Maths - I

→ Modular Arithmetic

%

$a \% m \Rightarrow$ Remainder when a/m

$12 \% 5 = 2$

$7 \% 9 = 7$

$\boxed{a \% m}$

Range:

Min of $a \% m$

0 is the least Remainder

Max of $a \% m$

$(a/m) \Rightarrow \boxed{(m-1)}$

$\boxed{\times a}$

$17 \% 8 = 1$

$15 \% 8 = 7$

$\Rightarrow$

$\Rightarrow (m-1)$

Range of $a \% m = [0, m-1]$

$\% m \Rightarrow [0, m-1]$

$a \% m =$

⇒ Map Infinite Numbers to finite set of buckets

$x \% 5$

x:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$x \% 5$:	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1

Sequence is repeating after every 5 numbers

$30 \% 7 = 2$

1) Repeated subtraction

$30 \to 23 \to 16 \to 9 \to 2$

$\%7 \Rightarrow [0, 6]$

2) Subtract largest multiple of 7 less than 30

$30 - 28 = \boxed{2}$

3) Remainder: smallest number to be subtracted from 30 to get a multiple of 7

$\Rightarrow 30 - 2 = 28$

2 → Remainder

$-30 \% 7$

Largest Multiple of 7 $< (-30) =$

$-42 \quad -35 \quad -28$

$\boxed{n \% m}$

$-30 - (-35) =$

$-30 + 35 = \boxed{5}$

Negative Remainder =

$m = 7$

$[0, 6]$

Puzzle: 7 balls out of which one is gold

→ the $\boxed{1000^{th}}$ ball is gold

→ Forward

→ Reverse

	A	B	C	D	E	F	G
	B1	B2	B3	B4	B5	B6	B7
(1-12) →	1	2	3	4	5	6	7
	14	2	3	4	5	6	7
	13	12	11	10	9	8	

14 15 16 17 18 19

25 24 23 22 21 20

1000 % 12 → 4

13

Pattern

B1 B2 B3 B4 B5 B6 B7 B6 B5 B4 B3 B2 B1 B2 B3 B4

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

13 24

Sequence of L = 12 will Repeat.

B1 B2 B3

1000 % 12 is 4

a % 12 → [0, 11]

12

B1 1, 13, 25, 37, 49, 61 . . .

B2 2, 14, 26, 38 . . .

12

Remainder with 12

% 12

100 % 12 = 4

100th item

Puzzles

InterviewBit

Real-life : clock

24 hr

2 AM

2 PM

1st part = [0 - 12]

2nd part [13 - 24]

>12

if (> 12)
PM
else
AM

14 % 12 = 2

AM, PM

(14/12) = 0 → AM

(14/12) = 1 → PM

5

2, 14, 26, 38 . . .

p = 2

p = 55

$2^2, 14^2, 26^2, 38^2$

$2^{55}, 14^{55}, 26^{55}$ . . .

$2^2 = 14^2, 26^2, 38^2$

$2^{100}, 14^{104}, 26^{100}$

8 + 11 = 19

35 + 8 = 43

= 12 × P_1 + 2

(196) % 12 = (4)

