SIAM FM21 Programming Challenge

Version 2.0[*]
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The programming challenge shall consist of optimizing a portfolio of stocks by constructing a trading strategy which uses historical stock prices. You shall use a market simulator provided to you to generate the stock price history. The specification of the problem together with the modeling and documentation requirements are provided below.

Definitions and terminology: Consider the problem of optimizing a portfolio in $d > 0$ exchange traded stocks over at each time period $t = 0, 1, \dots, T-1$. At each period, the proportional allocation of capital to each stock is represented by the weights

$$w_t \in \Delta^d := \{x \in \mathbb{R}^d : x^i \geq 0 \text{ and } \sum_i x^i = 1\} ,$$

where each w_t^i represents the proportion of the total capital allocated to stock i at time period t. Note for avoidance of doubt, that the weights are defined in terms of the position size (i.e. number of assets held), u_t^i, as $w_t^i = u_t^i S_t^i / P_t^i$, where the portfolio value $P_t^i = \sum_i^d u_t^i S_t^i$. Given a sequence of chosen weights $w_{0:T-1} = (w_t)_{t=0}^{T-1}$ historical stock prices $s_{0:T} = (s_t)_{t=0}^{T}$, where for each t we use the notation $s_t = (s_t^i)_{i=1}^d$ to indicate the vector of prices, we calculate the total return of the portfolio over the T periods as

$$R_T(w_{0:T-1}) = \prod_{t=0}^{T-1} \left(1 + \sum_{i=1}^{d} \{ w_t^i \, r_t^i - \eta \, |\Delta u_{t-1}^i| \} \right)^+ ,$$

where we use the notation $(\,\cdot\,)^+ = \max\{\cdot, 0\}$, $r_t^i = \frac{s_{t+1}^i - s_t^i}{s_t^i}$ and $\Delta u_t^i = u_{t+1}^i - u_t^i = w_{t+1}^i P_{t+1}^i / S_{t+1}^i - w_t^i P_t^i / S_t^i$ with the convention that $\Delta u_{-1}^i = 0$. We note that the above definition of the return encompasses two terms for each period t:

[*]Note that this revision updates the total returns and market simulator to use position changes in the trading fee term and impact term respectively. In this way, there are only trading fees and market impact when the position changes. In the previously version, the fee and market impact term depended on the changes of weights which allowed for fees and market impact even when there is no rebalancing.

first is the standard definition of the return on the portfolio in period t plus a transaction cost parameter that the portfolio manager must pay each time they rebalance (i.e. change) their portfolio positions, where $\eta > 0$ controls the scale of this cost.

Problem statement: The objective for the portfolio manager will be to construct a trading strategy which for any fixed $\lambda > 0$ (Risk aversion parameter) and $T > 0$, maximizes the mean-variance objective function

$$\mathcal{L}_T^\lambda(u_{0:T}) = \mathbb{E}[R_T(u_{0:T-1})] - \lambda\, \mathbb{V}[R_T(u_{0:T-1})] \ .$$

At each time t, the portfolio manager may only only use trading strategies which use historical stock price information in order to decide on positions u_t. In other words, the portfolio manager can not have knowledge of future stock prices nor can they use some other data, not provided. For avoidance of doubt, the stock symbols nor the historical dates shall not be provided.

Market Simulator: You may assume that stock prices randomly evolve over time and are dependent on how the portfolio is rebalanced. More precisely, letting $S_t = (\log s_t^i)_{i=1}^d$, the increments of S_t satisfy the relation

$$S_{t+1} - S_t = \mu \ + \kappa\,(\Delta u_{t-1}) + M\,\xi_t \ ,$$

where

- $\mu \in \mathbb{R}^d$ is a drift vector.
- $\kappa : \mathbb{R}^d \to \mathbb{R}^d$ is a market impact function which depends on the change in positions, Δu_t, which we define according to

  $$\kappa(x_i) = \Big(c_i\, \mathrm{sign}(x_i)|x_i|^{\frac{1}{2}}\Big)_{i=1}^d \ ,$$

  for constants $c_i > 0$, and where $\mathrm{sign} : \mathbb{R} \to \{-1, 0, 1\}$ is the sign of a number.
- $M \in \mathbb{R}^{d\times d}$ is a low-rank matrix
- $\xi_t = (\xi_t^i)_{i=1}^d$ is a vector of independent and identically distributed random variables with density p satisfying $\mathbb{E}[\xi_t^i] = 0$ and $\mathbb{V}[\xi_t^i] = 1$.

You may not assume that the portfolio manager has any knowledge of (μ, c, M, p) in advance. Hence, strategies must be devised without the use of these variables. For this reason, you shall generate the stock prices from a market simulator which shall be provided to you.

Modeling and Documentation Requirements: Each team shall find the optimal trading strategy for the generated training data, without assuming the data generation process. Such a trading strategy should therefore be robust if applied to test data generated from a modified data generation process in which (μ, c, M, p) are not the same.

To characterize the optimal trading strategy, each team shall plot the "efficient frontier", the plot of the expected portfolio return against the standard deviation of the portfolio return by varying $\lambda > 0$ at small increments and repeating the optimization. The approach implemented should be carefully annotated and a short report shall include the efficient frontier diagram together with a mathematical description of the solution approach.

Additionally each team shall submit their Matlab implementation of their trading strategy to the programming committee. See the "Programming Challenge Instructions" on the website for further instructions.