1. Encontrar a tangente de $\underline{y = x^3}$ em $\underbrace{x = 0}_{x_1}$ e $\underbrace{x = -1}_{x_2}$.

reta

Equação da reta: $y = ax + b$

coeficiente angular → a

" linear → b

$$y = x^3 \quad \begin{cases} y_1 = x_1^3 = 0 \\ y_2 = x_2^3 = -1 \end{cases}$$

$$y' = 3x^2 \quad \begin{cases} a(0) = 3 \cdot 0^2 = 0 \\ a(1) = 3 \cdot 1^2 = 3 \end{cases}$$

Equação 1: $y_1 = a(0)\, x_1 + b$

$0 = 0 \cdot 0 + b$ $\quad \therefore$ Logo: $y_1 = 0$ #

$b = 0$

Equação 2: $y_2 = a(1)\, x + b$

$-1 = 3 \cdot -1 + b$ $\quad \therefore$ Logo: $y_2 = 3x + 2$ #

$b = 2$

2. Encontre a reta tangente à curva $y = f(x)$ no ponto P:

a) $f(x) = \frac{1}{x}$; $P = \left(\frac{1}{2}, 2\right)$

$f(x) = x^{-1} \to f'(x) = -\frac{1}{x^2} \to a\left(\frac{1}{2}\right) = -\frac{1}{\left(\frac{1}{2}\right)^2} = -4$

$y = ax + b$

$2 = -\overset{2}{\cancel{4}} \cdot \frac{1}{\cancel{2}} + b$

$b = 4$

$y = -4x + 4$

b) $f(x) = 2x^2 + x + 2$; $P = (-1, 3)$

$2 \cdot 2x^{2-1} + 1 \cdot x^{1-1}$

$f'(x) = 4x + 1 + 0 \to a(-1) = 4 \cdot -1 + 1 = -3$

$y = ax + b$

$\cancel{3} = \cancel{-3 \cdot -1} + b$

$b = 0$

$y = -3x$

3. Seja $f(x) = x^{\frac{2}{3}}$, encontre $f'(x)$ pela definição (de limite)

racionalização de raiz cúbica:

$$\left(\sqrt[3]{a} - \sqrt[3]{b}\right) \cdot \left(\sqrt[3]{a^2} + \sqrt[3]{a} \cdot \sqrt[3]{b} + \sqrt[3]{b^2}\right) = a - b$$

implica

ou

$$\left(\sqrt[3]{a} + \sqrt[3]{b}\right)\left(\sqrt[3]{a^2} - \sqrt[3]{a} \cdot \sqrt[3]{b} + \sqrt[3]{b^2}\right) = a + b$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

racionalização

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^{\frac{2}{3}} - x^{\frac{2}{3}}}{\Delta x} \cdot \overbrace{\frac{\left[(x+\Delta x)^{\frac{2}{3}}\right]^2 + x^{\frac{2}{3}} \cdot (x+\Delta x)^{\frac{2}{3}} + \left[\left(x^{\frac{2}{3}}\right)\right]^2}{''}}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x\left[(x+\Delta x)^{\frac{4}{3}} + x^{\frac{2}{3}}(x+\Delta x)^{\frac{2}{3}} + x^{\frac{4}{3}}\right]}$$

$$= \lim_{\Delta x \to 0} \frac{(\cancel{x} + \cancel{\Delta x} - \cancel{x})(x + \underbrace{\Delta x}_{\to 0} + x)}{\cancel{\Delta x}\left[(x+\underbrace{\Delta x}_{\to 0})^{\frac{4}{3}} + x^{\frac{2}{3}}(x+\underbrace{\Delta x}_{\to 0})^{\frac{2}{3}} + x^{\frac{4}{3}}\right]}$$

$$= \frac{2x}{x^{\frac{4}{3}} + x^{\frac{2}{3}} \cdot x^{\frac{2}{3}} + x^{\frac{4}{3}}} = \frac{2\cancel{x}}{3\cancel{x}\sqrt[3]{x}} = \frac{2}{3\sqrt[3]{x}}$$

