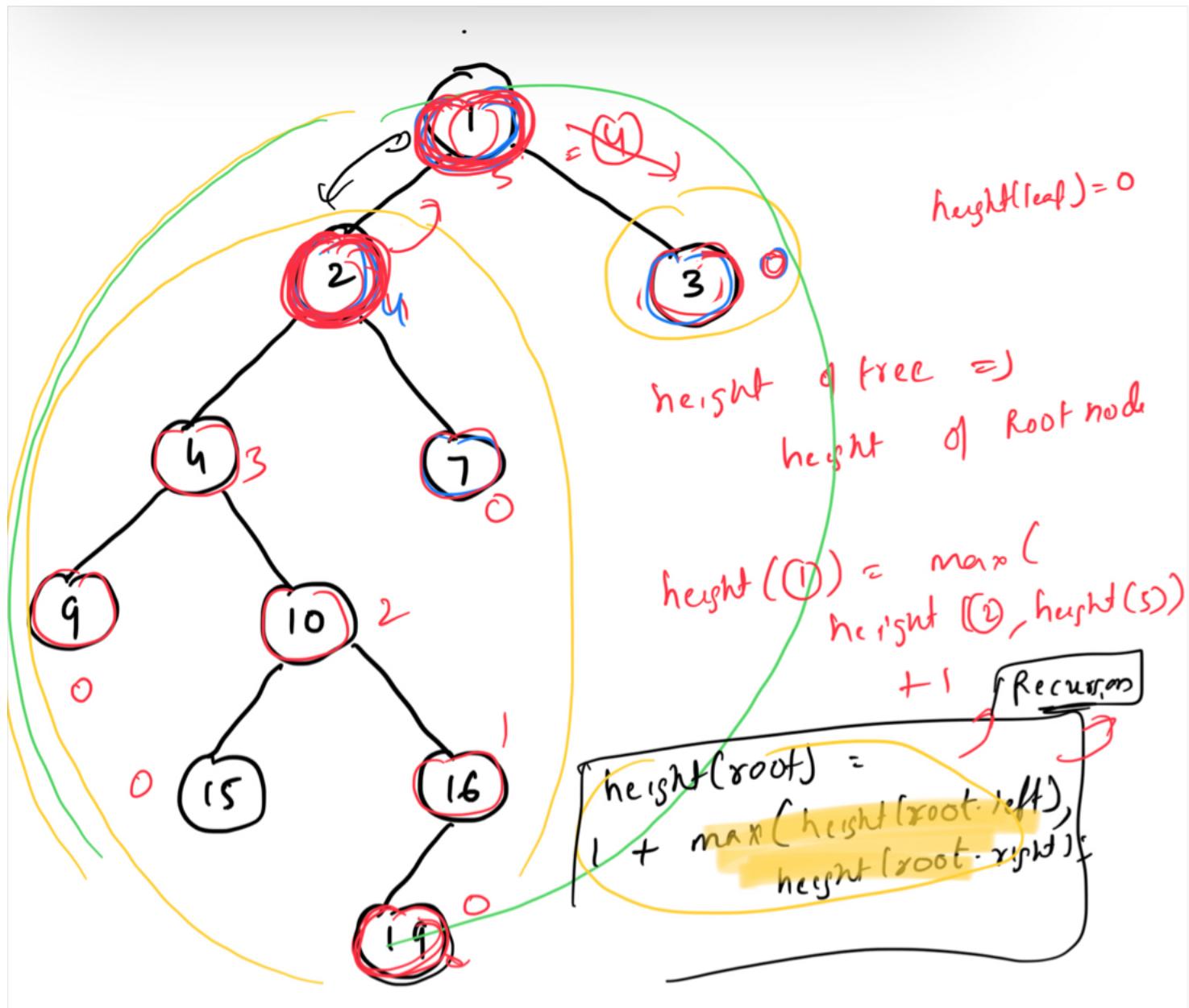
- Every node except root has an incoming edge = $S \frac{N-1}{2}$
- No node will have more than 1 incoming edge



2) Depth of a Node :
Distance (No. of edges) from root to the current node



3) Height of Node: Distance (No. of edges) from current node to the farthest Leaf Node.

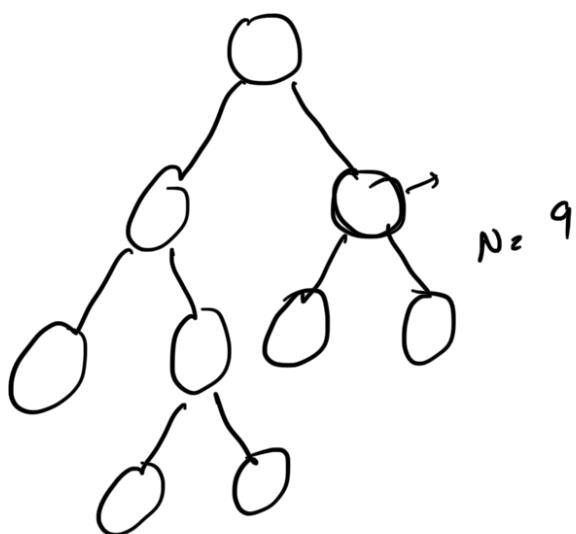
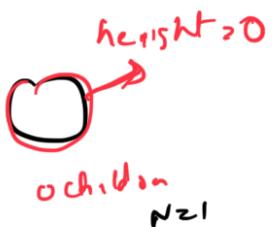
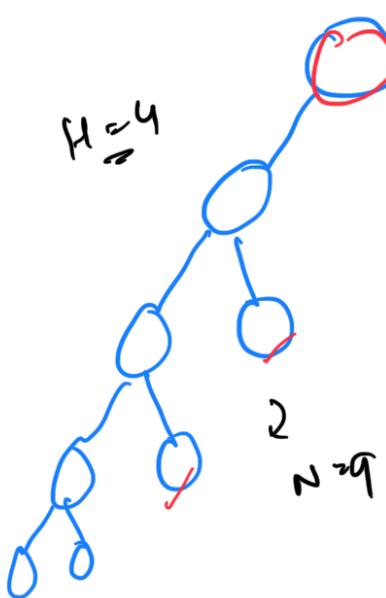


