Maths -2

Question:

N doors numbered 1 to N,

N=10

No. of open doors.

(N)

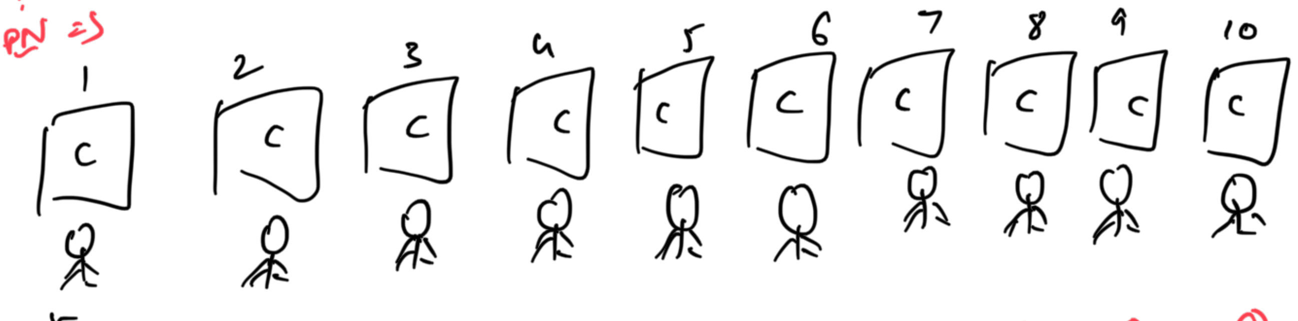
P1 => 1, 2, 3, 4 10

P2 => 2, 4, 6, 8, 10

P3 => 3 6 9

:

PN =>



	1	2	3	4	5	6	7	8	9	10
P1	O	O	O	O	O	O	O	O	O	O
P2	O	C	O	C	O	C	O	C	O	C
P3	O	C	C	C	O	O	O	C	C	C
P4	O	C	C	O	O	O	O	O	C	C
P5	$\boxed{O}$	C	C	$\boxed{O}$	C	C	C	C	$\boxed{O}$	C

P6

$\boxed{Ans = 3}$

< 10

$\boxed{1, 4, 9}$

state

Brute Force:

| c | c | c | c | c | c | c | c | c | c |

for(i=0; i<n; i++) {
for (   )
}

T.C: $O(n^2)$

=> Efficient Approach

Door (N) how many people will toggle this door. (38)

C → O

18 = 1, 2, 3, 6, 9, 18

All factors of number

Even => C
odd => O

C → O → C

{ 1 F : O
2 F : C
3 F : O
4 F : C

$N \times ($

1 => N
2 => $\frac{N}{2}$
3 => $\frac{N}{3}$
4 => $\frac{N}{4}$

$\boxed{N\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\ldots\right)}$

HP

18 : 1, 2, 3, 6, 9, 18

Factors will come in pairs

Even Factors

(1, 18) (2, 9) (3, 6)

O C

16 = 4^2 ⇒ 1, 2, 4, 8, 16 [odd Factor]

even Factor

(1, 16) (2, 8) (4,

$1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$

Odd No of Factor

N = 11

[1, 4, 9] → Open

Solution: No of Perfect squares ≤ N

N = 100 ⇒ 10

N = 110 ⇒ [floor($\sqrt{N}$)]

$\sqrt{110}$ = 10.5

$1^2, 2^2, 3^2 \ldots 8^2$ 9^2 $10^2 \le 100$ ⇒ 100

Question: Rearrange the array of [distinct] elements such that

a[i] → a[a[i]]

[0, n−1]

N distinct

$0 \le a[i] \le n-1$

O(1) space

0 … N−1

[N = 4]

→ Ex

	0	1	2	3
Arr =	3	2	0	1
Ans =	1	0	3	2

a[0] → a[a[0]]
→ a[3]

a[2] → a[a[2]]
→ a[0]
→ 3

$a[i] \to a[a[i]]$
$\to a[2]$
$\to 0$

$a[3] \to a[a[3]]$
$\to a[1]$
$\to 2$

for every 'i',
ans[i] = a[a[i]]

T.C : O(n)
S.C : O(n)

a[i] → a[a[i]]

a[i] → a[a[i]]

Ex

	0	1	2	3	4
Arr =	4	0	2	1	3
Ans =	3	4	2	0	1

temp = 3, 2

1 O(n)

	0	1	2	3
A =	1	0	0	1

Ans =

O(n) extra space

$\boxed{\text{old val, new Val}}$

Approach 2:

4 byte

[0, 1]

9 AM / 9 PM ?

9 → (21)

%12 => [0 - 11]

12 numb.