4. Derive:

a) $f(x) = \underbrace{\sqrt{5}}_{\text{constante}} + 2x + 3x^6$

$$f'(x) = 2 + 3.6x^5 = 2(1 + 3.3x^5) = 2(1 + 9x^5)$$

b) $g(x) = \frac{1}{\sqrt{x}} + \sqrt[5]{2x} + \sqrt{7}$

$$= \underbrace{x^{-\frac{1}{2}}}_{a} + \underbrace{\sqrt[5]{2}\, x^{\frac{1}{5}}}_{b} + \underbrace{7^{\frac{1}{2}}}_{\text{constante} = 0}$$

$$a' = -\frac{1}{2} . x^{-\frac{3}{2}} = -\frac{1}{2\sqrt[3]{x^2}}$$

$$b' = \frac{1}{5} . \sqrt[5]{2} . x^{\frac{1}{5} - 1} = \frac{\sqrt[5]{2}}{5\sqrt[5]{x^4}}$$

$$g'(x) = a' + b' = -\frac{1}{2\sqrt[3]{x^2}} + \frac{\sqrt[5]{2}}{5\sqrt[5]{x^4}}$$

c) $b(t) = \underbrace{(t^2 - 2t + 1)}_{u = (t-1)^2}\underbrace{(1 - 3t^{-5})}_{v = \frac{t^5 - 3}{t^5}}$

$$u' = 2(t-1).1$$

$$v' = 0 - 3.-5t^{-6} = \frac{15}{t^6}$$

$$b'(t) = u.v' + u'.v$$

$$= \frac{(t-1)^2 . 15}{t^6} + \frac{2(t-1)(t^5 - 3)}{t^5}$$

$$= \frac{(t-1)}{t^5}\left[\frac{15(t-1)}{t} + 2(t^5 - 3)\right]$$

$$= \frac{(t-1)}{t^6}\left[15(t-1) + 2t(t^5 - 3)\right]$$

$$= \frac{(t-1)(15t - 15 + 2t^6 - 6t)}{t^6}$$

$$= \frac{(t-1)(2t^6 + 9t - 15)}{t^6}$$

d. $f(r)=\frac{\overbrace{1+3r^2}^{u}}{\underbrace{r^2-r}_{v}}$

$u' = 0 + 3.2r^{2-1} = 6r$

$v' = 2r^{2-1} - 1r^{1-1} = 2r - 1$

$f'(r) = \frac{v.u' - v'u}{v^2}$

$= \frac{(r^2-r).6r - (2r-1)(1+3r^2)}{(r^2-r)}$

$= \frac{6r^3 - 6r^2 - (2r + 6r^3 - 1 - 3r^2)}{r^2(r-1)^2}$

$= \frac{\cancel{6r^3} - \overset{3}{\cancel{6}}r^2 - 2r - \cancel{6r^3} + 1 + \cancel{3r^2}}{r^2(r-1)^2}$

$\frac{\overbrace{-3r^2 - 2r + 1}}{r^2(r-1)^2} =$

$(-2)^2 - 4.-3.1 = 16$

$\frac{-(-2) \pm \sqrt{16}}{2.-3} =$

$-\left(\frac{1 \pm 2}{3}\right)$ → -1, $\frac{1}{3}$

$\frac{-3(r+1)\left(r - \frac{1}{3}\right)}{r^2(r-1)^2} =$

$\frac{-\cancel{3}(r+1)\frac{(3r-1)}{\cancel{3}}}{r^2(r-1)^2} =$

$\frac{-(r+1)(3r-1)}{r^2(r-1)^2}$

5. Derive:

a) $f(x) = \sqrt[3]{(x^2+1)^2} = (\underbrace{x^2+1}_{u})^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3} u^{\frac{2}{3}-1} \cdot u'$$

$$= \frac{2}{3} (x^2+1)^{-\frac{1}{3}} \cdot 2x$$

$$= \frac{4x}{3\sqrt[3]{x^2+1}}$$

b) $f(x) = \cos^2(1-x^2) = \underbrace{\Big[\underbrace{\cos(\underbrace{1-x^2}_{g})}_{h}\Big]^2}_{i}$

$g' = -2x$

$h' = -\operatorname{sen} g$

$i' = 2h$

$$f'(x) = i' \cdot h' \cdot g'$$

$$= 2 \cdot \cos(1-x^2) \cdot -\operatorname{sen}(1-x^2) \cdot -2x$$

$$= 2x \cdot \operatorname{sen}[2(1-x^2)]$$

c) $f(x) = \underbrace{\cos(\underbrace{(\underbrace{1-x^2}_{g})^2}_{h})}_{i}$

$g' = -2x$

$h' = 2g$

$i' = -\operatorname{sen} h$

$f'(x) = -\operatorname{sen}[(1-x^2)^2] \cdot 2(1-x^2) \cdot -2x$

$= 4x(1-x^2) \cdot \operatorname{sen}[(1-x^2)^2]$

d) $f(x) = \tan^3 x + \tan x^3 = \underbrace{(\tan x)^3}_{u} + \underbrace{\tan(x^3)}_{v}$