Height \approx tree = Depth of farthest leaf from root
 $\text{height}(\text{node}) = 1 + \max(\text{height}(\text{node.left}), \text{height}(\text{node.right}))$

Types:

1) Proper Binary

A Tree where every node has either 0 or 2 children.



$$[N \ 8 \ 1] \Rightarrow$$

$=1 \Rightarrow$ Odd No
 $0 \Rightarrow$ Even No

proper Height

$$N = N_1 + N_2$$

N_1 = No. of nodes with 0 children
 N_2 = No. of nodes with 2 children.
 $E_{\text{leaf}} = 0 + 2 \cdot N_2$

$$N_1 + N_2 - 1$$



Contd

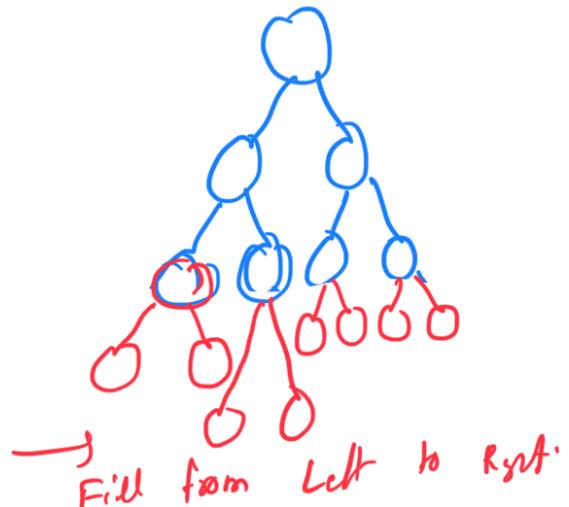
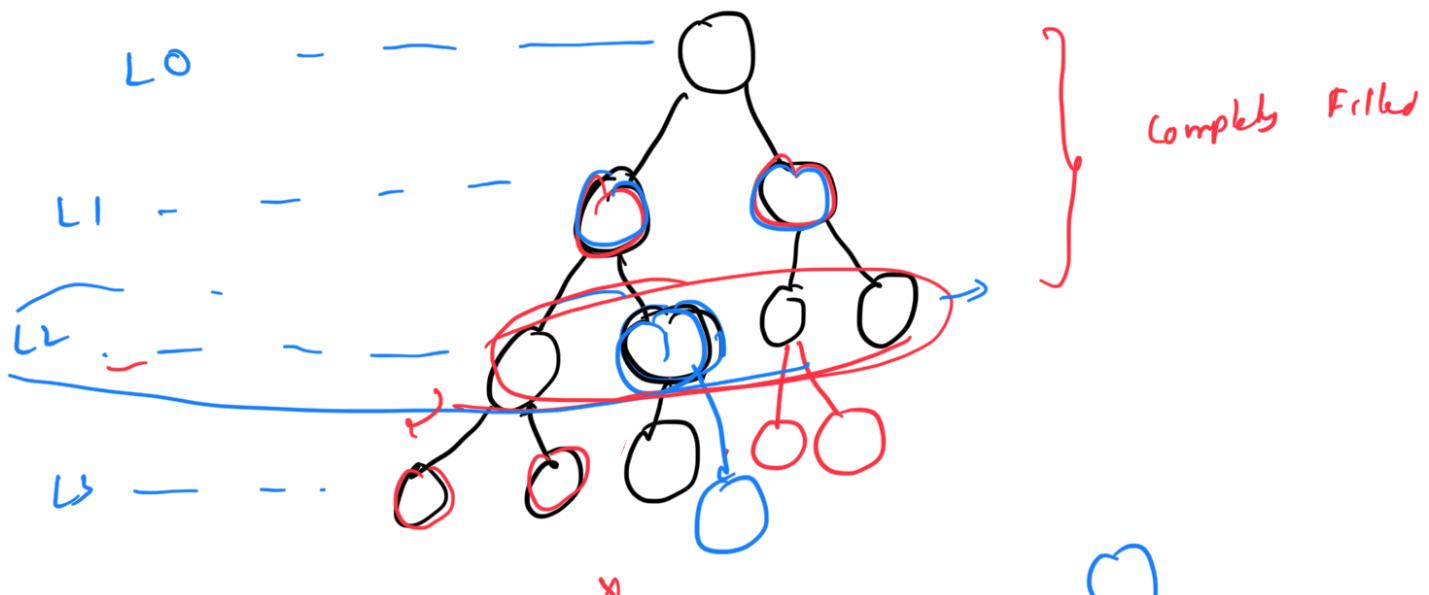
$$\text{Nodes} = \frac{\text{Edges} + 1}{2}$$
$$= \frac{i2N_2 + 1}{2}$$

Even

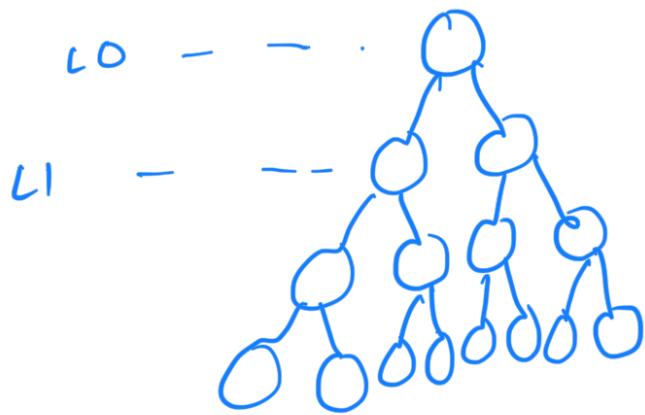
Odd Number

2) Complete Binary:

- Every level is completely filled, except the last level (possibly):
→ In the last level, all the nodes are possible.