21 = 1 × 12 + 9

0 1 2 , 1

(9) /

(21) → AM / PM
→ Time of the 12hr slot

(21) => 1×12 + (9)

21

21/12

21 → 21/12
→ 21 % 12

n/12 = 0 , 1
AM PM

(old xn + new val)

old val
New value

Org:	3	2	0	1
step:	1	0	3	2
	(3x4)+1	2x4+0		

N = 4

old = 3
new = 1

(N)

[0 ... n-1]

(N) % x

13 /4 → 3 → (/N)

%4 → 1 %N New val

8 /4 → 2
% → 4

	0	1	2	3
Arr =	3	2	0	1
step1 ⇒ Arr =	12	8	0	4
step2 ⇒	13	8	3	6
step3 ⇒ %N →	1	0	3	2

N = 4

a[0] → a[3]

(N)

a[0] → a[a[0]]
→ a[12/4]
a[12]

arr[0] → arr[arr[0]]
a[0] → ar[3]
→ a[

new val
a[0] → a[a[0]]

a[0] → a[12/4]

a[0] → a[3] /4

a [0] ... [...

Algo

1) Multiply by N

2) arr[i] += arr[arr[i]/N] / N %N

3) arr[i] = arr[i] % N

Arr = 1 0 3 2

	0	1	2	3
A =	3	2	0	1
A =	12	8	0	4
A =				

N = 4

a[3]

a[a[0]]

a[12/4]/4

3 - 5

Question. Given a prime number $\{p <= 10^{1000}\}$, $p > 3$

$(p^2 - 1)$

p = 2

p = "103 . . . . . . "

check if $(p^2-1) \% 24 = 0$?

p = 5, $(25-1) \% 24$ → True

p = 7, $(48) \% 24$ → 0

11 $121 - 1 = (120) \% 24$

24 nos 2120

24 = 2 × 2 × 2 × 3

2 2 2 3 (p^2-1)

$(a^2-b^2) = (a-b)(a+b)$

$(p^2-1) = (p-1)(p+1)$

x $x+1$ $x+2$

%3

$(p-1)$ p $(p+1)$

2

consecutive nos

2

odd no

4 = 2's x%3

p is not divisible by 3

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

$\boxed{\text{TRUE}}$

$\boxed{(p^2-1) \% 24 == 0}$

14 15 16

%3?

45 46 47

x $x+1$ $x+2$

(1) 2 0

% 4

/3

11?

$(p^2-1)\%24$

(11^2-1)

$(121-1)$

$(120)\%24 = 0$

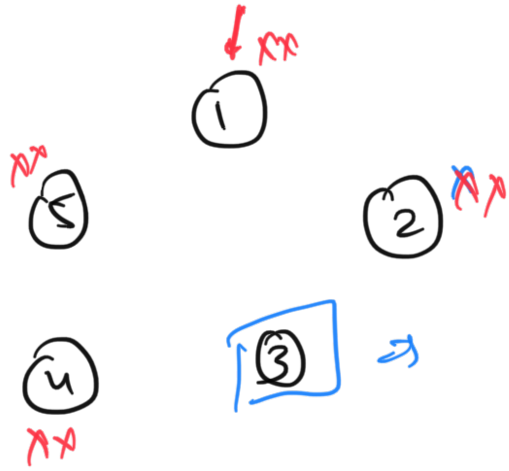
$24 \times 5 = 120$

Question:

N people → standing in a circle

$\boxed{\text{Hitlers}}$

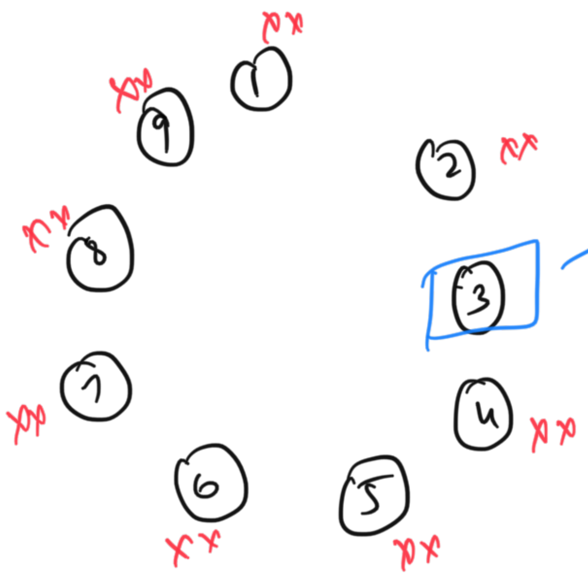
$\boxed{N = 5}$



$\boxed{\text{Ans} = 3}$

$N = 10^9$

$N = 9$



$\boxed{\text{Ans} = 5}$

$N = 1$

101
110
101

N	Winner	
N = 2	1	2^0
N = 3	3	
N = 4	1	2^2
N = 5	3	011
N = 6	5	101
7	7	
8	1	2^3
9	3	
10	5	
11	7	
12	9	
13	11	
14	13	
15	15	
16	1	2^4