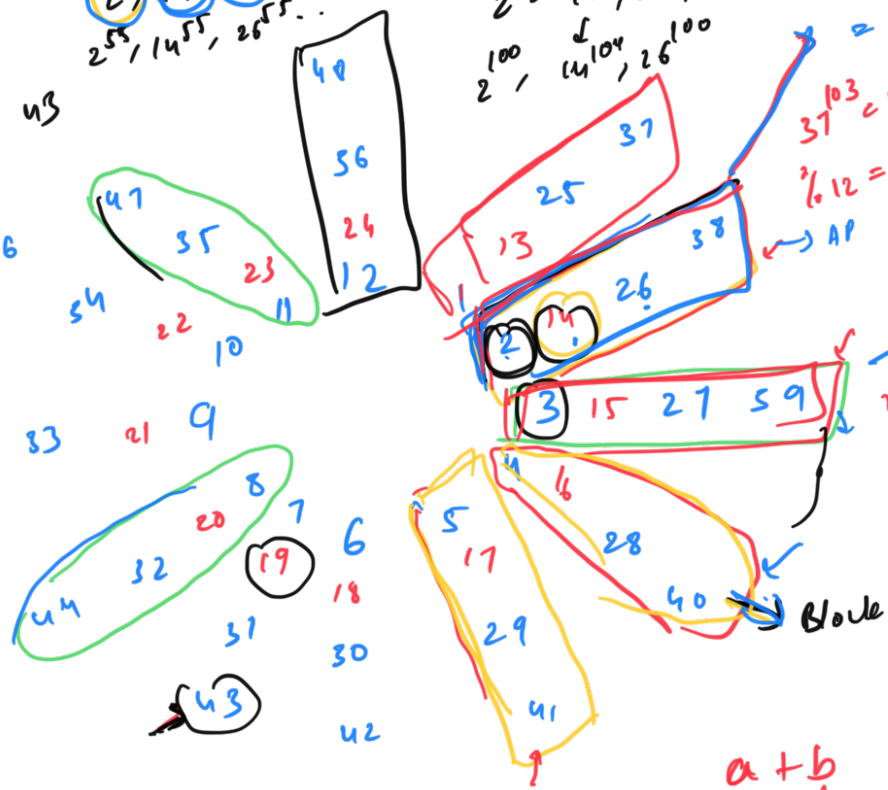
$37^{103} = 1^{103}$

% 12 = 2

→ AP

12 × P_2 + 3

rem = 3



3, 4

12

14^2 = 196

196 % 12 = 4

a + b

2 + 3 = 5

26 + 15 = 41

a = 12P_1 + 2

b = 12P_2 + 3

a + b = 12P_1 + 12P_2 + 5

a + b = 12(P_1 + P_2) + 5

% 12 = 5

(a.b) % m = (

(a, b)

(a/m)

$\boxed{(a+b) \% m}$ = (a % m + b % m) % m

r_1 = a % m

m . P + r_1 — (1)

$a = m \cdot P_1 + r_1$ ② $r_2 = b \% m$

$b = m \cdot P_2 + r_2$

$\Rightarrow (a+b) = m(P_1+P_2) + r_1 + r_2$

$(a+b)\%m = [m(P_1+P_2) + r_1 + r_2] \% m$

$(a+b)\%m = [m(P_1+P_2)]\%m + (r_1+r_2)\%m$

0

$(a+b)\%m = (r_1+r_2)\%m$

$$\boxed{(a+b)\%m = (a\%m + b\%m)\%m}$$

+ / *

$a = m \cdot P_1 + r_1$

$b = m \cdot P_2 + r_2$

$(a \cdot b)$

$\boxed{a/b}$

$$\boxed{(a \cdot b)\%m = (a\%m \cdot b\%m)\%m}$$

$(a\%m < b\%m)$

$(a-b)\%m = (a\%m - b\%m)\%m$

WA

C / C++ $(-7)\%9 \Rightarrow$ Return negative number

WA

$(a-b)\%m = (a\%m - b\%m + m)\%m$

$[0, m-1]$ $[0, m-1]$

$a\%m - b\%m$

Min if $a\%m = 0$, $b\%m = (m-1)$

$-(m-1)$

$\underline{\text{Max}}$ if $a\%m = m-1,\ b\%m = 0$

$= (m-1)$

$[-(m-1), (m-1)]$

$(+m)$ Good Practice

$25\%12 = 1$

$37\%12 = 1$

Extend it to Powers → Expansion

$(a+b)^p = {}^pC_0\, a^p + {}^pC_1\, a^{p-1} b + {}^pC_2\, a^{p-2} b^2 + \dots$

${}^pC_{p-1}\, a\, b^{p-1} + {}^pC_p\, b^p$

$a = 12q$
$b = 2$

$[2\ \ 14\ \ 26\ \ 38\ \ 40 \dots]$ 2 was Remainder (12)

$(a = 12 \times q + 2)$

a^p — p can be any number.