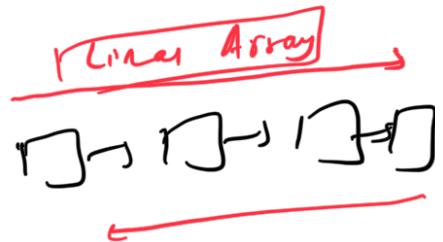


Perfect binary tree: All the levels are completely filled



$$L(n) = k \text{ nodes}$$

$$L(n+1) = 2k \text{ nodes}$$



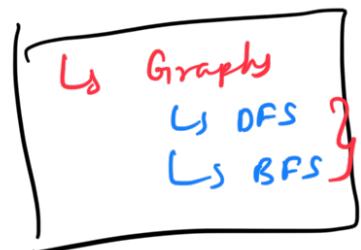
Traversals:

the process
in any

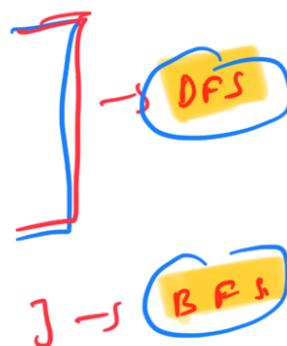
of
D.S

visiting

every node



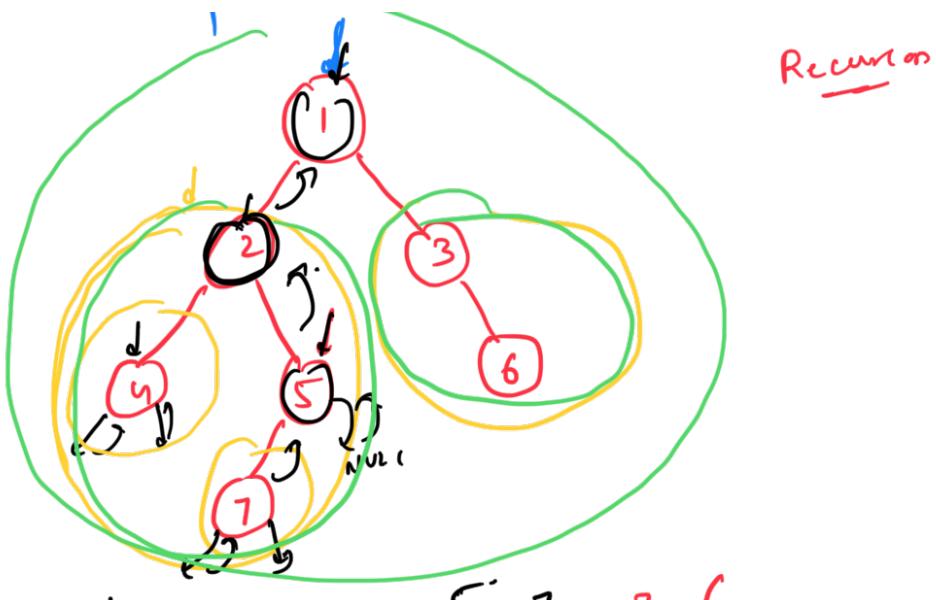
- 1) Preorder
- 2) Inorder Traversal
- 3) Postorder
- 4) Level Order



Preorder
root, Left, Right

Inorder
Left, Root, Right

Postorder
Left, Right, Root



Preorder:

1 2 4 5 7 3 6

Inorder:

postorder:

Recursion

1) Assumption: preordu (root) will point the pre-order traversal

2) Main Logic:

```
print(root.data);
preordu (root.left);
preordu (root.right);
```

3) Base Case:
 if (root == null) return;

```
void preordu (root) {
    if (root == null) return;
    print (root.data);
    preordu (root.left);
    preordu (root.right);}
```

7
void Inordu (root) {
if (root == NULL) return;
}

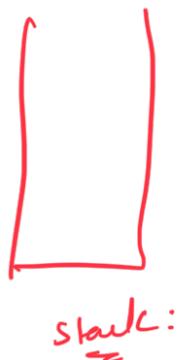
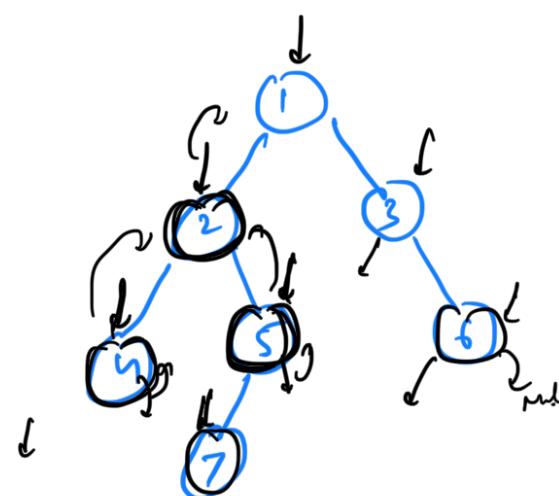
Inordu (root.left); ✓
print (root.data);
Inordu (root.right);

void Postordu (root) {
if (root == NULL) return;