$u' = 3\tan^2 x \cdot \sec^2 x$

$v' = \sec^2 x^3 \cdot 3x$

$f'(x) = 3\tan^2 x \cdot \sec^2 x + \sec^2 x^3 \cdot 3x^2$

$= 3(\tan^2 x \cdot \sec^2 x + \sec^2 x^3 \cdot x^2)$

e) $f(x) = -\dfrac{\operatorname{sen}^2 x}{x} = \underbrace{-(\operatorname{sen} x)^2}_{u} \cdot \underbrace{x^{-1}}_{v}$

$u' = -2\operatorname{sen} x \cdot \cos x = -\operatorname{sen} 2x$

$v' = -\dfrac{1}{x^2}$

$f'(x) = u \cdot v' + u' \cdot v = -\operatorname{sen}^2 x \cdot \dfrac{-1}{x^2} + \dfrac{(-\operatorname{sen} 2x)}{x}$

$$f'(x) = \frac{\sin^2 x - x \cdot \sin 2x}{x^2}$$

f) $f(x) = (2x^6 + 5x^3)^{\frac{3}{5}}$

$$f'(x) = \frac{3}{5}(2x^6 + 5x^3)^{-\frac{2}{5}}\underbrace{(12x^5 + 15x^2)}_{3x^2(4x^3+5)} = \frac{9x^2(4x^3+5)}{5(x^3(2x^3+5))^{\frac{2}{5}}}$$

g) $f(x) = \sin^7(\cos((2x+1)^{10})) = \underbrace{\left(\underbrace{\sin\left(\underbrace{\cos(\underbrace{(\underbrace{2x+1}_{a})^{10}}_{b})}_{c}\right)}_{d}\right)^7}_{e}$

$a' = 2$

$b' = 10a^9$

$c' = -\sin b$

$d' = \cos c$

$e' = 7d^6$

$$f'(x) = a' \cdot b' \cdot c' \cdot d' \cdot e'$$

$$= 2 \cdot 10(2x+1)^9 \cdot -\sin(2x+1)^{10} \cdot \cos[\cos(2x+1)^{10}] \cdot 7\sin^6(\cos(2x+1)^{10})$$

$$= -140(2x+1)^9 \cdot \sin(2x+1)^{10} \cdot \cos[\cos(2x+1)^{10}] \cdot \sin^6(\cos(2x+1)^{10})$$

h) $f(x) = \tan(5x^2 - x)$

$f'(x) = \sec^2(5x^2 - x) . (10x - 1)$

i) $f(x) = \dfrac{[x + \operatorname{sen} x]^{20}}{[\cos x]^{10}} = \underbrace{[x + \operatorname{sen} x]^{20}}_{u} \underbrace{(\cos x)^{-10}}_{v}$

$u' = 20(x + \operatorname{sen} x)^{19}(1 + \cos x)$

$v' = -10 \cos^{-9} x . - \operatorname{sen} x = 10 \tan x . \sec^8 x$

$f'(x) = uv' + u'v$

$f'(x) = (x + \operatorname{sen} x)^{20} . 10 \tan x . \sec^8 x + 20(x + \operatorname{sen} x)^{19}(1 + \cos x) . \sec^{10} x$

$= 10(x + \operatorname{sen} x)^{19} . \sec^8 x \, [\tan x (x + \operatorname{sen} x) + 2(1 + \cos x)\sec^2 x]$

j) $f(x) = \underbrace{(3x - x^{-1})}_{u} \cdot \underbrace{\cos 2x}_{v}$

$u' = 3 - (-1)x^{-2} = 3 + x^{-2}$

$v' = -\operatorname{sen} 2x \cdot 2 = -2\operatorname{sen} 2x$

$f'(x) = u \cdot v' + u'v$

$= (3x - x^{-1}) \cdot -2\operatorname{sen} 2x + (3 + x^{-2}) \cdot \cos 2x$

$= \cos 2x\,(3x - x^{-1}) - 2\operatorname{sen} 2x\,(3x + x^{-2})$

l) $f(x) = \operatorname{sen}\left(\frac{2x}{x^4 - 4x}\right) = \operatorname{sen}\left(\frac{2\cancel{x}}{\cancel{x}(x^3-4)}\right) = \operatorname{sen}[2(x^3-4)^{-1}]$

a, b, c

$a' = 3x^2$

$b' = -2a^{-2}$

$c' = \cos b$

$f'(x) = a' \cdot b' \cdot c' = 3x^2 \cdot -2(x^3-4)^{-2} \cdot \cos[2(x^3-4)^{-1}]$

