Logic and Truth Tables

August 17, 2017

Definition of SL

1. All statement letters are SL statements. $(A, B, ..., Z, A_1, B_1, ..., Z_1, A_2, B_2, ..., A_n, ...)$.
2. If p, q are SL statements, then the following are also SL statements:
   - $\neg p$
   - $(p \wedge q)$
   - $(p \vee q)$
   - $(p \rightarrow q)$
   - $(p \leftrightarrow q)$
3. Nothing except what follows from 1 and 2 is a statement in SL, except that we may use square brackets in place of parenthesis for clarity in grouping.

The Logical Operators and Their Truth Tables

"p"	"q"	Negation "not p"	Conjunction "p and q"	Disjunction "p or q"	Conditional "p implies q"	Biconditional "p if and only if q"
p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Replacement Rules

We use the symbol $\equiv$ to denote logical equivalence.

- Double Negation: $\neg\neg p \equiv p$
- Material Implication: $p \rightarrow q \equiv \neg p \vee q$
- Material Equivalence: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

- DeMorgan's Laws:
  1. $\neg(p \land q) \equiv \neg p \lor \neg q$
  2. $\neg(p \lor q) \equiv \neg p \land \neg q$

Homework: Construct a truth table for the following statements. (Due Thursday, 17 August)

1. $p \to \neg q$
2. $\neg(p \to q)$
3. $p \lor \neg q$
4. $\neg(p \lor q)$
5. $\neg p \leftrightarrow q$
6. $p \leftrightarrow \neg q$
7. $\neg p \land q$
8. $\neg(p \land \neg q)$