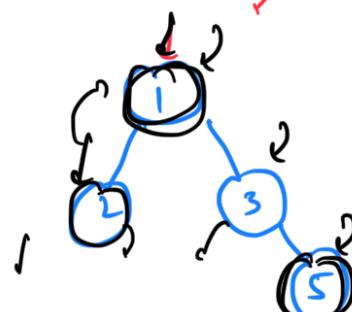
Postordu (root.left);
Postordu (root.right);
print (root.data);

Recursion Stack

height of tree



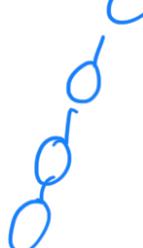
Inordu: 4 2 7 5 1 3 6



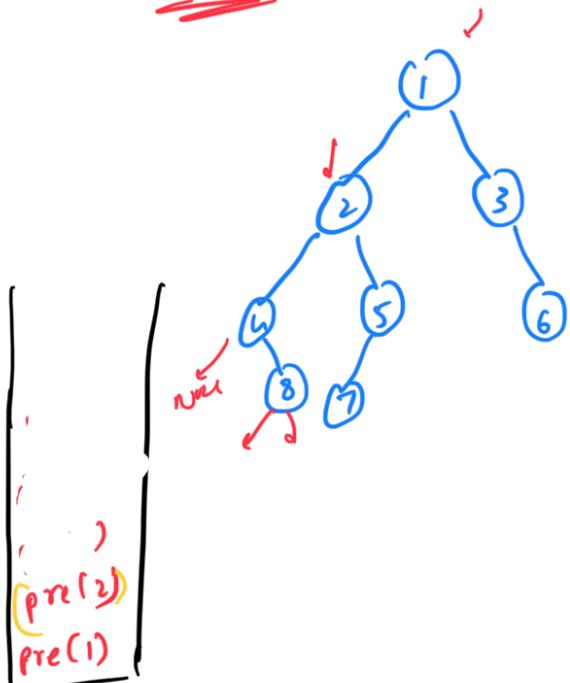
T.C: $O(N)$
S.C: $O(H)$

$$N = H + N - 1$$

Postordu: 2 5 3 1



Iterative:



Recurren stark.

gterakon:

shark



Stack:

$$\text{Cur} = \gamma_{00} t.$$

`curr = curr.left`

$\text{Carry} = \text{top-right};$

$$C_{\text{max}} = 7 \cdot \tau_{\text{light}}$$

Cart = 7 · right

$\text{c}_w.xf = \text{root};$)

```
cur8 = root; // cur8 != NULL  
while (!str.empty()) {
```

$\text{M}_{\text{outlet}} = \text{NULC} \{$

```

    print ( curr.data );
    stk.push ( curr );
    curr = curr.left;

```

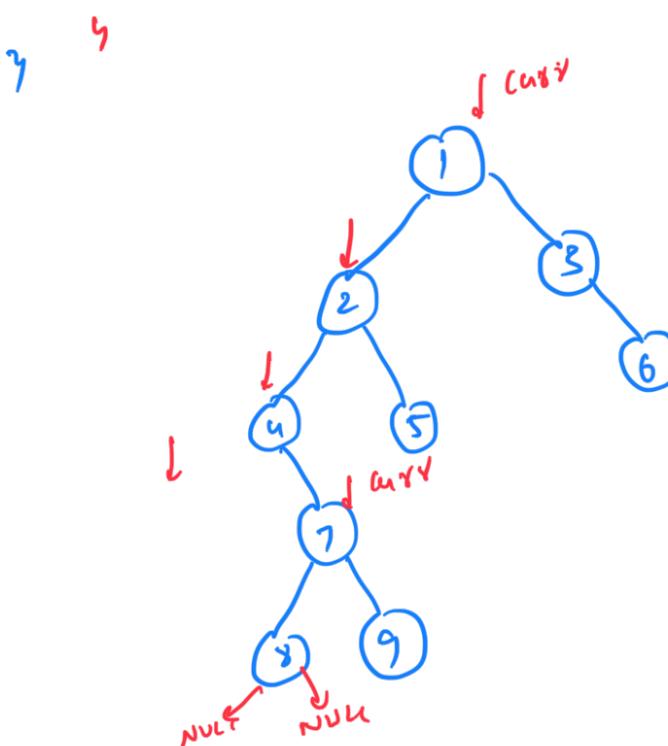
y

```

else {
    top = stk.top();
    stk.pop();
    curr = top.right;
}

```

y



stack:

Output: 4, 8

Postorder:

Preorder:

Postorder:

Reverse(Post Order) =



root = root-right

i) Find Reverse of Post order ↴

-> Print Reverse (Ans) ↴

v)

.....

$$Tc: O(n)$$

$$S.C: O(H)$$

$$\downarrow$$

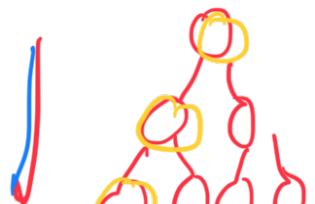
$$H \Rightarrow O(n)$$

Doubtly

When to use Preorder (Postorder)?

Preorder: First, root is visited.