$= -6x^2(x^3-4)^{-2} \cdot \cos[2(x^3-4)^{-1}]$

k) $f(x) = \underbrace{(x^2+4)^{\frac{5}{3}}}_{u} \underbrace{(x^3+1)^{-\frac{3}{5}}}_{v}$

$u' = \frac{5}{3}(x^2+4)^{\frac{2}{3}} \cdot 2x = \frac{10x}{3}(x^2+4)$

$v' = -\frac{3}{5}(x^3+1)^{-\frac{8}{5}} \cdot 3x = -\frac{9x}{5}(x^3+1)$

$f'(x) = uv' + u'v$

$= (x^2+4)^{\frac{5}{3}} \cdot -\frac{9x}{5}(x^3+1) + \frac{10x}{3}(x^2+4)(x^3+1)^{-\frac{3}{5}}$

$= \frac{10x}{3}(x^2+4)(x^3+1)^{-\frac{3}{5}} - \frac{9x}{5}(x^2+4)^{\frac{5}{3}}(x^3+1)$

6. Encontre a reta tangente para $y = \dfrac{8}{\sqrt{x-2}}$ no ponto $p=(3,2)$

$\hookrightarrow y = ax + b$

$y = 8(x-2)^{-\frac{1}{2}}$

$\rightarrow y' = -\dfrac{8}{2}(x-2)^{-\frac{3}{2}} \cdot 1$

$= -4(x-2)^{-\frac{3}{2}}$

$\rightarrow a(3) = -4(3-2)^{-\frac{3}{2}} = -4$

$y = ax + b$

$2 = -4 \cdot 3 + b$

$b = 14$

$y = -4x + 14$

7. Vide enunciado

$$s(t) = \sqrt{1+4t} = (1+4t)^{\frac{1}{2}}$$

$$s'(t) = \frac{1}{2}(1+4t)^{-\frac{1}{2}} \cdot 4 = 2(1+4t)^{-\frac{1}{2}}$$

$$s''(t) = -\frac{\cancel{2}}{\cancel{2}}(1+4t)^{-\frac{3}{2}} \cdot 4 = \frac{4}{\sqrt{(1+4t)^3}}$$

$$s''(6) = \frac{4}{\sqrt{(1+4 \cdot 6)^3}} = \frac{4}{5^3} = \frac{4}{125}$$

8. Vide enunciado

$$\beta = 10 \log_{10}\left(\frac{I}{10^{-16}}\right)$$

$$= 10 \log_{10} 10^{16} I$$

$$= 10 \left[\log_{10} 10^{16} + \log_{10} I\right]$$

$$= \underbrace{160}_{\text{constante}} + \log_{\underbrace{10}_{a}} \underbrace{I^{10}}_{b}$$

$$\beta'(I) = 0 + \frac{b'}{b \cdot \ln a}$$

$$= \frac{10\cancel{I}^{9}}{I^{10} \ln 10}$$

$$\beta'(10^{-4}) = \frac{10}{10^{-4} \ln 10}$$

$$= 10^5 \log_{10} e$$

9. Vide enunciado

$$I_0 = 1$$

$$R(I) = \frac{\ln I - \ln I_0}{\ln 10}$$

$$= \frac{\ln I - 0}{\ln 10} = \log_{10} I$$

Propriedade dos logarítimos

Logarítimos: $\log_a b = \dfrac{\log_c b}{\log_c a}$

a) Vide enunciado

$$8{,}3 = \log_{10} I$$

$$I = 10^{8{,}3}$$

$$\log_a b = c$$
$$\Downarrow$$
$$a^c = b$$

b) Vide enunciado

$$I = 10^{6{,}3}$$

c) $R(I) = \log_{10} I$

$$R'(I) = \frac{1}{I} \cdot \log_{10} e \cdot 1$$

$$= \frac{\log_{10} e}{I}$$