$(12q+2)^p = [\, {}^pC_0 (12q)^p + {}^pC_1 (12q)^{p-1} 2 + {}^pC_2 (12q)^{p-2} 2^2 \dots$

${}^pC_{p-1} (12q)\, 2^{p-1} + {}^pC_p\, 2^p \,]$

$p = [1, 10^9]$

Multiple of 12

$((12q+2)^p = (12 \times () + 2^p)\%12$

0 Remainder $= (2^p)\%12$

Same Remainder → $(x^p)\%12$

Common for all number

in same seam

$\boxed{\begin{matrix} 37\%12 = 1 \\ 37^n\%12 \end{matrix}}$

$(37^{103} - 1)\%12$

$(37^{103}\%12 - 1\%12 + 12)\%M$

$(1 - 1 + 12)\%12$

$37^{103}\%12 = (0+12)\%12 = \boxed{0}$

$\boxed{2^{103}}\%M$

Advanced lab

$$(a+b)\%m = (a\%m + b\%m)\%m$$
$$(a \cdot b)\%m = (a\%m \cdot b\%m)\%m$$
$$(a-b)\%m = (a\%m - b\%m + m)\%m$$

ans% 1000007 $\boxed{10^9+7}$

(Sum of array elements)% M

$A = [10^5\ 10^5\ 10^5 \ldots]$ $N = 10^6$

```
sum=0;
for(i=0; i<n; i++)
    sum= (sum+ a[i])%M
return sum%M;
```

$sum = 10^6 \times 10^5 = \boxed{10^{11}}$

INT

overflow

%M

Less collision

$\boxed{10^9+7}$

$[0, 2 \times 10^9]$

$\boxed{10^9+7}$

Congruence Modulo

... to each other

x and y are congruent w.r.t n

$(14-6) \% 4$

$x \% n = y \% n$

$14 \equiv 6 \bmod 4$

$(15 \equiv 7) \bmod 8$

$(15-7)$

$\Rightarrow \boxed{x \equiv y \pmod{n}}$

$(x-y)$ is divisible by n?

p_1, r_1 are random num

$r_1 \neq r_2$

$x \% n = y \% n$

$$\left\{\begin{array}{l} x = p_1 . n + r_1 \\ y = p_2 . n + r_1 \end{array}\right\}$$

$(x-y) = (p_1 - p_2) n + 0$

$(x-y)$

$(x-y) = \boxed{(p_1 - p_2) n}$.

$\rightarrow$ If difference of 2 number is divisible by n,

$(a-b) \% n = 0$

$a \equiv 6 \bmod n$

$a \equiv b \bmod n$ $[a \% n = b \% n]$

$c \equiv d \bmod n$ $[c \% n = d \% n]$

$(a+c) \equiv (b+d) \bmod n$

$a = p_1 . n + r_1$, $b = p_2 . n + r_1$

$c = q_1 n + r_2$, $d = q_2 . n + r_2$

$a+c = (p_1 + q_1) n + (r_1 + r_2)$

$(b+d) \Rightarrow (p_2+q_2)n + (r_1+r_2)$

$\boxed{(a+c) \equiv (b+d) \bmod n}$?

$(b+d) - (a+c) \Rightarrow n(p_2+q_2-p_1-q_1) + (\cancel{r_1}+r_2) - (\cancel{r_1}+r_2)$

$\Rightarrow n(p_2+q_2-p_1-q_1)$

Question: (Amazon)

check if a pair whose sum is divisible by 'k'

A = 3 7 5 13 4 6 9

K = 10
(3, 7) (6, 4) (13, 7)

K = 8
(3, 5) (7, 9) (13, 3)

Brute Force:

consider all the pairs

2 Nested loops

T.C: $O(n^2) \Rightarrow$

S.C: $O(1)$

for(i = 0 to n-1)
for(j = i+1 to n-1)
if (a[i] + a[j]) % k == 0
return True

$\boxed{n = 10^6}$

$(10+20) \% 10 = 0$

Efficient Approach:

A = 3 7 5 13 4 6 9 10 20

$\boxed{k = 10}$

$p_1 = a[i] / k$

$r_1 = [0, k-1]$

$a[i] = p_1 \cdot K + r_1$

$a[j] = p_2 \cdot K + r_2$

$$\boxed{[a[i] + a[j]] \% K = 0}$$

$(a[i] + a[j]) \% k = (K(p_1 + p_2) + r_1 + r_2) \% k$

$= [\cancel{K(p_1+p_2)}] \% K + (r_1 + r_2) \% k]$

$(r_1 + r_2) = 2k, 3k, 4k$

$(a[i] + a[j]) \% k = \boxed{(r_1 + r_2) \% k} = 0$

$r_1 + r_2$

$[\boxed{r_1 = r_2 = 0}$

$\boxed{k} \Rightarrow [0, K, 2k, 3k \ldots]$

$\Rightarrow$ Find 2 Numbers with their sum of remainder $= [0, K]$

$\boxed{K = 8}$

A =	3	7	5	13	4	6	9
Rem =	3	7	5	5	4	6	1

$r_1 = [0, k-1]$ ✓

$r_2 = [0, k-1]$ ✓

$r_1 + r_2 = [0, 2k-2]$

$\boxed{0, K}$

$0, K, 2K, 3K, 4K$

Find a pair of numbers whose sum = $\{0, K\}$

(Hashing) [Hash set] $\Rightarrow$ $\boxed{\text{Hashing - 1}}$

$O(1)$

(Subarray)

$\boxed{\text{Prefix}}$

$O(1)$

T.C: $O(n)$

S.C: $O(n)$ [Hash set]

$\boxed{\text{Hash map : } \{\text{val, frequency}\}}$ $\Rightarrow$ $\boxed{\text{Hashing - 1}}$

$\boxed{\text{Surprised}}$

Question: Given array of size N

... subarray with sum

Check if there is a subarray
divisible by N

(N = 4) Arraysize
subarray => Contiguous

A = 7 5 3 7

N = 4
[7, 5], [5, 3]

A = 3 7 14

N = 3 [3], [7, 14], [3, 7, 14]

n!, 2^n,

Brute Force:

Consider all subarrays
→ n^2 subarray
→ $\boxed{O(n)}$
T.C: $O(n^3)$

Approach 2:

Intro to Arrays

→ n^2 subarrays.
→ O(1)

$\boxed{pre[R] - pre[L-1]}$

T.C: $O(n^2)$

Approach 3:

(Subarray whose sum is divisible by N)
(subarray sum % N == 0)

(7+5+3) = (15) % 4 = 3

A = 7 4 3 7 [4]

→ pre = 7 11 14 21

pre' = pre % N = 3 3 2 1

N = 4

(7+4) % 4 =

[4]

Return True, if you find a 0.

If you find duplicate pre', then return True;

T.C: O(n) $\boxed{O(n)}$

S.C: O(n) (Hashset) (surprised)

Efficient Approach: O(1)

A = 7 4 3 7

N = 4

→ pre = 7 11 14 21 → size = (N)

pre' = 3 3 2 1 $\boxed{n}$

pre[i] % N => [0, N-1]

N unique value

$\boxed{\text{N Slots}}$ N value

size N

N value

10 Pigeons

$\boxed{\text{Case 1}}$: If all the value in the range [0, N-1] are coming exactly only [0, N-1]

[0, n-1]

N = 4

TRUE

[0, 1, 2, 3]

[0, N-1]

[0, 3]

4 slots ↓

{0, 1, 2, 3}

Case 2 : 0 is not there

(N-1) number

N slots

| 1 | 3 | 2 | 2 |

True

sum % n == 0 ?

| 1 | 2 | 3 | 2 |

{1, 2, 3}

O(1) T.C

O(1) S.C

func(){
return True;
}

TRUE

N = 4

Ex

A =	1	2	3	2
Pre =	1	3	6	8
Pre % N =	1	3	2	0

[1, 2, 3, 2]

True

N = 5

A = 3 3 3 3 3

3 × 5

n n n n n

sum = (n × n) % n

N = 1

A = [1, 0, 0, 0] N = 4

Pre = 1 1 1 1