Control is going from top to bottom

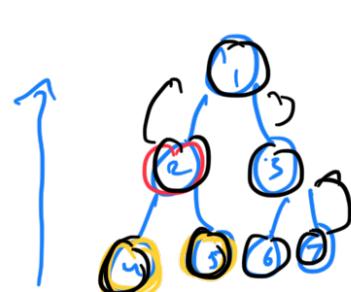


- { 1) If we don't need info of Left & Right Subtree
 2) Apply first - last method



Postorder:

Control is from bottom to top



Question:

Find sum of nodes in a B.T



root

$$\text{sum}(3) = 3 + \text{sum}(2) + \text{sum}(5)$$

```
int sum(root) {
    if (root == NULL) return 0;
```

```
return root.data + sum(root.left) +
```

```
sum(root.right);
```

$$3 + 2 + 5 + 1 + 2 + 3 + 7$$

Postorder Traversal

$$\text{sum} = 0$$

inorder,
preorder,
postorder

~~$\text{sum} = \text{root.data}$~~

Questions:

Max value in

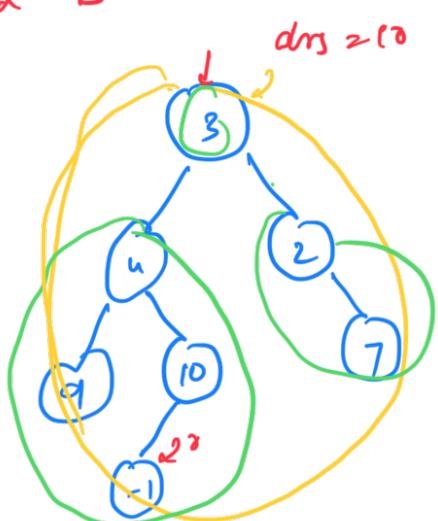
a B.T.

```
int maxm(root) {
    if (root == NULL)
        return -INFINITE;
```

```
return maxm(root.data,
            maxm(root.left),
            maxm(root.right));
```

?

Postorder



$$\max(-1, 0, 0) = 0$$

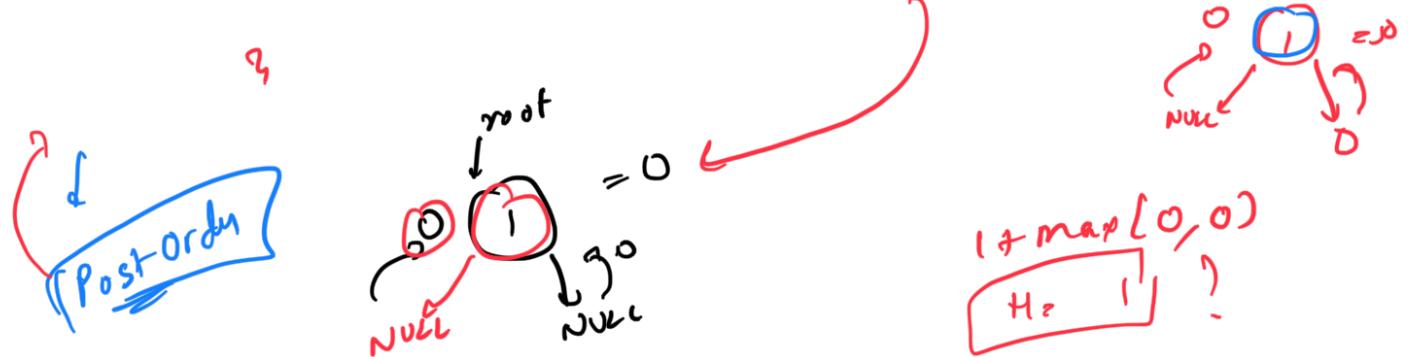
$$\underline{\underline{H}} = \underline{u}$$

... of Tree

Question: Height

```

int height( root ) {
    if( root == NULL) return -1;
    if( root.left == NULL && root.right == NULL) return 0;
    return 1 + max( height( root.left ),
                     height( root.right ) );
}
    
```



Count

```

count( root ) = 1 + count( root.left ) + count( root.right );
if( root == NULL) return 0;
    
```

Question: Depth of node

$$\text{depth}(\text{parent}) = d$$

$$\text{depth}(\text{child}_m) = d+1$$

$$\text{depth}(\text{root}) = 0$$

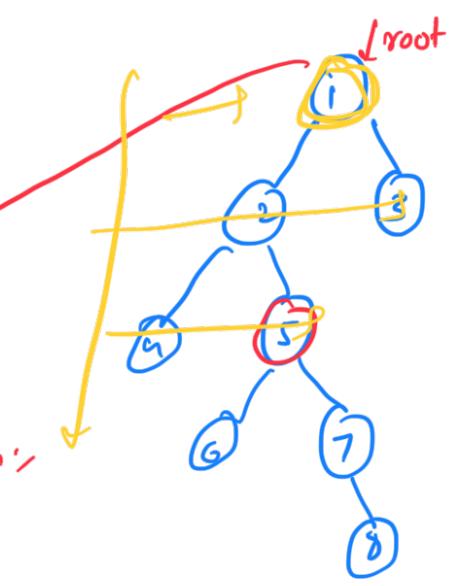
$$\text{depth}(1, 0)$$

```

void depth( root, depth ) {
    if( root == NULL) return;
}
    
```

$$\text{root.depth} = \text{depth};$$

$$\dots \text{if } \text{depth} + 1$$



Node 5

Preorder

depth (root.left, curDepth);
depth (root.right, depth + 1);

int data
Node left, right,
int depth = 0;

Question:

2D array

{
 [1]
 [2, 3]
 [4, 5, 6]
 [7, 8, 9]]