011

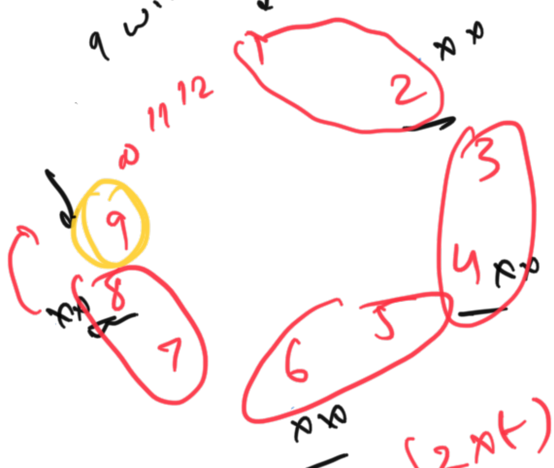
100 → 000 → 001

N = 8

$N \to \frac{N}{2} \to \frac{N}{4} \to \frac{N}{8} \dots$ 1

N = 12

9 will win



Winner = 9

t = 4

Ans = 9

(2t+1)

$(2 \times t) + 1$

$\left(\frac{N}{2}\right)$ are killed

$\left(\frac{N}{2}\right)$ people are alive

$\left(\frac{N}{4}\right)$ people

$\boxed{N = 2^k + t}$

$\boxed{12 = 8 + 4}$

2^k

kill 't' people

t = 4

$\boxed{12 = 2^3 + 4}$

$21 = 2^4 + 5$

$\boxed{t}$

$\boxed{2^3}$

$28 = 16 + 12$

$2 \times 12 + 1 = \boxed{25}$

$28 = 2^4 + 12$

t

$(2 \times 12 + 1)$

25 will be winner

2^4

2^k

$(2t+1)$

$32 = 2^5 + 0$

$t = 0$

$2 \times t + 1 = 0 + 1 = \boxed{1}$

Algo

(1)

(N)

$t = N - 2^k$

$16 = 2^4$

$\log_2^{16} = 4$

$(\log_2^{18}) = 4....$

$18 = 2^4 + \_\_$

$\text{Floor}(\log_2^n)$

t

$18 = 10010$

10000

(F)

$18 = 16 + 2$

New $= 00010$

t

$2t+1$

t

$\boxed{2 \times t}$

$2t+1$

$+1$

Algo

(1) Remove MSB

2) Left Shift
3) 1 in LSB

Question: N^{th} Magic Number

→ Magic No can be either represented as a power of 5 or sum of unique powers of 5.

$5,\ 5^2,\ 5+5^2,\ 5^3,\ 5^3+5,\ 5^3+5^2,\ 5^3+5^2+5,$
5, 25, 30, 125, 130, 150, 155, 625, 630, 650, 655, 750

5^2

Ascending order

N=10

Binary Rep

N=1 → 5
N=2 → 5^2
N=3 → $5^2 + 5$
N=4 → 5^3 → $5^3 + 0 \cdot 5^2 + 0 \cdot 5$
N=5 → $5^3 + 5$ → $5^3 + 0 \cdot 5^2 + 1 \cdot 5$
N=6 → $5^3 + 5^2$ → $5^3 + 5^2 + 0 \cdot 5$
N=7 → $5^3 + 5^2 + 5$
N=8 → 5^4
N=9 → $5^4 + 5^1$
N=10 → $5^4 + 5^2$
N=11 → $5^4 + 5^3$

$1010 \rightarrow 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 0$

$\boxed{K=2}$

Magic No : power of 2 or sum of unique powers of 2

N

001	1 =	2^1
010	2 =	2^2
011	3 =	$2^2 + 2^1$
100	4 =	2^3
101	5 =	$2^3 + 2^1$
1 1 0	6 =	$2^3 + 2^2$
1 1 1	7 =	$2^3 + 2^2 + 2^1$
1000	8 =	2^4

$\Rightarrow 2^3 \times 0 + 2^2 \times 0 + 2^1 \times 1$

2

$2^0, 2^1$

$\Leftrightarrow 1 \times 2^3 + 0 \times 2^2 + 1 \times 2$

$8 = 2^3$

1 0 1 1 0

$= 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 1 \times 2^5$

$5^5 \times 1 + 5^4 \times 0 + 5^3 \times 1 + 5^2 \times + (5^1) \times 0$

5^1

Problem 2

T.C :

$(N) \to O(\log_2^N)$

$N = \underbrace{10110}_{6 \text{ bits}}$

%2

14 $\Rightarrow$

(n % 2)

1 1 1 1 = (4)

N = N/2
= 7

N
ans = 0
bit = 1

$5^1, 5^0, 5^2$?