Level Order Traversal

queue =

L0

L1

L2

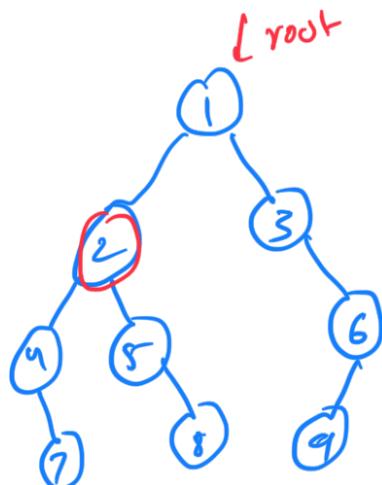
L3

Level Order:

1 2 3 4 5 6

First In First Out

currDepth = 3



Approach::

Maintain

a queue

q

{Node, depth}

[7, 8, 9]

[4, 5, 6]

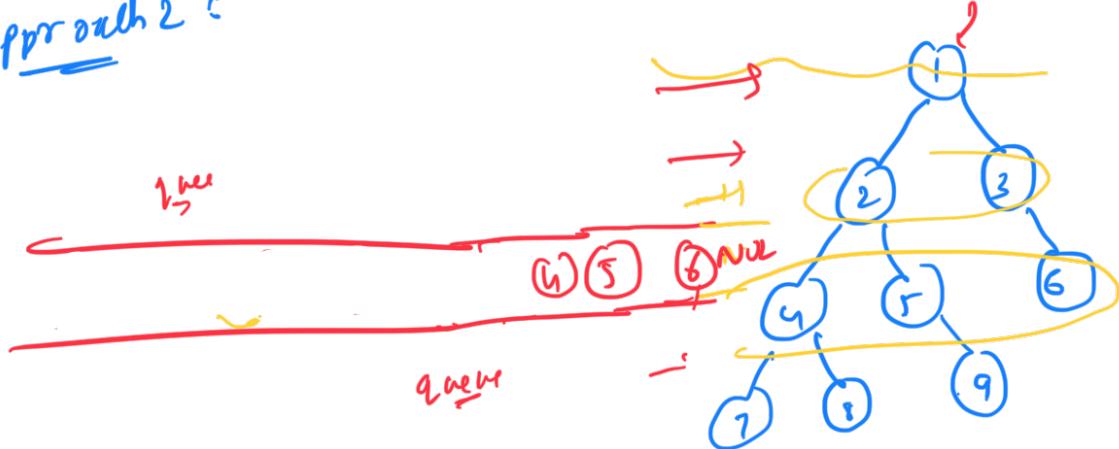
[2, 3]

[1]

queue

(7, 3), (8, 3), (9, 0)

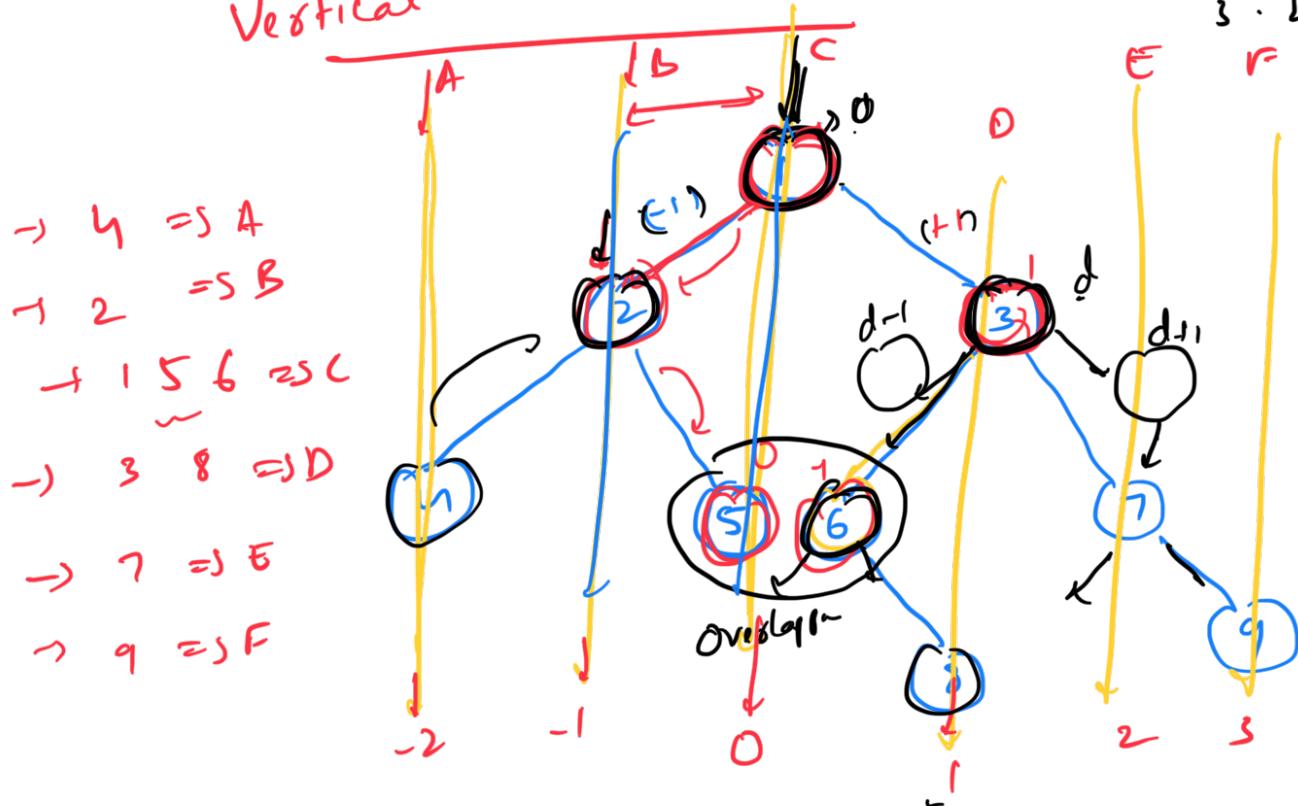
Approach 2:



$\rightarrow \cdot \downarrow \curvearrowright$

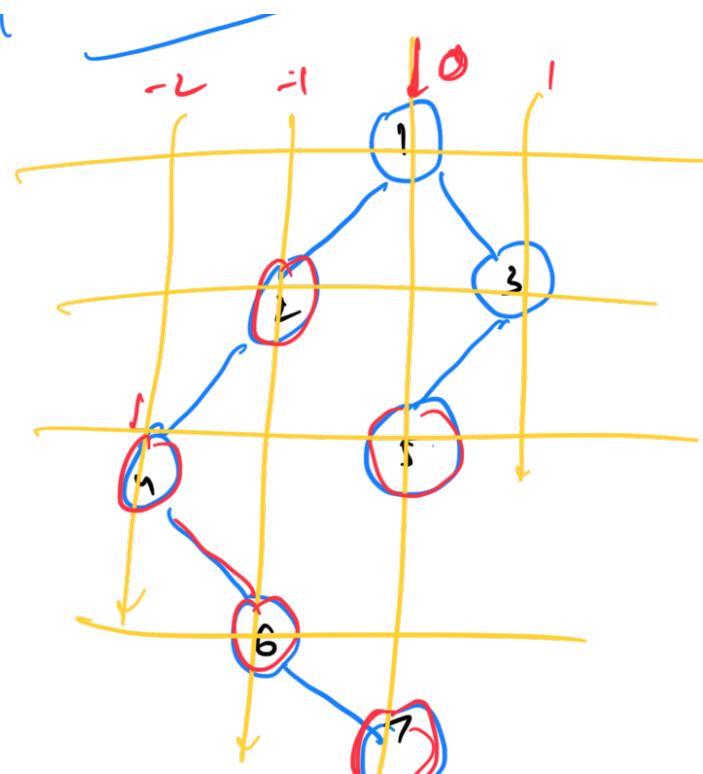
vector<Node>
 Hashmap <int, ?>
 Hashmap : {
 0 : [1, 5, 6],
 -1 : [2,
 -2 : [4],
 1 : [3, 8],
 2 : [7],
 3 : [9] }

Vertical Order Traversal



prefix: root, left, right

from root



$\delta-1$

$\delta+1$

$\{1, 5, 7\}$

HashMap = $\begin{cases} 0: [1, 7] \\ -1: [2, 6] \\ -2: [4, \dots] \end{cases}$

Level Order Traversal

vertical Order (root, δ)

vertical Order (root, left, δ)
n. 11
(root, right, δ)

n^{δ}

$d-1$

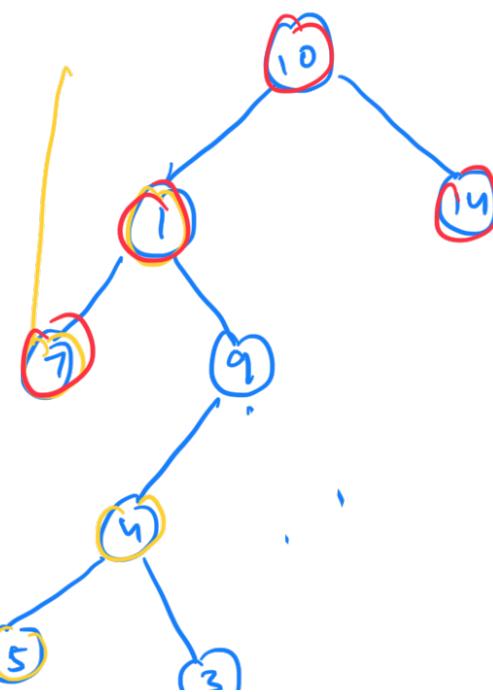
$d+1$

(left, $\delta-1$) (right, $\delta+1$)