7 % 2

7 = 7/2 = 3

3/2 = 1

while (N > 0) {
  LSB = N % 2;
  ans = ans + LSB * pow(5, bit)
  bit++;
  N = N/2;
}

(Binary Exponent)
O(log(

T.C: $\log_2(n)$

1 0 1 1 1 0

→ O(No. of bits)

$\log_2 16 = 4$
$= 2^4$

a^b ⇒ O(log b)

Binary Exponent

18 = 1 0 0 1 0
16 = 1 0 0 0 0
17 = 1 0 0 0 1

1 0 0 0 0

5

→ log(n)

⌊log(17)⌋
= 4...
5

32

floor(log(n))

No. of bits = $\boxed{O(\log n)}$

O(32)
O(1)

O(32)(2) O(1)

$\boxed{O(\log n)}$

$\boxed{32 \text{ bits}}$ ⇒ O(32 bits)

O(1)

(2×10^9)

O(...

Ans = 3 4 2 0 1

N = 5

	0	1	2	3	4
A →	4	0	2	1	3
step1 →	20	0	10	5	15
step2 →	20+3	0+4	10+2	5+0	15+1
step3 → /5	3	4	2	0	1

arr[i] = arr[i] + arr[arr[i]/N]/N

a[4]/5

a[1] = a[1] + a[a[1]/N]/N
+ a[0]/N
20/1

a[2] = a[2] + a[a[2]/5]/5
= 10 + a[2]/5
10 + 2

$\boxed{\text{Array} = O(n) \text{ space}}$

$O(n^2)$ space

a[3] = a[3] + a[a[3]/5]/5
= 5 + a[1]/5
= 5 + 0/5
= 5 + 0

a[4] = a[4] + a[a[4]/5]/5

$a[24]/2 \quad 4 \ldots$

$= 15 + a[15/5]/5$

$15 + a[3]/5$

$15 + 5/5$

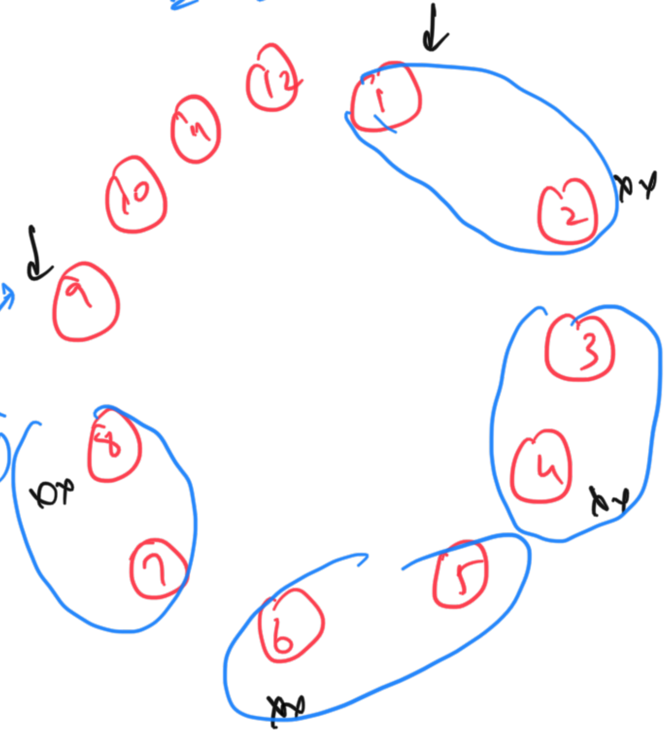
$= 15 + 1$

$12 = 2^3 + 4$

$t = 4$

8

(2k+1)



Even Factors => C

18 =

(1, 18)

(2, 9)

(3, 6)

16 =

(1, 16)

(2, 8)

(4, 4)

1, 2, 4, 8, 16

$1^2, 2^2, 3^2, 4^2$

$N \le 100$

$1^2, 2^2, 3^2, 4^5, 5^6, 6^2, 7^2, 8^2, 9^2, 10^2$

Nth Magic Number

```
ans = 0;
int bit = 1;
while(N > 0){
      ans += N%2 * pow(5, bit);
      N = N/2;
      bit++;
}
```

Rearrange the array

```
for(int i = 0; i<n; i++)
      arr[i] = arr[i] * n;
for(int i = 0; i<n; i++)
      arr[i] += arr[arr[i]/n]/n;
for(int i = 0; i<n; i++)
      arr[i] = arr[i] % n;
```