Views of a Binary Tree



First Node of
every level
is in level order

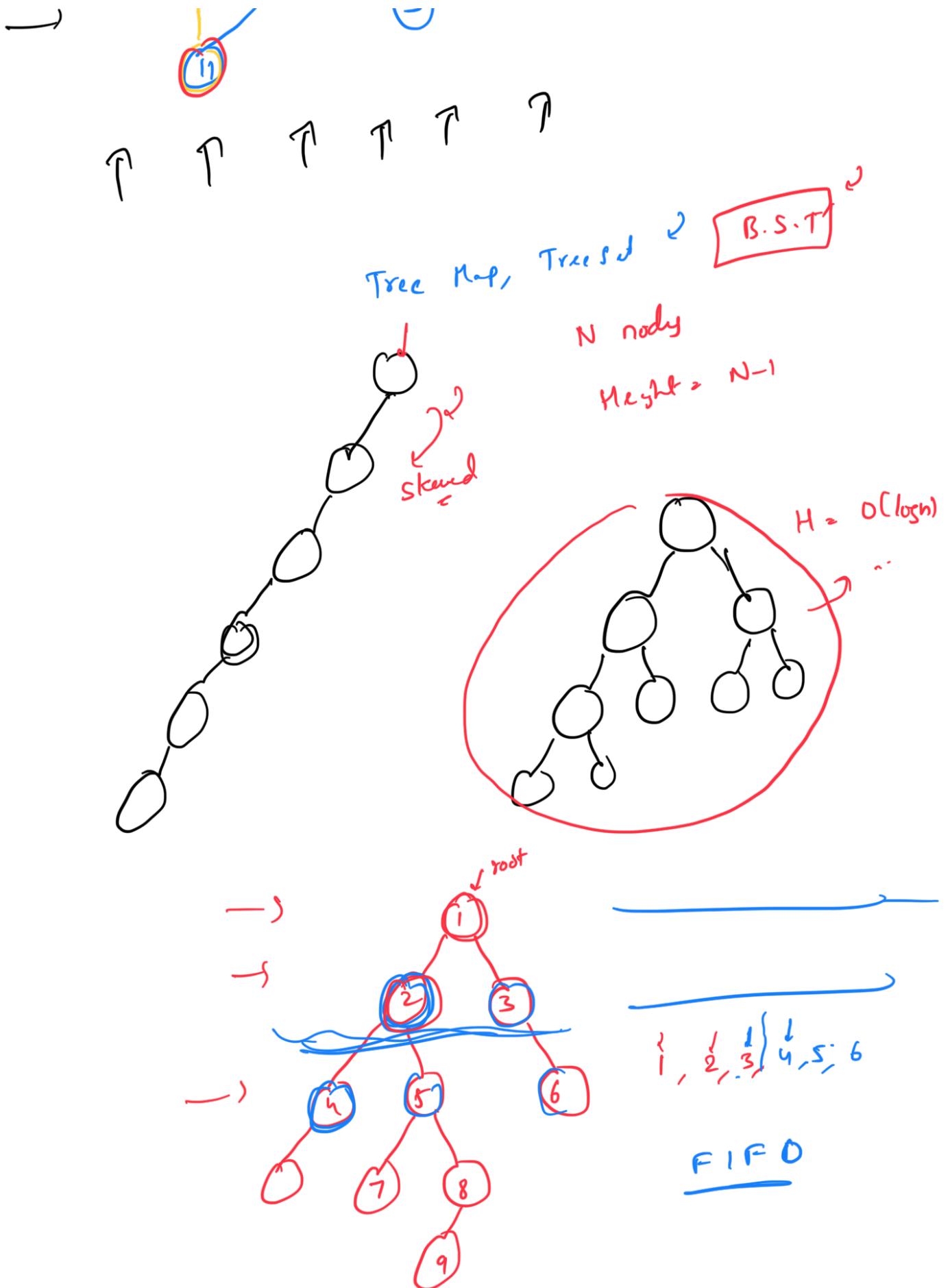


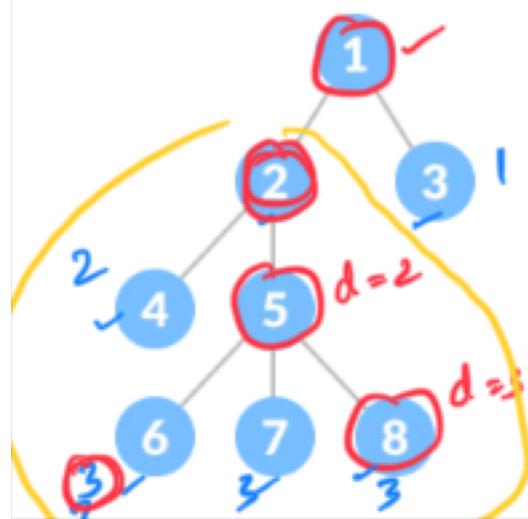
$11 7 1 10 14$

map<int, vector<nodes>>

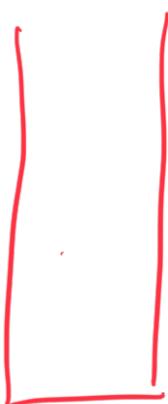
map<int, Node*>

Last node of a level
level order Trans





Postorder Traversal



stark

Output: 7 4 5 2 6 3 | Postorder: left, right, root
 |
 |
 |
 |
 |
 |

Output: 7 4 5 2 6 3

$curr = \text{stk.top()}.sign()$

carry s

Vertical Order Traversal

Do it using level order traversal to preserve the order in vertical order traversal.

```
verticalOrder(Node* root, int dist){  
    if(root == NULL) return;  
  
    // Preorder traversal  
    mp[dist].push_back(root->data);  
    verticalOrder(root->left, dist - 1);  
    verticalOrder(root->right, dist + 1);  
}  
print(map);
```

Level Order Traversal : By storing level of every node

```
level_order_pair(Node* root){  
    queue<pair<Node*, int>> q;  
    cur_level = 0;  
    q.push(<root, 0>);  
  
    while(!q.empty()) {  
        top = q.front();  
        q.pop();  
        if(top.level != curr_level){  
            cout << "\n";  
            curr_level++;  
        }  
        cout << top->val << endl;  
        if(top.first->left) q.push(<top.node->left, top.level+1>)  
        if(top.first->right) q.push(<top.node->right, top.level+1>)  
    }  
}
```

Level Order Traversal : By using a delimiter

```
levelOrder(Node* root){  
    queue<Node*> q;  
    q.push(root);  
    q.root(NULL);  
  
    while(q.size() > 1){  
        Node* f = q.front();  
        q.pop();  
  
        if(f != NULL){  
            cout << t->val;  
            if(t->left) q.push(t->left);  
            if(t->right) q.push(t->right);  
        }  
        else{  
            cout << "\n";  
            q.push(NULL);  
        }  
    }  
}
```